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Discount Rates in General Insurance Pricing

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Abstract

There is no generally accepted practice for discounting cash flows (or allowing for future investment earnings) when setting premium rates in Australian general insurance. While rates are generally set at each insurer's discretion, some statutory classes require rate filings and are reviewed by actuaries.

This paper covers the following:

- A review of various premium setting models and insights from financial economics.
- Current practice in the industry (including input gathered from interviews with practitioners).
- A review of academic papers on this topic.
- Considerations for various classes of business.
- Projections of the risk free yield curve, to allow for the delay from rate setting to the average date of premium collection.
- Discussion of possible practical approaches and criteria to assess each.
- Recommendations on good practice, based on various criteria which may apply in different situations.

While we have attempted to do justice to the existing literature on the subject, this paper also attempts to emphasise approaches and results useful for practitioners.

Keywords: Regulated classes, Bond yields, Risk free rates, Investment returns, Profit margins, Premium setting, Return on Equity, Total Shareholder Return

Contents

1. Introduction

What do we mean by discount rates?

Premium setting for general insurance contracts involves the consideration of cash flows which occur at different times and face different risks. Typically the policy premium is collected around the start of the policy term and, if the class of business is long tailed, the claim payments may occur many years after the policy term has finished. During the time lag between premium payment and claim settlement the insurer earns investment income on the balance of the premium.

The timing differences in the cash flows involved in the insurance process means that the discounted cash flow approaches developed for investment appraisal in the field of financial economics (also termed capital budgeting) have become standard tools for formal premium setting exercises. Central to these cash flow approaches are discount rates which are used to calculate the present value of a variety of cash flows. These discount rates are the topic of this paper.

A variety of rates are used to calculate present values in general insurance pricing. Examples include:

- the projected investment earning rate on technical funds or capital
- the discount rate assumed to project outstanding claims through the life of a cohort of business. This assumption is made when the business is priced
- the target return on equity to shareholders, as this effectively sets the net present value of shareholder transfers to zero.

In this paper we refer to all such instances as discount rates.

Motivation for this paper

Discount rates are sometimes viewed as one of the less interesting inputs when setting premium rates in general insurance. This may be due to the challenges in setting other assumptions (e.g. claim costs). It may also be because discount rates are set within a firm using long established practices or relatively uncontroversial techniques. Often it is because what competitors are doing in relation to price matters far more than any technical pricing exercise.

However, since the global financial crisis (GFC) interest rates in most countries have been lower than historical norms. Many economists are projecting that they will remain low for some years. In theory lower interest rates should lead to higher premium rates, if profit margins or projected returns are to be maintained. Higher premium rates may lead to reduced demand for insurance, which is an issue for competitive insurance markets. Affordability is an important issue for regulated statutory classes such as CTP. Faced with such pressures insurers and regulators have increasingly questioned the investment earning allowances included in premiums. This naturally leads to questions about the appropriateness of the discount rate assumptions used in premium setting models.

A further motivation was to explore the challenges in yield curve projection from the date premium rates are set to the average date of premium collection (i.e. the investment date). Many actuaries currently use the forward rate curve for this purpose. This is one alternative but is not always the best option. This is described further in the "Background" section which follows.

Objective

The objective of this paper is to provide guidance for selecting discount rate assumptions for general insurance pricing. Our intended audience is practitioners involved in the premium setting process, to guide them in the issues they should be considering when pricing. Students of general insurance will also find the paper useful. With this in mind the approaches considered have been chosen to be:

- Theoretically sound
- Intuitive and easily implemented without undue cost
- Flexible enough to deal with the real world complexities of the insurance process.

Background

Much of the academic literature in relation to discount rates in insurance pricing dates back to the 1980s and 1990s. We are not aware of any significant changes to the theory since then. Our recommendations, in general, are an exposition of long established work.

The one exception to this relates to the challenge presented by the need to project risk free discount rates from the date rates are set to the period the rates are charged (and premium collected). This problem is referred to as yield curve projection and in Section 5 of this paper we present a new approach to this problem that is relatively simple and which improves upon the common actuarial practice of using forward rates for this purpose. We compare this approach to existing practices of using forward rates or the current spot rate curve.

In some privately underwritten markets there are restrictions on acceptable pricing assumptions, including discount rates. Compulsory statutory lines (e.g. CTP) are one example in Australia. Classes of business written by insurers in the United States requiring rate filings are another example. In these regulated markets there is often an emphasis on producing prices that are considered fair. In this context fair usually means that insurers receive sufficient compensation for the risks they bear while customers do not pay excessive premiums.

Price regulation of insurance markets and the issues of fair premiums are complex topics. Robb et al (2012) is a recent paper which discusses some of the issues in these areas. Our paper provides some consideration of the role that discount rates play in producing fair premiums. However we have not attempted to present a complete framework for producing fair premiums in regulated markets, as this is not an objective for this paper.

This paper focuses on Australian general insurance markets. To support our analysis and understand other approaches, we considered the setting of discount rates and investment earning loadings in Australian life insurance and in overseas insurance and pensions markets (primarily the United Kingdom and United States).

While some of the concepts and material in this paper will be familiar to many readers, we have covered these for a complete view of the role of discount rates in general insurance pricing in recent years. We acknowledge that a range of views exists about what constitutes best, or even appropriate, practice in this area.

Outline of this Paper

We begin this paper with a review of the two main discounted cash flow models used in practice over the years for pricing general insurance contracts – the Internal Rate of Return (IRR) model and the Myers-Cohn model (**Section 2**). The review covers both the theory and practical issues associated with each model, including any challenges in using these models in practice. This is an obvious precursor to our later discussion on best practice for setting discount rates, because the discount rate assumptions will depend on the pricing model used.

Section 3 then provides a practical illustration of these models, the aim being to illustrate some of the sensitivities and differences between the models.

In **Section 4** we provide some guidance on best practice in relation to selecting discount rate assumptions when using the pricing models. This continues to **Section 5** where we present a new approach to projecting the yield curve. Our conclusions are set out in **Section 6**.

Acknowledgement

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However, the views and opinions given, and any remaining errors, remain the responsibility of the authors.

2. Summary of Pricing Models

In this section we provide an overview of the two main discounted cash flow techniques used in general insurance pricing in recent years – the Internal Rate of Return (IRR) model and the Myers-Cohn model. However before commencing this overview we present some background on the importance of these models in practice and in the academic literature.

Relevance of pricing models in practice

In preparing this paper we interviewed a number of practitioners involved in general insurance pricing both in Australia and overseas, to gain an appreciation of current practice (as noted in Section 1). We also spoke to people working in both life insurance and pensions. This section provides a brief summary of the relevance of the IRR and Myers-Cohn models in practice.

For long-tailed classes of business, for both general insurance and life insurance, the IRR model can be considered the standard discounted cash flow model that is used. The Myers-Cohn model has been used by some practitioners but only in the context of providing advice on fair premiums to the regulatory authorities of compulsory statutory lines. Its limited use reflects some practical challenges in calibrating and using it. Some also have a fundamental objection to its underlying assumptions and the fact that insurers are not compensated for some business risks. The reasons for this will become clear when we discuss the models in detail below. Other pricing models, for example those based on option pricing methods, are not commonly used in general insurance but are used to price some guaranteed life insurance policies.

For short tail classes, where the average lag between premium collection and claim payment is less than a year, the IRR model is still used. However other simpler models are also used, such as adding a loading to the expected value of claims and expenses.

IRR models are used by insurers in the following two slightly different ways:

- They can be used to establish the price for a portfolio of policies that meets a particular return on capital target. This output is often referred to as the technical price. Competitive and strategic factors may result in the implemented price being different to the technical price.
- Alternatively, IRR models can be used to determine the implied return on capital for a given price. This is relevant for some compulsory statutory lines (such as Queensland CTP) where prices are controlled by the regulator. It is also relevant when the implemented price has departed from the technical price, due to competitive pressure.

Prices at the individual risk level will also typically vary from the average portfolio price set using an IRR approach. For example, in short-tail retail portfolios of comprehensive motor policies a combination of risk relativity analysis and price optimisation may be used to alter individual policy prices.

For many commercial risks, such as Fire and ISR policies, underwriter discretion is also a source of price variation at the individual risk level.

Pricing models in the academic literature

Much of the academic literature on financial models for insurance pricing has focused on fair prices, defined in those studies as those that would arise in a freely competitive and rational market (e.g. Cummins and Harrington, 1985; Taylor, 1994). Cummins (1990, 1991) provides an excellent overview of developments in these models up to 1991.

Cummins identified that by 1991 the pricing models that had gained the most prominence were the IRR model and the Myers-Cohn model – both discrete time discounted cash flow models. The prominence of these two models was attributed to each model being:

- based on sound financial theory
- relatively straightforward and intuitive
- adaptable to real-world insurance processes.

Cummins also discussed a number of financial models based on option pricing methods e.g. Doherty and Garvin (1986) uses discrete time risk-neutral valuation theory and Cummins (1988) uses continuous time option pricing approaches. In 1991 it was noted that, while these models provided valuable insight into the insurance pricing problem and promised to provide a more sophisticated basis for pricing in the future, they were not yet developed enough to be useful at a practical level (at least in a regulatory context).

The model by Doherty and Garven deserves special mention because unlike the IRR and Myers-Cohn models it recognises default risk. Doherty and Garven's model adjusts the IRR approach to recognise the default risk of the insurer. Thus dividends to shareholders can be viewed as a call option on the profits of the insurer. For insurers with more marginal solvency, the model has two impacts (with opposite signs) on premiums:

- The increased default risk means that shareholders require a higher return on capital to compensate for the reduced value of the insurer, increasing the premium.
- The "option" that shareholders hold to limit losses in the event of adverse experience means that they face less downside risk, and thus require a lower premium.

For well-capitalised insurers with a low probability of default, this should not give materially different results to the IRR approach.

Also of interest in this early literature are the two spiritual precursors to the IRR and Myers-Cohn approaches. These are the approaches by Fairley (1979) and Hill-Modigliani (1987, although originally from 1981). They are both one-period models, which simplifies many of the assumptions. Both attempt to define the "fair" profit margin on a premium with reference to the reserves, claims, expenses, the risk free rate and the Capital Asset Pricing Model (CAPM).

Since the reviews by Cummins in 1990 and 1991 there appears to be very few theoretical developments in pricing models. However, of particular note is the work by Johnston (2004). This describes a model of equilibrium insurance prices but which does not assume a freely competitive market. Johnston argued that pricing should have reference to the consumer market. Because consumers are risk averse, with insurers (i.e. management and shareholders) being less so, the mismatch in utility creates a range of feasible prices that are acceptable to both the consumer and the insurer. The model however has not been extended to practical applications, due to the challenge of estimating an appropriate utility function for consumers.

Internal Rate of Return

Overview of approach

We begin our overview of the main discounted cash flow models (Cummins, 1990) with a discussion of the IRR model, as it is the one most commonly used by practitioners (see Feldblum (1992) for one detailed practical example). The focus of our discussion will be on the cash flows and discount rates required for the model. Discussion on how to choose appropriate discount rates is left for Section 4.

The method is based on the concepts of capital budgeting and investment appraisal for a project. Writing a portfolio of insurance business is implicitly viewed as a project under consideration by the insurer. The appraisal is undertaken from the point of view of the shareholder. The method is used to determine a price for the project that will provide a suitable rate of return on the shareholders' equity, taking into account the riskiness and timing of the future cash flows.

More formally, it is the price that sets the net present value (NPV) of shareholders' cash flows to zero. The NPV is evaluated using a chosen rate of return on equity that captures both the time-value of money and overall riskiness of the project.

Analysis of cash flows

The analysis is performed in discrete time periods ($t = 0,1, \ldots, T$) where $t = 0$ denotes policy inception. For simplicity we will assume that the premium is received at policy inception. Upon receipt of the premium (P_0) the insurer will commit a portion of the shareholders' capital to provide security that claims will be met. The insurer will invest total assets for the cohort of policies (A_0) in securities. Those assets will earn investment income and will be used to pay claims, expenses and taxes. As the value of the insurer's liability for future claims runs off, capital will be returned to the shareholders. The cash flows involved in this process can be summarised as (borrowing from Taylor, 1994):

- P_t - the gross premiums. As noted above, typically it is assumed there is a single premium at time 0.
- C_t - claim payments

- E_t - expenses, including policy acquisition costs, underwriting and policy administration costs and claims handling expenses
- I_t - investment income
- T_t - tax payments[1]
- F_t - the net cash flow to shareholders

The balance of these cash flows will change the value of the insurer's assets as follows:

$$A_t - A_{t-1} = P_t - C_t - E_t + I_t - T_t - F_t. \quad [1]$$

And a simple re-arrangement gives:

$$F_t = P_t - C_t - E_t + I_t - T_t - (A_t - A_{t-1}). \quad [2]$$

And since assets will only be held at a level needed to provide the appropriate security needed to make future claims we can write:

$$A_t = L_t + K_t, \quad [3]$$

where L_t is the sum of the insurer's premium liability and outstanding claims provisions (including the risk margin, which is contributed capital) and K_t is contributed capital in excess of the risk margins. This allows one to split F_t into the profit (first term in square brackets in Equation [4]) and return of contributed capital (second term in square brackets):

$$F_t = [P_t - C_t - E_t - (L_t - L_{t-1}) + I_t - T_t] + [K_{t-1} - K_t]. \quad [4]$$

Application of the IRR model involves the projection of each component in equation [4] to each time point into the future and then setting P_0 so that the NPV of the cash flows to shareholders (F_t) is zero, based on the targeted shareholder rate of return. This model appeals to most practitioners because all important cash flows impacting the insurer, shareholders and policyholder are explicitly modelled. In determining the net cash flow all components are included at their expected values.

For actuaries the process of projecting most of these components is well understood. However K_t (the contributed capital in excess of the risk margins) and I_t (investment income) deserve some comment.

Projection of capital

In the Australian context K_t is usually estimated by considering the capital requirements of the prudential regulator, APRA. This will mean issues such as the line of business, the insurer's asset mix and amounts of capital to be held in excess of the regulatory minimum should all be considered. In general the considerations needed to determine future capital requirements are

[1] Tax payments are sometimes reduced in Australian applications to allow for the value of dividend imputation to shareholders.

complex. Hitchcox et al. (2006) presents an excellent discussion of these complexities.

Typically the analysis excludes intangible assets. The market value of a firm consists of both tangible and intangible assets. The excess of the market value of a company's equity (i.e. the market capitalisation for listed companies) over the fair value of its accounting net assets is sometimes referred to as franchise value (Hitchcox, 2006). Loosely speaking, franchise value is the market value of a firm's intangible assets.

Franchise value results from intangible assets such as brand value and having well-trained staff, such as underwriters and claims managers. These intangible assets allow the firm to achieve higher returns on their tangible assets. The value of these assets is part of the overall insurance company valuation. It is reflected in the price investors pay for shares in a general insurer. As such, investors in the insurer reasonably require a return on these assets.

Some further subdivide franchise value into:

- "economic capital", as defined by Robb et al. (2012). This covers intangible assets which are justified (and required to operate an insurer) in a real-world freely competitive market, and
- monopoly value, due to some constraint on the level of competition in the market. This leads to higher profits (and a higher market value) than would exist in a freely competitive market, under the formulation in Robb et al (2012).

In regulated insurance markets the regulator needs to determine to what extent intangible assets should be included in any determination of fair premiums (See Robb et al. 2012 for a useful discussion of the issues).

It is possible to explicitly allow for intangible assets in the capital flows and target returns used in IRR models. These assets would be treated as producing zero investment income, but would require the specified return on capital. However in our view it seems preferable to restrict capital within these models to tangible net assets (i.e. the book value) and add a loading to the required return on this capital in order to produce a suitable total shareholder return. The idea here is to focus on the return on the book value of the firm – a typical use of the term "ROE" – rather than the return which shareholders achieve on their investment (which is based on the market value of the firm, the market capitalisation for listed companies). Hitchcox et al, (2006) contains a thorough discussion of the complexities involved. For internal targets, which need to be applied across many lines of business, a focus on ROE is probably easier.

In this paper expressions such as "return on equity" and "return on capital" should be read in context. Either may refer to the return on book value (i.e. the accounting net assets) or to the return on the market value of the firm, including franchise value. In the remainder of this paper, where we wish to avoid confusion we use the following abbreviations with specific meanings:

- "ROE" refers to the return achieved on accounting net assets. For example, many companies have a ROE target of around 15% per annum.
- Total Shareholder Return (or "TSR") refers to the investor's return, reflecting the price to buy and sell shares over the investment period and any dividends received. A number of asset pricing models (and historical return studies) suggest that a reasonable TSR target is around 10 to 12% per annum across economic cycles for a share investment with average risk.

Different ROE and TSR targets with values similar to the examples above can be shown to be consistent, due to the presence of franchise value in the company's market capitalisation.

The two definitions above are consistent with those used in Hitchcox et al (2006).

Projection of investment income

A large proportion of a general insurer's assets is usually invested in low risk assets such as cash and fixed interest securities. For practical reasons there will usually be a delay between the date that premium rates are set and the average date that future premiums will be collected and invested in these assets. This delay will depend on the lag between pricing analysis and rate implementation and the amount of time that the premium rates remain in force before being updated. It also depends on premium payment terms.

The final technical premium depends on the investment return assumptions; unless the yield curve (and the implied investment return) remains completely stable over time, then a premium calculated using the yield curve at the point of the pricing exercise will be different to what would be calculated at the premium collection date. This interest rate risk is material for some classes and in some interest rate environments. This is discussed further in Sections 3 and 5. If accurate forecasts of premium volumes can be made, then it is possible to "lock in" the yield curve at the time of the pricing exercise by hedging (e.g. using derivatives). However, to date this does not appear to be common practice in the general insurance industry.

Myers-Cohn

Relationship to the IRR model

The Myers-Cohn model (Myers & Cohn, 1987) is related to the IRR model. Like the IRR model, the Myers-Cohn model aims to find a premium such that the NPV of cash flows to shareholders is zero. This does not imply that shareholders should not earn a return on their capital. Where the Myers-Cohn model differs from the IRR model is the approach used to determine that NPV.

While the IRR model is focused on a single discount rate (the return on equity) to apply to the shareholders' net cash flow, the Myers-Cohn model determines the NPV of each cash flow stream separately using a risk-adjusted discount rate appropriate for that cash flow stream. From the perspective of financial economics, the Myers-Cohn approach to net present values is to be preferred. This is because it considers the riskiness of each cash flow on its own merits, rather than using a single discount rate that approximates the average riskiness of all cash flows as the IRR model does. However, as we shall discuss below there are practical reasons why the IRR method is used more extensively. Also, many prefer the IRR method because past practical applications of the Myers-Cohn model have used CAPM to determine the discount rates for NPV calculations. By using CAPM business risks not correlated with overall market risk are not priced. However, as noted by Myers and Cohen (1987), the Myers-Cohn method does not need to be used with CAPM. Any justifiable approach to determining NPV can be used.

Another objection that some practitioners have to Myers-Cohn is that it does not allow for revenue risk i.e. the risk taken by insurers about how much business they will write in a particular market when premium rates are set.

Another difference, as observed in common constructions of the two models, is the cash flows which are projected. The Myers-Cohn model projects cash flows between the insurer, its policyholders and other external parties (for the payment of expenses, taxes and receipt of investment income).

The IRR model includes these cash flows as well, but also includes cash flows between the insurer and its shareholders. A fuller description is given in Robb et al (2012).

In the Myers-Cohn model any cash flows between the insurer and its shareholders are implicit. Ensuring that these are appropriate (i.e. the shareholders' receive an adequate return on their investment, considering the risks taken) relies on appropriate calibration of the model.

The two methods can give reasonably similar premiums if assumptions about the riskiness of cash flows are treated consistently across both methods. In practice a common cause for difference is inconsistent assumptions. For example when applying the Myers-Cohn method it is common (but not necessary) to assume that the owners and management of firms are risk neutral in relation to diversifiable risks[2] that are not correlated with market risk (in other words CAPM is often used). These risks are potentially broad, encompassing uncertainty around premium revenue, claims costs and expenses. When applying the IRR method risk aversion around these diversifiable risks is often assumed. Consistency in assumption setting across both methods is discussed again in section 4. In Section 4 we discuss some of the frictional costs included in IRR calibrations (often implicitly) which explain this risk aversion. These frictional costs are measured relative to the ideal of a freely competitive market.

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[2] These are risks which in theory can be diversified by investors in the company by holding a sufficiently diverse investment portfolio

Derivation of the Myers-Cohn model

Starting with the net cash flows to shareholders (equation [2]) and assuming we set the premium so that the net present value of these cash flows is zero we obtain:

$$0 = NPV[P_t] – NPV[C_t + E_t] – NPV[T_t] + NPV[I_t - (A_t – A_{t-1})] \quad [5]$$

Where NPV refers to an estimate of the net present value using:

- The risk-free discount rate (R_F)for premiums
- The risk adjusted rate for claims and expenses (R_L). In some constructions different rates are determined for claims and expenses, but here we use the same rate.
- R_L for that component of tax relating to underwriting profits [assuming provisions are reserved on an APRA basis]. For the component of tax relating to investment returns the NPV value depends on the asset pricing model used for discounting. If the CAPM asset pricing model is used then the NPV is determined using Myers' Theorem[3] and the risk free rate, R_F, is used.
- The risk adjusted rate for the asset portfolio (R_A) for investment return on the total assets supporting this cohort of business.

Now the last NPV term represents the present value of the investment of premiums and shareholder assets in securities and its run-off over the life of the policy. Because these assets earn investment income at rate R_A and the present value is evaluated using R_A the term equates to zero. So setting the last term in [5] to zero, and continuing with our illustrative assumption that premiums are collected at inception, we get the Myers-Cohn model result:

$$P_0 = NPV[C_t + E_t] + NPV[T_t]. \quad [6]$$

Or in words, the Myers-Cohn premium is equal to the risk premium for claims and expenses (evaluated at R_L) plus the discounted value of tax costs (to allow for double taxation of the insurer's net profit, once at the insurer and once in the hands of its shareholders[4]).

Interpretation of the Myers-Cohn model

So in the absence of taxes, the Myers-Cohn premium is equal to the risk adjusted discounted value of claims and expenses. Depending on the approach taken for determining R_L this may be higher, lower or equal to the discounted value of claims and expenses evaluated using the risk free rate. Or in other words this approach can lead to positive, zero or negative profit margins. Models for determining R_L are discussed in Section 4.

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[3] See Derrig (1994) for a discussion of Myers' Theorem.

[4] The adjustment for tax costs is sometimes reduced for Australian applications to allow for dividend imputation.

In the presence of taxes an additional loading is applied to the premium to compensate the shareholder for the tax paid on their capital investment – had the shareholder simply invested directly in shares and bonds then the additional tax levied on the insurer would not have been paid.

Because of the different approaches to producing net present values the IRR and Myers-Cohn models will give different results in practice, unless the right consideration goes into setting assumptions. An illustration of the difference is given in Section 3. There are some conditions where the two methods will give the same results (see Taylor, 1994) but these will not typically occur in practice.

Reasons for the dominance of the IRR model in practice

At the beginning of this section we commented that interviews with insurance practitioners indicated that in their experience the IRR model was the most commonly used discounted cash flow method. We noted there has been limited use of the Myers-Cohn method in relation to the compulsory insurance classes in Australia. However, the use of the Myers-Cohn method for regulated business appears to be fading.

The Myers-Cohn model was first used for regulating automobile and workers' compensation insurance rates in the US state of Massachusetts. It was used from the early 1980s; however it stopped being used for rate regulation in this state in 2003. In other industries in Australia such as electricity and telecommunications, where price is regulated, the regulator relies on methods related to the IRR model – often weighted average cost of capital (WACC) methods – to determine an appropriate return on equity (e.g., see IPART, 2013).

There appear to be three main reasons for the dominance of the IRR approach:

- The first reason relates to the use of CAPM to determine NPV in the Myers-Cohn model in most practical applications. Many have a fundamental objection to the use of CAPM with Myers-Cohn because doing so treats insurance as a security. It implicitly assumes that insurance contracts can be traded in deep, liquid markets with few (or no) transaction costs. It also assumes that the supply of insurance is unconstrained.

  In reality insurance is a product, not a traded security. Under Australian law insurance policies have to be issued by APRA licensed insurers, from which it is difficult to withdraw capital. Insurers face a number of business risks which are not compensated under CAPM (e.g. revenue and expense risks). There are also frictional costs, which we explore in Section 4.

  However the Myers-Cohn approach is not tied to CAPM (Myers and Cohn, 1987) and one can use NPV approaches which allow for a variety of business risks and frictional costs and a desired return on economic capital (i.e. required intangible assets to support the insurance operation). The critics of Myers-Cohn argue that it is easier (and better) to allow for these business risks and frictions, often implicitly, within an IRR model. Robb et al. (2012) present one framework for allowing for these.

- The second reason for the dominance of the IRR approach is a practical issue. It is far easier to make objective estimates of ROE than it is to make objective estimates of a risk adjusted rate of return for claims and expenses. For listed companies which are actively traded there are many estimation techniques and refinements that have been developed to estimate ROE and TSR, and their relationship. These are useful benchmarks for unlisted companies. However in the absence of an active market in liabilities, risk adjusted rates of return for claims and expenses are hard to estimate (Cummins and Harrington, 1985). In section 4 we comment further on approaches to determining the risk adjusted rate of return for claims and expenses.

- The third reason relates to the IRR model's focus on ROE (and by extension the explicit consideration of insurer-shareholder cash flows). This is an advantage because it makes the model easier to interpret – ROE is a measure that is familiar and important to managers. Also, adjustments to the models, such as allowing for intangible assets, can be made to the IRR simply by adjusting the required return on equity. These types of adjustments can also be made to the Myers-Cohn model; they just require a bit more thought.

However in some contexts the general Myers-Cohn framework has an advantage over the IRR method because it focuses on the risks inherent in the individual cash flows (notwithstanding the challenges in quantifying these risks). There is less chance in the Myers-Cohn method of giving an excessive return to shareholders, after allowing for the capital supplied and risks taken. Nonetheless if a Myers-Cohn framework is used it is important that the capital supplied does receive a fair expected return for the risks taken.

3. Practical Model Examples

In this section we describe the application of the IRR and Myers-Cohn models to one long tail class of business (CTP). We also consider an IRR model applied to a short tail class (Domestic Motor). We have focused on the IRR model due to its practical advantages discussed in Section 2.

Background and Approach

We calibrated the premium rates for the two classes using broadly realistic industry data and projections at the time of writing this paper. However, the examples are hypothetical and do not represent any particular insurer or state compensation scheme. Also, they are not meant to be a benchmark for fair premiums – they are simply an illustration.

In the IRR model we made two related projections:

- The run-off for a cohort of policies written at time 0.
- The steady state, long run position after 20 years where policies have been written for a number of years. This assumes:
    - o no changes in real premium rates over this period – nominal premium rates only increase in line with claim inflation
    - o policy volumes grow at 2% per annum, recognising the fact that most Australian general insurance markets are now mature.

We tested the sensitivity of the IRR model outputs to changes in the model parameters. We assumed that the gross premium did not change in each case. Instead we allowed the implied profit margin, ROE and other model outputs to vary. We effectively modelled a shock to the portfolio once the premium rates were set.

An alternative approach would be to vary the gross premium following changes to the input parameters, ensuring the same profit margin or ROE is achieved. While this approach has merit for testing insurance affordability (in the context of regulated markets such as CTP) or competitiveness (e.g. comparing technical premium rates to what can be charged in the market in unregulated markets), we focused instead on understanding the sensitivity of model outputs to changes in each parameter. We were interested to see how the sensitivity of the discount rate assumption compared to other inputs.

The base scenario uses market yields as at 30 September 2014. For each class we assumed investment returns were a constant margin above the risk free rate (though the risk free rate is used to project outstanding claim provisions).

Further information on our approach and assumptions is set out in Appendix A.

IRR Base Scenario

Table 1 sets out the IRR results for our base scenario for each class of business.

The measures shown are:

- ROE (IRR) – the internal rate of return (per annum) on shareholder capital transfers to support a single cohort of business.
- Profit Margin – the percentage of the current gross premium which represents the insurance profit, again for a single cohort of policies.
- Net loss ratio – the inflated and discounted net loss ratio in the pricing basis for policies written at time 0, ignoring risk margins and CHE loadings.
- Gross expense ratio – the expense loadings in the premium as a percentage of the gross premium.
- Capital Base to GWP – the long run ratio of target capital to GWP, once the portfolio stabilises.
- The net accounting loss ratio – also the long run position, measuring the impact of prudential margins held in the insurer's accounts.
- ROE (NPAT / Net Assets) – this alternative measure of ROE compares the insurer's long run net profit after tax to net assets at the start of the year.
- Insurance Margin – this compares the long run insurance profit (i.e. underwriting result plus investment income on funds backing technical provisions) to net earned premium. This alternative presentation of the profit margin is often quoted by securities analysts and insurers.

		Class of Business	
Result	Context	CTP	Motor
ROE (IRR)	Cohort	9.4%	30.1%
Profit Margin	Cohort	12.6%	7.3%
Net loss ratio (inflated & discounted)	Cohort	75%	68%
Gross expense ratio	Cohort	12%	19%
Capital Base to GWP	Steady state portfolio	134%	29%
Net Accounting Loss Ratio	Steady state portfolio	92%	73%
ROE (NPAT / Net Assets)	Steady state portfolio	9.4%	30.1%
Insurance Margin (Insurance Profit / NEP)	Steady state portfolio	12.7%	9.8%

Table 1: IRR Model Results – Base Scenario

We note the discrepancy between the projected ROE for CTP (around 9% per annum) and Motor (30% per annum). The ROEs for the two classes straddle a common ROE benchmark of 15% per annum. These are broad indications of industry profitability for these classes at present, assuming insurers target an APRA Capital Base of 200% of the Prescribed Capital Amount (PCA) i.e. close to the current industry average. They are not value judgments on the suitability (or otherwise) on the returns achieved by these classes.

Furthermore, the ROE bears no relation to the Target Shareholder Return (TSR) for investors in these companies (assuming that these CTP and Motor portfolios are written by monoline insurers). We can reasonably expect that investors will recognise the higher ROE expected for Motor in future years, based on the latest projections, and bid up that company's share price (and hence its market capitalisation and franchise value) relative to the CTP insurer. This should happen until the target TSR is reached, which allows for the risks to investors in each company.

Other interesting observations from Table 1 are:

- The two alternative ROE measures within each class are near identical, despite their alternative calculation methods.
- Our assumptions produce a higher ROE for Motor than for CTP. Despite this Motor has a lower profit margin (7.3%) than CTP (12.6%). The lower profit margin for Motor is due to its lower capital requirement (29% of GWP in the steady state, compared to 134% for CTP).
- Given the long tailed nature of CTP, the valuation strain of a steadily growing portfolio is material. This is the main explanation for the difference between the long term net accounting loss ratio for CTP (92%) and the pricing loss ratio (75%). A small part of the gap is due to the inclusion of CHE expense in the net accounting loss ratio. The gap for Motor is smaller.

These results will be unsurprising to anyone experienced in Australian general insurance.

IRR impact of using the spot yield curve

In the base scenario we projected investment earnings using a single rate (set at a margin above the risk free rate). Similarly we projected the outstanding claims provision using a single discount rate based on Commonwealth Government Bond (CGB) yields (consistent with the cohort's liability cash flows and yields as at 30 September 2014).

In Table 2 we show alternative results for the CTP IRR model, but using CGB spot rates rather than a single discount rate for OSC provision calculations. Similarly the investment earning rate in each future year is set relative to CGB spot rates (instead of using a single rate). We considered two spot curves – one with a positive (upward) slope and one inverted.

These alternative scenarios use projected spot rates at the time the premium is expected to be received. For consistency with our base scenario, we used two spot rate curves which produce a single discount rate of 3.1% per annum (based on the assumed claim payment pattern).

The scenario for the upward sloping curve uses actual spot rates as at 30 September 2014. The scenario for an inverted curve uses hypothetical spot rates, set around the same broad level as the actual curve but with plausible slope changes varying by term (i.e. decreasing more at the short end and flattening out at the long end).

		CTP sensitivity		
			Using spot rates	
Result	Context	Base - single rate	Upward curve	Inverted curve
ROE (IRR)	Cohort	9.4%	9.9%	8.5%
Profit Margin	Cohort	12.6%	12.6%	12.6%
Net loss ratio (inflated & discounted)	Cohort	75%	75%	75%
Gross expense ratio	Cohort	12%	12%	12%
Capital Base to GWP	Steady state portfolio	134%	132%	135%
Net Accounting Loss Ratio	Steady state portfolio	92%	92%	93%
ROE (NPAT / Net Assets)	Steady state portfolio	9.4%	10.1%	8.4%
Insurance Margin (Insurance Profit / NEP)	Steady state portfolio	12.7%	13.5%	11.3%

Table 2: CTP – single rates in base scenario vs spot curves

The upward sloping curve scenario gives a higher ROE and insurance margin compared to the base scenario. Conversely, the inverted curve gives a lower ROE and insurance margin. The higher ROE for the upwards sloping curve simply reflects lower returns on capital in the earlier years compensated by higher returns in the long run. The opposite occurs for the downward sloping curve. The implication is that the way interest rates are parameterised in the model can impact key model outputs.

In most situations the curve will not be as steep as it was at 30 September 2014, so the differences shown in Table 2 will not be as great. We expect in practice the greater uncertainty around other inputs (especially claims costs) means that many insurers will not bother making this adjustment. But it is a result which users of IRR models should consider. The potential impact of using a single rate versus a spot curve is discussed further in Section 4.

IRR sensitivity

The following two tables summarise the ROE sensitivity for plausible changes in the model inputs. We focused on ROE as this drives Total Shareholder Returns (TSR), the metric which ultimately matters for the insurer's shareholders.

We classified the inputs as having one of three ROE sensitivities – High (Red), Medium (Amber) and Low (Green). The thresholds were lower for CTP than for Motor, reflecting the lower expected ROE for CTP in the base scenario.

Actual ROE results and other key outputs are summarised in Appendix A. So too are definitions of the assumptions listed in the tables below.

			ROE sensitivity[1]		
			Red	Amber	Green
Assumption	Base value	Variation	>2%	1-2%	<1%
Net claim cost per $100 gross premium (inflated & disc.)	75	+ / - 10%	●		
Gross expense rate	12%	+ / - 2%		●	
Net reinsurance cost	2%	+ / - 2%			●
Inflation	7%	+ / - 1%	●		
Discount rate	3.1%	+ / - 0.5%		●	
Excess investment return	0.5%	+ / - 0.5%		●	
Equities share for investments backing technical provisions	0%	20%			●
Equities share for investments backing shareholders' funds	0%	50%			●
Outstanding claims risk margin (APRA)	10%	+ / - 4%			●
Premium liability risk margin (APRA)	14%	+ / - 4%			●
Outstanding claims risk margin (accounts)	20%	+ / - 10%			●
Target capital ratio (Capital Base to PCA)	200%	+ / - 25%		●	
APRA average asset risk charge - cash and fixed interest	2.8%	+ / - 1.0%			●
APRA average asset risk charge - equities	26.6%	20%, 30%			●
R/I counterparty default charge	4%	+ / - 2%			●
ICRC per $100 gross premium	0	10			●
Payment pattern (undisc. mean term)	4.1 yrs	3.6, 4.6 yrs		●	

[1] Measures absolute value of change in ROE

Table 3: CTP ROE Sensitivity – IRR Model

Table 3 shows that the discount rate is a highly sensitive (though not the most sensitive) assumption for ROE changes in the CTP IRR model. This demonstrates the importance of getting the discount rate right for long tail classes. However, as it is not the most important assumption it probably explains insurers' reluctance to hedge the interest rate risk between the pricing analysis and average premium collection dates.

Assumption	Base value	Variation	ROE sensitivity[1] Red >4%	Amber 2-4%	Green <2%
Net claim cost per $100 gross premium (inflated & disc.)	57	+ / - 10%	●		
Gross expense rate	6%	+ / - 2%	●		
Net reinsurance cost	2%	+ / - 2%	●		
Inflation	3%	+ / - 1%		●	
Discount rate	2.6%	+ / - 0.5%			●
Excess investment return	0.5%	+ / - 0.5%			
Equities share for investments backing technical provisions	0%	20%	●		
Equities share for investments backing shareholders' funds	0%	50%	●		
Outstanding claims risk margin (APRA)	6%	+ / - 4%			●
Premium liability risk margin (APRA)	8%	+ / - 4%			●
Outstanding claims risk margin (accounts)	20%	+ / - 6%		●	
Target capital ratio (Capital Base to PCA)	200%	+ / - 25%	●		
APRA average asset risk charge - cash and fixed interest	2.8%	+ / - 1.0%			●
APRA average asset risk charge - equities	26.6%	20%, 30%		●	
R/I counterparty default charge	4%	+ / - 2%			●
TP recovery rate	30%	+ / - 2%			●
Payment pattern (undisc. mean term)	0.6 yrs	1.1 yrs	●		

[1] Measures absolute value of change in ROE

Table 4: Motor ROE Sensitivity – IRR Model

As expected, the discount rate is a less important assumption for a short tail class such as Motor. For this reason, for the remainder of this section we will focus on our CTP example.

IRR – Impact of Investment Strategy

As a further sensitivity we tested the impact of varying the investment strategy on the CTP IRR model output.

Most pricing exercises allow for the best estimate of actual investment returns on technical provisions and capital. While most insurers have relatively conservative investment strategies, these typically include some fixed interest securities with credit risk (e.g. semi-government and corporate bonds). Also, most insurers hold nominal bonds and few (if any) index linked bonds. Most insurers do not have perfect cash flow or duration matching either.

In other words, compared to a replicating portfolio of cash flow matched index linked bonds, the typical investment strategy has:

- some more risk (but not a lot)
- a higher expected return
- a higher APRA capital asset risk charge (thus requiring more capital to maintain the target capital ratio).

To compare these two approaches we ran an alternative CTP scenario, investing in a risk free replicating portfolio. The results (including the previous base scenario) are shown in Table 5.

		Investment scenario	
Result	Context	Base - modest risk	Risk free - replicating
ROE (IRR)	Cohort	9.4%	8.6%
Profit Margin	Cohort	12.6%	12.6%
Net loss ratio (inflated & discounted)	Cohort	75%	75%
Gross expense ratio	Cohort	12%	12%
Capital Base to GWP	Steady state portfolio	134%	125%
Net Accounting Loss Ratio	Steady state portfolio	92%	92%
ROE (NPAT / Net Assets)	Steady state portfolio	9.4%	8.5%
Insurance Margin (Insurance Profit / NEP)	Steady state portfolio	12.7%	10.8%

Table 5: CTP IRR Model – alternative investment strategies

Many of these metrics are unchanged in the alternate scenario. However, the ROE has decreased by 0.8% to 0.9% (depending on which ROE measure is used). The insurance margin is nearly 2% lower. However the Capital Base to GWP ratio has declined from 134% to 125%.

These changes came from assuming that investment returns were 0.5% per annum lower, but the average APRA capital asset risk charge on cash and fixed interest investments decreased from 2.8% to nil. Changing the relativity between these two assumptions will lead to different results to those shown above.

Myers-Cohn Results

In Table 6 we show results for the classic Myers-Cohn model applied to our CTP example above. We retained the claims cost and expense assumptions from the IRR base scenario. Within the Myers-Cohn model we have used a CAPM model to determine R_L and have varied the liability beta, the risk free rate and the market (equity) risk premium. The table below shows the implied profit margin measured consistently with the IRR model approach (i.e. comparing the Myers-Cohn premium to the NPV of claims and expenses discounted at the risk free rate).

	Liability Beta										
	-0.5	-0.4	-0.3	-0.2	-0.1	0	0.1	0.2	0.3	0.4	0.5
Equity Risk Premium											
3%	11%	10%	9%	8%	7%	6%	5%	4%	3%	2%	2%
4%	13%	11%	10%	9%	8%	6%	5%	4%	2%	1%	0%
5%	14%	13%	11%	10%	8%	6%	5%	3%	2%	0%	-2%
6%	16%	14%	12%	10%	8%	6%	4%	2%	1%	-1%	-3%
Risk Free Rate											
2.1%	13%	12%	10%	9%	8%	6%	5%	4%	2%	1%	0%
3.1%	13%	11%	10%	9%	8%	6%	5%	4%	2%	1%	0%
4.1%	13%	11%	10%	9%	8%	6%	5%	4%	3%	1%	0%
5.1%	12%	11%	10%	9%	8%	6%	5%	4%	3%	1%	0%

Table 6: CTP Myers-Cohn Model results – implied profit margins

In the top half of the table we have used the base scenario single risk free rate (3.1% per annum) and varied the liability beta and equity risk premium. In the second part we used the base scenario equity risk premium (4% per annum) and varied the liability beta and risk free rate.

The rows shaded grey show the same results, assuming the base risk free rate and risk premium. The only variation across each row is changes in the liability beta.

Table 6 shows that the implied profit margin varies from:

- -3%, for a liability beta of 0.5, the base risk free rate (3.1%) and a 6% market risk premium, to
- 16% for the same risk free rate and market risk premium, but using a beta of -0.5.

These results are unsurprising, once we consider the construction of the classic Myers-Cohn pricing formula. The sensitivity of the gross premium rate and profit margin to the liability beta increases as the equity risk premium increases. This may have consequences in low interest rate environments, where the risk free rate is low but the equity risk premium is higher than usual. This negative relationship between the risk free rate and equity risk premium is explored further in Section 4.

Achieving Equivalence between Myers-Cohn and IRR

In Table 1 we showed that the CTP profit margin in our base IRR scenario is 12.6%. What Myers-Cohn liability beta would achieve the same result?

We retained the relevant assumptions from the IRR base scenario, covering claims costs, expenses, the risk free rate and the equity risk premium.

By simple interpolation of the results shown in Table 6 (and allowing for the rounding of results in Table 6) we find that the equivalent liability beta is -0.49[5]. This equates to a NPV of the liabilities 9% higher than the central estimate of the liabilities discounted at the risk free rate. This could be interpreted as the value of the liabilities to a management averse to diversifiable liability risk.

Many applications of the Myers-Cohn model in the past have assumed liability betas close to zero, based on an analysis of the relationship between claims costs and share market returns. As the results above show, this usually produces a profit margin (and by extension a shareholder return) lower than most IRR model calibrations.

In Section 4 we explore the real world frictional costs which often justify higher profit margins than those produced using Myers-Cohn with a liability beta around zero. In a Myers-Cohn context some of these frictional costs can be allowed for by selecting a negative liability beta. In effect this adjusts premiums for the costs of the diversifiable business risks assumed by shareholders.

In other circumstances it may be possible to achieve an equivalent premium by including an additional loading for these additional frictional costs, by instead treating them as another form of “expense” for which the insurer should be compensated.

[5] An alternative and more formal way to achieve consistency between the assumptions of the two methods is via equation 10 in section 4 below. Using the assumed R_E and R_A from the IRR approach a liability beta of -0.5 is estimated.

4. The Selection of Discount Rate Assumptions

The purpose of this section is to provide some guidance around the selection of the discount rates used in the IRR and Myers-Cohn pricing models described in Section 2 (and illustrated in Section 3).

There are three discount rates we need to consider across the two pricing models. They are:

- Risk free interest rate (R_F)
- Return on equity (R_E)[6]
- Risk adjusted rate for liabilities (R_L).

For two of these discount rates, R_E and R_L, the approach to selecting assumptions will depend on whether the model is being used to determine fair profits in a regulated line of business or not. In other contexts the pricing exercise will not be constrained by considerations of "fairness".

We will start with a discussion of the risk-free rate, as this is the foundation of discount rate setting.

Risk free interest rate

The risk free interest rate (R_F) is an important input into both the IRR and Myers-Cohn methods.

As a discount rate it is used directly in the Myers-Cohn model to determine the NPV of premium cash flows. However it is also used indirectly in both models as it is required to discount the liability provisions (L_t) that in turn impact capital requirements.

Discounting general insurance liability provisions

For discounting general insurance liability provisions the Australian Prudential Regulation Authority (APRA, 2010) takes the view that the risk free rate should reflect the return that can be earned on assets that:

- Have no credit risk
- Match the term and currency of the future liability cash flows
- Are readily realisable or liquid.

In Australia we have a deep and liquid market in AAA rated Commonwealth Government securities. Both APRA and the Reserve Bank of Australia (2007) believe that returns on Commonwealth Government securities provide the best proxy for the risk free rate for insurers. We agree with this conclusion.

[6] We have not used the ROE abbreviation in this context because we wish to explore a broader idea of equity (and its return) than our definition of ROE given earlier. In some situations these will cover the same thing.

We do note that the requirement for risk free assets to be liquid is an issue of some debate. APRA's rationale for requiring liquidity is that if the insurance liabilities are not certain – as is the case with virtually all general insurance liabilities – it is possible that some assets may need to be sold in unfavourable market conditions to meet claim payments. In these circumstances holding liquid assets will be less risky than holding illiquid assets.

However, a general insurer's future claim payments can be estimated with reasonable confidence in most situations. Most insurers could allocate their fixed interest investments to a mixture of CGBs and more illiquid assets (e.g. semi-government bonds and highly rated corporate bonds) and have no adverse impact on liquidity, if cash flows are monitored regularly and the investment portfolio is adjusted as required.

For some types of Australian life insurance annuity products, where liabilities are fixed in terms of amount and timing, APRA considers that liquidity is not a necessary condition for the assets used to benchmark the risk free discount rate. For these products a "liquidity premium" can be added to the risk-free rate.

We understand that most UK life insurers use swap rates to set risk free discount rates. This corresponds to a common Australian life insurance approach before the LAGIC reforms (which commenced on 1 January 2013). As per the current Australian approach, UK life insurers sometimes add an illiquidity premium to the risk free rate for annuity business.

Notwithstanding the case made above to include a liquidity premium, setting the insurer's risk free rate based on CGBs has strong appeal. It deals with liquidity prudently, does not require a subjective allowance for a liquidity premium and is easy to estimate.

For many formal general insurance liability estimation exercises, liabilities are discounted using the implied spot rates at different terms to maturity in the yield curve for Commonwealth Government securities available on the balance date. However a common alternative is to convert the collection of spot rates into a single equivalent discount rate. This simplification may introduce a small error in pricing projections (as explored in section 3).

Also recall from our discussion in Section 2 on the projection of investment income that the risk free rates that we require are not those at the analysis date, but rather those at the average date of premium collection. It is only once the premiums are invested that the interest rates embedded in the yield curve are locked in. Approaches to projecting risk free rates are analysed in Section 5.

Discounting premiums

It is common practice to discount premiums (if at all) using the same risk free rate that is used for liabilities. It is possible to argue for the use of a different discount rate, such as a rate that includes a liquidity premium to allow for the near-certain premium cash flows, but no insurers seem to do this. This is probably because the average time from policy inception to premium receipt is usually short, so any adjustment would be immaterial.

Return on equity

Non-regulated lines of business

For non-regulated lines of business, insurers are free to choose any target (or hurdle) ROE that they want, limited only by market realities (i.e. competition, as well as what policyholders will pay in premiums). In general however the target should be set by considering the TSR, which in turn should be based on the opportunity cost of alternative investments of equivalent risk available to shareholders (Hitchcox et al. 2006).

Our interviews with practitioners indicated that insurers tend to set ROE targets that are stable over many years and do not change in response to changes in the risk free rate.

While this approach is fairly simple, it has a natural appeal in that a hurdle rate can be set above the ROE which is consistent with shareholders' expected TSR. This means that new projects generally have to be shareholder value positive before being approved. Its main disadvantage is it will tend to favour riskier projects with higher projected returns, unless risk is properly allowed for in the capital contribution.

Regulated lines of business

The general objective to set fair profit margins in regulated lines of business means that the ROE (and implied TSR) should ideally be determined with reference to an appropriate asset pricing model. This is because a sound asset pricing model will determine an appropriate return for the risks taken. The identification of risks requiring compensation is important. Suitable models for consideration include the Capital Asset Pricing Model (CAPM), Aribtrage Pricing Models and Multifactor Models that are described in most financial economic textbooks (for example Bodie et al. 2002). The choice of model may lead to different results.

For example Cummins and Phillips (2005) found that the Fama-French three-factor model (Fama and French, 2002, 2003) produced higher costs of capital (i.e. hurdle rates of ROE) than those determined with CAPM (which is a one factor model). The Fama-French three-factor model extends CAPM to include the effects of firm size and the ratio of the book value of equity to the market value of the firm.

The Fama-French three-factor model predicts that smaller firms and those with higher book to market values – often an indication of financial distress – have higher costs of capital. However in the Australian context we are unsure of the impact of using the three factor model over CAPM. One suggestion is that most Australian insurers are large (compared to the diverse US market studied by Cummins and Phillips). Also, the Australian insurance market is currently tightly regulated and supervised, so financial distress may not be such an important factor.

The CAPM is the oldest of these asset pricing models, and despite being extensively criticised for its failure to predict return differentials in the stock market, it appears to be the most commonly used asset pricing model across a variety of regulated industries (e.g. IPART, 2013). This may be because it is one of the simplest models, so it is relatively easy to understand and parameterise.

CAPM states that the expected return on equity is:

$R_E = R_F + \beta_E(R_M - R_F)$, where [7]

> R_E = rate of return on the equity stock
> R_F = risk free discount rate
> R_M = rate of return on the share market as a whole
> $\beta_E = Cov[R_E, R_M]/Var[R_M]$.

Estimation of CAPM parameters

The estimation of β_E will usually start with an analysis of historical stock price movements. This is covered in many standard financial economics textbooks. Insurer equity betas have generally been observed to cluster around 1 in the US market (Cummins and Phillips, 2005).

The other two parameters that need to be determined are the risk free rate (R_F) and the equity (i.e. market) risk premium (R_M – R_F). Care needs to be taken to ensure that both are estimated in a consistent manner.

For example, the equity risk premium is often estimated by taking an average of the historical values of R_M – R_F over a number of decades. Such an approach will give an estimate of the risk premium in the long run.

In contrast R_F is often set using the spot rates from the CGB yield curve as discussed in the previous section. This is a point in time estimate of R_F. There is an inconsistency in using a long run estimate of the equity risk premium and a spot estimate of the risk free rate if the equity risk premium varies over time.

There is some indirect evidence to suggest that the equity risk premium may be negatively correlated with the risk free rate.

One source of evidence is the credit spreads on corporate bonds. Credit spreads represent the risk premium for investing in corporate debt. It seems reasonable to expect that changes in the risk premium for investing in corporate debt would also be reflected in the equity risk premium.

If we look at the correlation between the 3 year yield on CGBs and the credit spread on BBB bonds there is some evidence of a negative correlation between credit spreads and the risk free rate – when yields are low the credit spreads tend to be higher (See Figure 1). The rationale for this observation is that in periods of high investor risk aversion there is a flight from risky assets to highly rated government bonds. This tends to push down yields on CGBs. For this reason, low risk free rates tend to be associated with high risk premiums for credit and equity (and vice versa).

Another source of evidence comes from attempts to measure the equity risk premium embedded in current stock prices. This involves trying to back out a forward looking equity risk premium from current stock prices and expected cash flows. A number of these analyses suggest that the current low interest rate environment is associated with high equity risk premiums relative to the long run average (Damodaran, 2013; Bank of England, 2010).

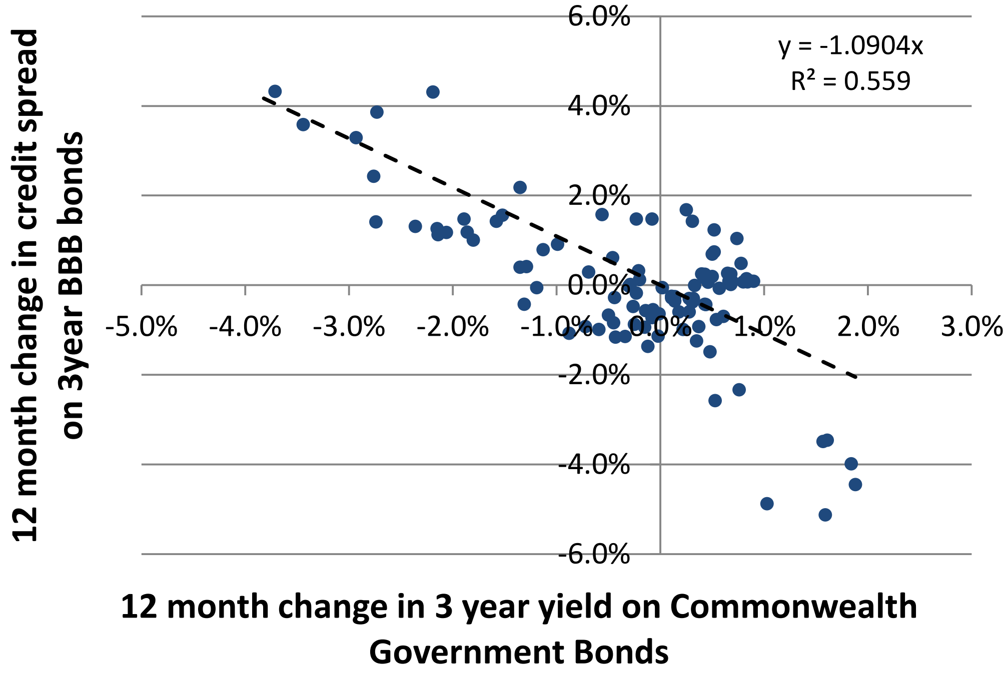

Figure 1: Negative correlation between credit spreads and the yield on 3 year Commonwealth Government Bonds (Source RBA statistical tables, F2 and F3; monthly data from Jan 2005 to Sep 2014)

In the Australian regulatory environment both IPART (2013) and the Australian Competition Tribunal (2009) have recognised the sometimes inverse relationship between the equity risk premium and the risk free rate.

We also note that the practice of many Australian insurers when applying the IRR method is to adopt a required return on capital that is independent of the current risk free rate. Many companies appear to leave their targets fixed for many years. This is consistent with the view that there is a negative correlation between the risk free rates and the equity risk premium.

Another issue that has also arisen in the Australian regulatory environment is whether CGBs are always the best proxy for risk free rates for use in CAPM models. The issue has arisen in energy markets, in submissions by regulated entities stating their views on required returns on equity. In dealing with these submissions the Australian Energy Regulator, making reference to advice from both the Reserve Bank of Australia (RBA) and the Australian Treasury, stated that CGBs remain the best proxy for the nominal risk free asset (Australian Energy Regulator, 2008).

Ground-up estimation of R_E

In the previous section we looked at estimating R_E directly from stock price movements using an asset pricing model. An alternative approach that is occasionally used is to make a ground-up estimate of R_E. The usual starting point in a ground-up approach is to estimate R_E in a freely competitive insurance market. Adjustments for market "frictions" or imperfections relative to the ideal of a freely competitive market are then made if they are assessed as material.

Here "freely competitive market" is used to describe a market that has many buyers and sellers and a single (largely undifferentiated) product. All participants are price takers, and the price is assumed to move (over some undefined time period) towards the average cost of all inputs, including capital. We are also assuming efficient capital markets.

The first step of most ground-up analyses is to determine R_E by valuing the insurer's financial assets and liabilities individually with the CAPM model. While we acknowledge the objections that some have to applying CAPM to insurance liabilities (for reasons given previously), we set out the methodology to document this approach that has been used in some places.

Such an approach leads to an R_E which is appropriate in a freely competitive market. If we accept that the requirements for a freely competitive market are met, then this R_E is a lower bound that a regulator may want to impose on insurers. If this is not the case – because in practice insurance markets have imperfections (or frictions) that have associated costs – then the regulator may (some say should) allow adjustments for these costs.

To understand this approach we start with the cash flows assumed in our IRR model. We have (re-arranging equation [3]):

$$K_t = A_t - L_t \qquad [8]$$

where K_t was defined as the contributed capital in excess of the risk margins. Now total contributed shareholder equity at any time (E_t) will also include the risk margins, so we can write:

$$E_t = A_t - L_{ce,t} \qquad [9]$$

where $L_{ce,t}$ is the central estimate of the liability provisions. Then, multiplying each item in equation [9] by its expected return under CAPM, and rearranging gives :

$$\beta_E = (A_t \, \beta_A - L_{ce,t} \, \beta_L) / \, (A_t - L_{ce,t}). \qquad [10]$$

where β_A is the asset beta and β_L is the liability beta (defined in the same way as an asset beta).

To get an idea of the β_E determined using this approach, firstly assume that the investments supporting the central estimate of the insurer's liabilities ($L_{ce,t}$) are risk free and the balance of the insurers assets are invested in a market average share portfolio. Also assume that the liability cash flows are not correlated with the market (i.e. $\beta_L = 0$). These assumptions give $\beta_E = 1$ which is consistent with studies of the US market.

However, more conservative asset allocations within this market framework produce a lower $\beta_{E,}$ consistent with taking less risk. For instance, if all investments were in risk free assets then $\beta_E = 0$. At present around 80% of Australian general insurers' total investments are allocated to cash and fixed interest, including risk-free CGBs, bank deposits, corporate bonds and other loans (APRA, 2014). In our experience, most of the credit exposed assets that insurers hold are high quality (A rated or higher). Such an asset mix is consistent with a β_E of around 0.5 or less, under the approach described above.

We are not aware of any publically available contemporary studies estimating β_E for Australian general insurers. We have seen recent financial analyst estimates for listed general insurer betas, which range from 0.5 to 0.8. Some consider a β_E of around 0.5 or less too low and attribute this to the fact that the analysis so far has ignored market frictions – in other words evidence, according to some, that the CAPM without adjustment for frictional costs does not match actual experience. This is explored further in the following section on market imperfections.

Equation [10] implies that an insurer will only be rewarded for claims risk if β_L is less than zero, which requires claim payments to be negatively correlated with market returns. This is likely in a few classes of business, such as credit insurance and D&O insurance, but is likely to be close to zero for most lines of business. Attempts to measure β_L have been made by analysing historical underwriting profits (e.g., Fairley, 1979, Cummins & Harrington, 1985). Estimates are generally close to zero but Cummins and Harrington note that methodological issues mean the estimates should be treated with caution.

Stating this result in another way is that, under CAPM, insurers only get compensated for claims risk that is negatively correlated with the market. Any claims risk that can, in theory, be diversified away is not compensated. While this result may seem at odds with the realities of real insurance markets it is understandable in the context of CAPM. CAPM assumes that the ownership of the corporation is infinitely divided. Each owner of the corporation is only allocating an infinitely small portion of their diversified portfolio and because of this all risk that is not correlated with the market is diversified away. The implication of this is that firms (i.e. their management) under CAPM display no risk aversion to non-market diversifiable risk. In the following section we explore why there is often aversion to this diversifiable risk in practice.

Market imperfections (frictions) for ground up estimates of R_E

In practice the insurance market will have imperfections with associated costs. The regulator of a price-controlled class of business will make some adjustments to ground-up estimates of R_E for these imperfections to reflect real world insurance markets.

The frictions we are interested in here are those that would impact the required R_E. They are the frictions that increase the required compensation for the business risks faced by insurer. For example, in reality insurers are likely to show risk aversion to diversifiable risks because:

- Ownership will not be infinitely divided

- There is a preference from management and shareholders (and indirectly from APRA) to limit variability in profits
- Managers may view their risk from the insurer as undiversified even if shareholders do (a form of agency cost). This is because managers' employment with the insurer will be their main source of income.

Risk aversion to diversifiable risks, as we discussed in the previous sub-section, would lead to a larger required R_E. There is in general no agreed way for allowing for such risk aversion in a ground-up estimate of R_E and this is one reason why the ground-up approach to estimating R_E is often criticised and in general why direct estimates of R_E from stock price movements are often preferred.

The desire for shareholders and management to maintain franchise value is often referred to as a market friction. As discussed in section 2 we prefer to think of the issue of franchise value as an issue about the appropriate capital base to use in the IRR calculation. A question for the regulator is then how much of a firm's (or industry's) franchise value to include in the capital base (whether directly or by adjusting R_E as discussed in section 2).

The impact of the risk aversion of shareholders' and management, along with the maintenance of franchise value, are expected to be the largest frictional costs faced by insurers.

Other frictional costs faced by insurers and their management are likely to have a smaller impact, but include:

- The costs of financial distress. In particular, in the event of financial distress the regulator may constrain management to a course that is sub-optimal with respect to shareholder value.
- The costs of tax asymmetries. If a company makes a loss, then the tax credit in respect of the loss will be realised only when it can be offset against future profit. Delays in the realisation of tax credit will carry a cost.
- The desire of management and shareholders for business growth.

Hitchcox et al (2006) provide a detailed discussion on how some frictions may be allowed for, including quantifying possible adjustments for frictional costs for UK insurers in 2005. We are interested in exploring the size if these frictions in the Australian context in the future.

Risk adjusted rate for expected claims costs and expenses (R_L)

R_L is required to determine the NPV of future claims costs and expenses in the Myers-Cohn model. We saw in the previous section that applications of the CAPM to determine premiums in a freely competitive market using a ground-up approach also require a R_L such that:

$$R_L = R_F + \beta_L(R_M - R_F). \qquad [11]$$

Using the CAPM model to determine R_L for a Myers-Cohn model will also produce premiums consistent with freely competitive insurance market. However, as noted above, the insurance market also has frictions requiring adjustment.

The Myers-Cohn model is independent of CAPM and can be applied using other pricing models and approaches. For example an alternative way of allowing for uncertainty in cash flows in capital budgeting decisions is the certainty equivalent method. Under this approach the value of claim and expense payments would be scaled up to reflect the risk aversion of the insurer (strictly speaking risk aversion of shareholders or management) before being discounted using a risk free discount rate. Because insurers' utility functions are not known in practice such an adjustment is necessarily subjective. A related approach would involve subjectively choosing R_L lower than the risk free rate to reflect risk aversion. Using these approaches the insurer would achieve compensation for diversifiable claims risk associated with the portfolio. A result consistent with most IRR applications could be achieved.

It is the need for such subjective adjustments in some circumstances that lessens the appeal of the Myers-Cohn model in practice. However one advantage of the general Myers-Cohn framework is that it focuses on the risks inherent in the individual cash flows. There is less chance in the Myers-Cohn method of giving an excessive return to shareholders, after allowing for the capital supplied and risks taken.

However, external factors may ensure that shareholders do not receive excessive returns in practice (after allowing for risks taken). In regulated markets the regulator typically aims for "fair premiums" and sets premium rates so that insurers are not expected to earn excess returns over the longer term. In unregulated markets competition usually contains insurer returns over the insurance cycle.

5. Projecting the Yield Curve

In this section we explore the challenge presented by the need to project the yield curve, allowing for the delay from the date premium rates are set to the period that premiums are received.

Some pricing approaches make direct use of the risk free yield curve (e.g. assuming the insurer invests in a replicating portfolio). In other applications the assumed investment return (or return on assets, R_A, as defined in Section 2) will be closely related e.g. set at a constant margin above the risk free yield curve. In both cases there is a need to project the risk free yield curve.

In Section 2 we found that it is possible to "lock in" the yield curve at the point of a pricing exercise by hedging via derivatives. However, this practice still appears to be in the minority for Australian general insurers. For this reason, it is still important to consider how best to project the yield curve from the date of the pricing exercise to the point of expected premium collection.

A common approach is to use forward rates derived from the risk free curve. An alternative is to assume no change in yields i.e. use today's spot curve. This section explores these and an alternative projection methodology, and recommends a relatively simple approach to curve projections which in our view is superior to the forward rate approach, particularly for projecting the shorter end of the yield curve (terms up to two years). We explore the relativities between the different methods in different circumstances.

The analysis has particular relevance to regulated classes, where pricing is often done with reference to the risk free rate and there are controls around most pricing assumptions.

Motivation for yield curve projection

Much of the preceding discussion on pricing requires an estimate of the risk free rate. This is a cornerstone of most asset pricing frameworks (including CAPM). Thus risk free rates are often an input into regulated pricing as well as technical pricing exercises.

In Australian general insurance, as noted previously the risk free rate is generally taken from yields on CGBs. These yields are non-constant in the sense that they vary:

- Over **term** – longer dated bonds will attract a different (usually higher) yield than shorter dated bonds
- Over **time** – as time passes, the level and shape of the yield for a given term will change in response to changes in interest rate speculations and term preference.

These changes are important from a pricing perspective. In particular, while the pricing exercise may be performed at a given time with a particular yield curve available, the prices will be implemented and premium collected at a future point in time. In this case the yield curve at the premium collection date is more relevant than the curve at the point of the pricing exercise. If the time gap between pricing and premium collection is short, or if the bond market is stable, the difference between the two may be small.

In other contexts, the updated yield curve may be significant enough to make a material difference to the calculated premium and the resulting ROE. We illustrate this in Figure 2, which shows the actual ROE from the CTP IRR model of Section 3, tested using historical yield curves and calibrated to target a 15% ROE. This tests pricing performed one year before premium collection and risk free rates of return set based on forward rate expectations, without hedging (i.e. common industry practice). The interest rate risk arising from the one year delay creates significant volatility in the ROE, routinely +/-5% compared to the targeted 15% per annum return.

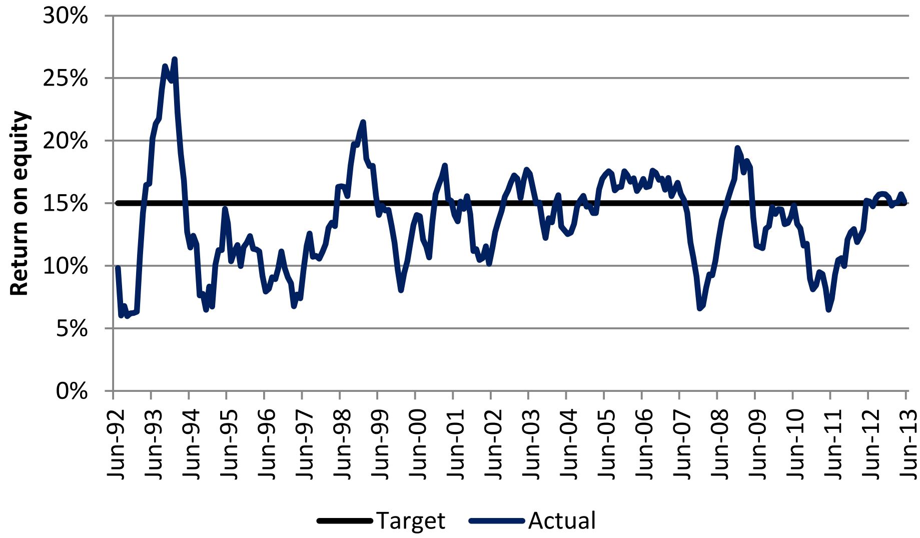

Figure 2: Return on equity variability when risk free rates are set one year before premium collection

In most practical pricing exercises the gap from rate setting to premium collection is less than one year. Nevertheless there is still significant risk of volatility in achieved ROEs (compared to target) from interest rate changes.

These results motivate the question addressed below: how should we project the risk free yield curve into the future?

Theories of the yield curve

Here we give brief coverage of yield curve theory. For a more comprehensive discussion of components of the yield curve, readers are referred to Mulquiney and Miller (2014) or general texts such as the Discount Rates textbook from the *International Actuarial Association* (2013).

One key yield curve theory is the **expectations hypothesis**. This theory states that the forward rate curve is the expected short term interest rate (effectively the cash rate) at each of those future points in time. If this were the case then it would be possible to 'left truncate' the forward rate curve to obtain the expected yield curve at some future time period, as shown in Figure 3.

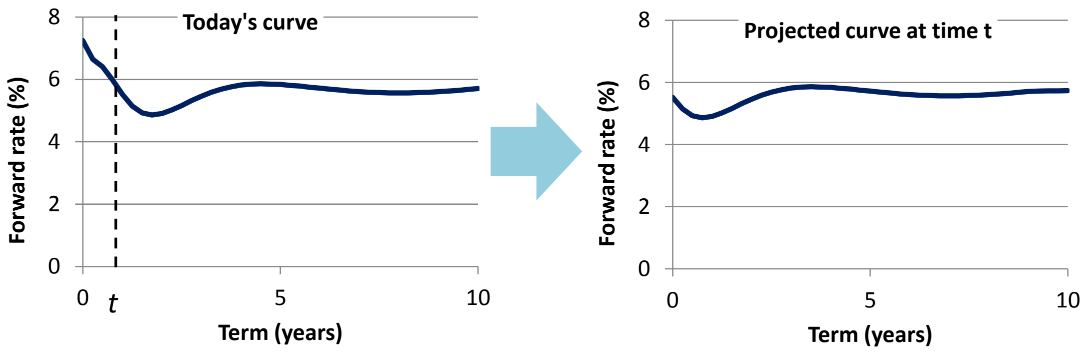

Figure 3: Illustration of the yield curve projection under the expectations hypothesis

While the expectations hypothesis can explain much of the movement in yield curves observed over time, academic studies have repeatedly shown that the theory does not hold as a single explanation for future interest rate movements, for a number of reasons:

- It ignores interest rate risk. Investors should demand a premium in the yield for long dated bonds due to the greater capital risk for holding these bonds, compared to shorter dated instruments.
- It ignores term preference. Demand for long-term bonds from large institutional investors such as insurance companies and pension funds can drive down long-term forward rates. This is because these long-term bonds offer a closer match to insurer and pension fund liabilities and are less risky investments to these investors.
- Estimating future short-term interest rates over extended periods of time is notoriously difficult (Guidolin and Thornton, 2008).

The more conventional approach to explaining the yield curve is that rates are a combination of interest rate expectations and (usually slowly moving) term premia, resulting in a more subtle term structure of the yield curve. This means a more sophisticated projection of the yield curve than the approach of Figure 3 is justified. In the next section we discuss approaches to yield curve projection in the literature. We then analyse the usefulness of two simple and commonly used predictors (the forward rate and current spot rate), using historical yield data. The subsequent section discusses a heuristic approach to yield curve projection that we have developed using Australian data.

Academic approaches to yield curve projection

One of the most popular frameworks for modelling the evolution of the yield curve over time is the Heath-Jarrow-Morton (HJM) framework (Heath et al, 1992). It places a stochastic structure on the yield curve and then the no-arbitrage condition imposes a series of rules on the evolution of the curve over time. One of the key consequences is that the no-arbitrage assumption "drift" of the forward rate at any given term is a function of the volatility of the curve. This is analogous to bond-pricing formulae under Black-Scholes, where the value of an option depends only on the volatility of the stock and its current level and not on expected movements in the underlying security. The dynamics of the yield curve will then be driven by the volatility (variance) structure imposed on it.

The HJM approach is very flexible, but it has two important limitations:

- Its flexibility means that parameterisation can be an issue, and a fair bit of uncertainty can be hidden in the model assumptions
- The distribution of interest rates is assumed to be normal, which permits negative interest rates, an issue which has to be worked around in practice.

The HJM structure also includes a number of special cases. For instance a constant volatility structure across term and time gives the Ho-Lee Model (Ho and Lee, 1986).

Another popular approach to yield curve modelling is the Libor Market Model, also called the Brace Gatarek Musiela (BGM) Model (see Brace and Musiela, 1997). Although generally seen as more practical compared to HJM, one key difference is that it models the evolution of observable market forward rates (rather than instantaneous yield curves). Each of these instruments is assumed to follow a lognormal process, with conditions governing how they evolve simultaneously. The complexity of these models generally requires something like Monte Carlo simulation to make predictions. This model appears to be a relatively popular way of modelling exotic interest rate derivatives that depend on other observable interest rate instruments. As its name implies, it is used in modelling the evolution of the Libor rate curve in London, where a significant amount of interest rate derivative trading takes place.

Initial Analysis – Performance of Common Simple Estimators

In the remainder of this section of the paper we test alternative approaches to the projection of the risk free yield curve. We first conduct an empirical analysis of the performance of two common yet simple approaches for estimating the future yield curve, namely:

- The forward yield curve – that is, using the expectations hypothesis.
- Using today's curve as a proxy for what it will look like in the future – that is, assume that tomorrow's yield curve will be identical to today's. We refer to this as the 'static' approach. The motivation for this approach is that it solves the issue of term premia tending to increase as term increases.

We tested these two simple estimators on projections of the single discount rate applicable to a long tail portfolio such as CTP. This approach tests the ability to project spot rates at different terms, with the weight given to each term determined by the expected cash flow at that term. A six month projection period for yields is used. This roughly matches the time from price setting to rate implementation in many regulated classes. However it falls short of the average time between price setting and premium collection. We tested projections at each month end from June 1997 to January 2008, as during this period there is no overall trend in discount rates which could distort our conclusions. However, the main conclusions from our analysis did not change when we considered the wider time period of data from June 1992 to November 2013.

We considered both the bias (i.e. average difference between actual and expected) and the estimation error (Root Mean Square Error, or "Root MSE") from the two simple approaches. There was no clear trend over time. However, we did observe a pattern depending on the shape (slope) of the yield curve at the estimation date. We defined the slope as the gap between the 10 year and 3 month spot rates. The results for prediction bias are shown in Table 7 and the results for prediction error are shown in Table 8.

			Average bias (basis points)		
Yield curve	Slope	No. months	Forward	Spot	Diff.
Inverted	< -0.25%	26	0.21	0.16	0.04
Flat	-0.25% to 0.25%	25	-0.00	0.00	-0.00
Normal	> 0.25%	77	-0.15	-0.02	-0.13
Normal - steep	> 1%	37	-0.33	-0.13	-0.20
Total		128	-0.05	0.02	-0.07

Table 7: Average bias in single discount rate estimation: June 1997 to January 2008

			Root MSE (basis points)		
Yield curve	Slope	No. months	Forward	Spot	Diff.
Inverted	< -0.25%	26	0.43	0.40	0.02
Flat	-0.25% to 0.25%	25	0.53	0.54	-0.00
Normal	> 0.25%	77	0.64	0.61	0.03
Normal - steep	> 1%	37	0.72	0.66	0.06
Total		128	0.58	0.56	0.02

Table 8: Average prediction error in single discount rate estimation: June 1997 to January 2008

Both tables show the different slope bands we used to define inverted, flat and normal (i.e. positively sloped) yield curves at each estimation date. We also show a subset of the normal yield curves, where the curve is steep (greater than 1%).

We observe the following from Tables 7 and 8:

- The static (spot) projection method seems to perform better than the forward rate method for most yield curve shapes, whether this is measured using the average bias or root MSE.

- Over the period tested the forward curve on average projected a single discount rate that was 0.05% higher than the actual single discount rate. This bias is probably a reflection of the fact that term premia tend to be larger at longer terms. For a CTP premium with a mean term of payments of about 4 years, the premium would be underestimated by about 0.2% on average. This effect is more pronounced when the yield curve is steeper than usual, where the premium might be underestimated by about 1% on average by using forward rates.
- However, over all yield curve shapes, the performance difference between the two methods is relatively small. Over projection periods up to one year, the forward and static approaches give very similar results. In general, the relative performance of these two methods will depend on whether the yield curve shape is dominated by changes in term premia or by interest rate expectations. If term premia are dominant then a static approach should perform better and vice versa.

For shorter-term interest rates (terms up to 2 years) the average prediction error of the static projection approach is slightly higher than the forward rate approach. However in periods of interest rate volatility, such as the GFC in 2008, the errors attached to static curve predictions are usually considerably worse than the forward rate predictions.

Further analysis of the Australian yield curve

We sought an alternative approach which added more rigour than the simple projection methods used above, namely the forward rate (expectations) method and the static method.

Alternatively, approaches such as HJM and BGM involve a level of complexity that will generally be beyond the requirements of an insurance pricing exercise. These sentiments were echoed in our discussions with practicing actuaries. For this reason we performed an alternative analysis on yield curve projection, based on empirical results on Australian data over the past 20 years. In terms of complexity this lay between the simple estimators and the more complicated methods of HJM and BGM.

We used instantaneous forward rate curves produced by the RBA. These are fit to bond prices using the Merrill Lynch Exponential Spline Method as presented in Finlay and Chambers (2008). Our analysis used the end of month forward rate curve for every month from January 1996 through to June 2014. While earlier dates were also available, the bond crash of 1994 and recovery in 1995 tended to distort the analysis so was omitted.

For the nearly twenty years of data, we took an average yield curve $\bar{f_t}$, being the average yield at term to maturity t. This is shown in Figure 4. Let $f_t(s)$ be the forward rate at term t and time s. Then we considered a simple model for estimating the yield curve at time $s+\delta$, given its state at time s:

$$f_t(s+\delta) = \alpha_{t,\delta} + \beta_{t,\delta} f_{t+\delta}(s) + (1-\beta_{t,\delta})\bar{f_t} + error$$

Thus the future forward rate is a function of the expectations hypothesis (the second term above) and a mean reversion term (third term). The intercept term is included to recognise any bias arising from differences in term premia and long term trends in the yield curve. The α and β terms are permitted to vary by term t and time gap δ; hopefully they vary in a smooth and interpretable way.

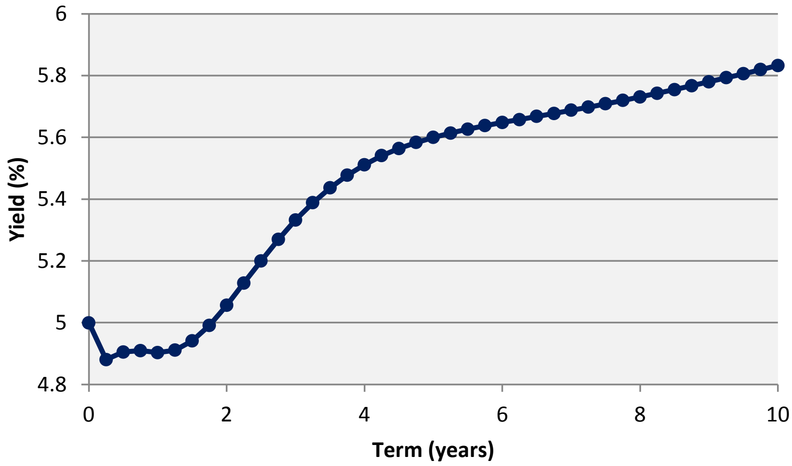

Figure 4: Average Australian forward rate curve $\bar{f}_t$ 1996-2014

We also considered the addition of an additional term to the equation above, being an extra parameter based on $f_t(s)$. This would be consistent with the theory that the yield curve exhibited stationarity over time, as would be the case if expectations were constant and term premia fixed. However we found $f_t(s)$ and $f_{t+\delta}(s)$ to be so highly correlated that having both terms in this model was unnecessary. However for future work we intend to look at using $f_t(s)$ instead of $f_{t+\delta}(s)$.

For any combination of t and δ we can estimate the corresponding α and β terms via a linear regression. Details of this linear regression are given in Appendix B. Figure 5 shows the estimated fits for various t (in years) when $\delta = 1$. The main features are:

- The β terms are lower at shorter terms, suggesting more volatility not explained by the expectations in the forward curve. Beyond term three the β estimates are fairly stable around 70%.
- The 95% confidence interval for the β terms are very wide. These were found by bootstrapping annual blocks of the yield curve and re-estimating the parameters. This suggests that the estimates are quite sensitive to the historical periods used to estimate them.
- The α terms are generally negative, signalling that the yield tends to be lower than a weighted sum of the forward curve and average curve would suggest. Overall a 1 year projection should allow for about a 0.3% fall in yields, and slightly more around term 2. This partly reflects the generally decreasing yield curve over the study period (in which case users might choose to mute these terms going forward), and partly reflects the change in term premia across the yield curve.

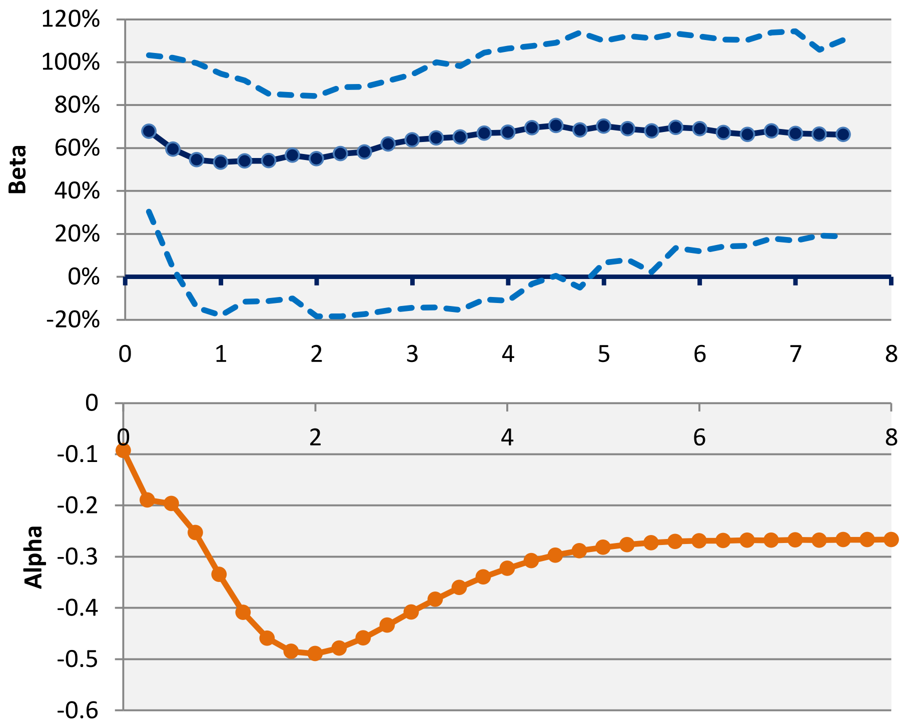

Figure 5: Estimated parameters for δ=1. 95% Confidence interval shown for β estimate

We have estimated β at other values of δ. The resulting table of estimates is shown in Appendix B. Our other main observation is that the β tend to fall as δ grows. This is intuitive as the long term mean reversion effect is likely to be more significant for projections of the yield curve further into the future.

The observed shapes and the relatively wide confidence intervals motivate a simpler formulation for the β values. We have found that the following formula:

$$\beta_{t,\delta} \approx \frac{0.6143}{1 + 0.6364 \times \delta \times 0.4166^t} + \frac{1 - 0.6143}{1 + 1.0288 \times \delta \times 1.0653^t}$$

gives a reasonable empirical fit to the estimated β values, giving due consideration to the relative confidence in each estimate.

In theory we would also need a corresponding formulation for the $\alpha_{t,\delta}$ values. However the negative estimates observed mainly reflect a decreasing yield curve over the historical period. Therefore we believe that it is plausible to set $\alpha_{t,\delta} = 0$ and use a mean yield curve that has linearly decreased over time, to reflect the evolving 'new normal'.

This permits some judgement as to what the average yield curve should look like over the short to medium term. Our approach is to set December 2010 mean curve as the average curve observed between July 2009 and June 2012 (given in Appendix B), and modify it linearly depending on the time elapsed. This approach gave superior empirical results compared to a number of other candidates considered – it tends to better allow for medium term shifts in the yield curve that can take extended periods of time to resolve. The resulting December 2013 curve, as shown in Appendix B, also looks to be a reasonable choice for $\overline{f_t}$ for the current interest rate environment.

We found that the recommended heuristic formula (summarised in Appendix B) produces a 20-30% lower prediction error on average compared to an expectations only projection (i.e. using the forward curve alone). Further, using this decreasing mean yield curve as $\overline{f_t}$ provides better yield curve estimates than the expectations hypothesis in about 65% of cases historically.

We illustrate this improvement in Figure 6, where we compare the forecast accuracy of our formula based projection compared to the expectations hypothesis and static curve assumption. Similar results are seen over various terms and projection intervals.

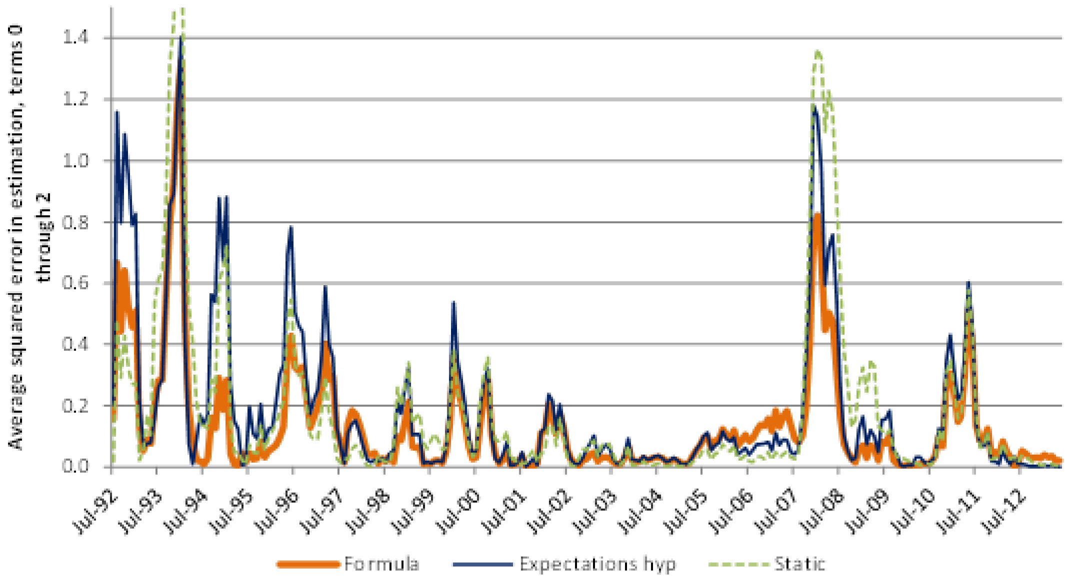

Figure 6: Error for recommended yield projection curve formula versus the expectations hypothesis and static curve assumption, for a time gap of one year (δ=1)

While the mean yield curve $\overline{f_t}$ has been decreasing over the period studied, it is reasonable to expect interest rates to stabilise and start rising soon. We believe that the yield curve as at 31 December 2013 is a reasonable ongoing choice for this long term average if the method is to be applied, as at the time of writing. This curve is shown in Appendix B.

A final question is: to what extent does our formula-based projection alter the premiums set? Using the IRR pricing model of Section 3 for CTP premiums, we can measure the difference in premiums over time using the expectations hypothesis, static curve assumption and the new formula, for a fixed target ROE. Premiums regularly vary in the order of 1-2%. Over the last year or so the formula gives a premium about 2% lower than the expectations hypothesis, which reflects some of the risk of upwards yield curve movement towards its long run mean (as incorporated in the formula).

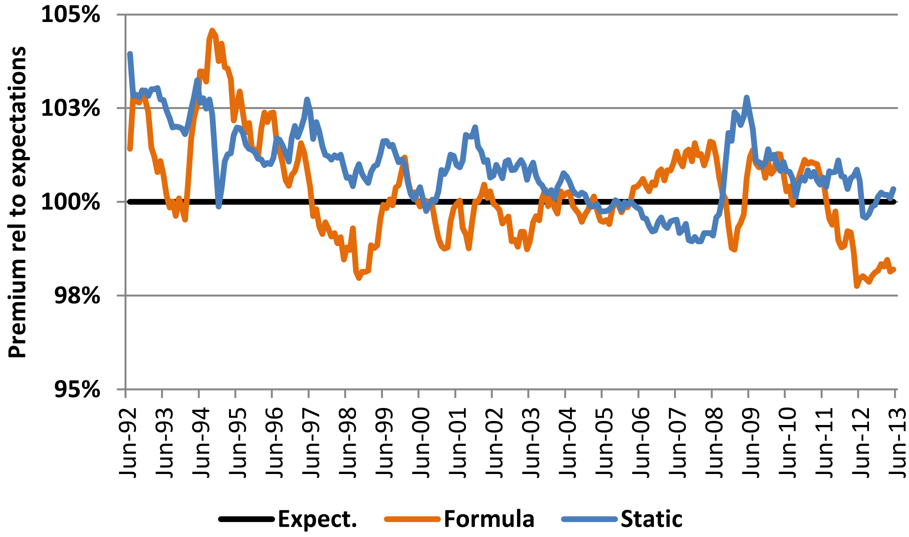

Figure 7: Relative premium using yield curve projection formula and static assumption as a fraction of the premium using expectations hypothesis. Time gap of one year (δ=1)

Relevance of time period

It is important to note that the time period considered may impact the conclusions or parameter estimates of any projection methodology. While we have tested the models in different periods and found they broadly hold true, large changes in bond markets (e.g. GFC type shocks) may lead to different results than those shown above.

Regulated versus unregulated considerations

The formula above is most useful in unregulated contexts when a company wants a best estimate of the yield curve at a future point in time.

The considerations for a regulator in statutory classes are somewhat different:

- If a regulator allows choice in setting discount rates, this may allow insurers more scope to vary their premiums via assumption setting, which may not be desirable for the regulator. However this can be controlled in other ways (e.g. reviewing the construction of floor and ceiling premiums).
- If a regulator allows a formula such as the one derived above, and this gives a higher premium than the expectations hypothesis, then the regulator may be permitting a form of arbitrage (if we ignore the costs of hedging). This is because an insurer can lock in the pricing basis of the expectations hypothesis via hedging. This would represent a transfer from policy holders to insurers, which may not be desirable. The converse is true if the formula gives a lower premium than the expectations hypothesis. Any approach should be consistent across all yield curve scenarios.

These points seem to suggest that a regulator should require prices based on the current forward rates (or expectations hypothesis) for premium filings, plus the cost of hedging. Insurers could then decide to hedge or accept the interest rate risk.

The cost of hedging can be approximated by the credit spread associated with borrowing a premium amount from the market today, to be repaid upon collection of actual premiums. As at June 2014 these spreads were about 0.6% per annum for an A-rated company. However such borrowing costs should receive a tax offset, reducing the cost to about 0.4% per annum. Also requiring consideration is the additional management time to oversee and arrange the hedge and the uncertainty about how much premium income will be written by the insurer, both of which must be included in any cost estimate. For a six month gap from price setting to premium collection, these costs might add up to nearly 0.25% of the premium. However, it may be possible to reduce these costs by hedging with derivatives. These costs can be thought of as the price of interest rate risk in a regulated setting, which must either be borne by the insurer or policyholder.

We summarise our findings and recommendations in the next section.

6. Conclusions

The discounted cash flow models used in practice

Myers-Cohn and IRR models are the two main discounted cash flow techniques used for setting premiums. However the IRR model has many practical advantages and this is the reason that the IRR model is the dominant pricing model used in practice.

Considerations for determining discount rates for the IRR model

For the IRR model the key discount rate is the ROE. For non-regulated insurance markets insurers can choose any ROE they want, constrained only by the competitive realities of the market place, the price sensitivities of consumers and their own internal hurdles.

For regulated markets the regulator's objective to set fair yet technically sound premiums means that formal discount rate models are needed. The CAPM is the most common asset pricing model used in most regulatory contexts. When parameterising the CAPM consideration needs to be given to the possibility of negative correlations between the risk free rate and the equity risk premium. Keeping the equity risk premium constant over time, but adjusting the risk free rate in response to yield curve movements may give incorrect results.

The relationship between ROE and TSR is important and should be considered in any holistic review of discount rates for an IRR model.

If CAPM is used to determine a ground-up return on equity then the return on equity determined using this approach will approximate that required in a freely competitive market. This return on equity should be considered a lower bound that a regulator may want to impose on insurers. In practice the insurance market will have imperfections with associated costs, for which the regulator may (some say should) allow adjustments. These imperfections include the risk aversion of insurers to diversifiable business risks and various factors contributing to franchise value (e.g. regulatory costs from having capital tied up within insurers).

Considerations for determining discount rates for the Myers-Cohn model

For the Myers-Cohn model typically the most contentious discount rate is the risk adjusted rate for claims and expenses. If a CAPM approach is used for setting this discount rate the premiums determined should be increased to allow appropriately for market frictions. Other approaches can be used. For example if an insurer is risk averse to non-market diversifiable risk then a certainty equivalent method could be used. Under this approach the value of claim and expense payments would be scaled up (e.g. by selecting a negative liability beta) to reflect the risk aversion of the insurer before being discounted using a risk free discount rate. Notwithstanding these possible adjustments, many practitioners take issue with some of the assumptions underlying the Myers-Cohn approach, as applied in the past.

Yield curve projection

If a company is prepared to hedge their yield curve exposure (and incur any associated costs), then it is appropriate to use the assumptions implied by the expectations hypothesis. Outside of this, the expectations hypothesis can lead to small inaccuracies when projecting future yields, with a general bias towards over-estimating short term rates. The bias is greatest when the yield curve is steep. Among simple estimators, a "static" approach using the current spot yield curve is generally superior to the expectations hypothesis. Across most yield curve shapes, there is little to distinguish these two simple estimators. The static approach is notably better if the yield curve is steep and upward sloping.

We can improve on both the static assumption and the expectations hypothesis, by taking a weighted average of the expectations hypothesis and a slowly evolving average yield curve,

$$f_t(s+\delta) = \beta_{t,\delta} f_{t+\delta}(s) + \left(1-\beta_{t,\delta}\right)\bar{f_t}$$

With:

$$\begin{aligned}
s &= \text{Current time} \\
\delta &= \text{Projection interval} \\
t &= \text{Term required at projection date} \\
\beta_{t,\delta} &= \frac{0.6143}{1+0.6364\ \times\delta\ \times 0.4166^t} + \frac{1-0.6143}{1+1.0288\times\delta\ \times 1.0653^t} \\
\overline{f_t} &= \text{Current estimate of medium term yield curve (see Appendix B)}
\end{aligned}$$

This approach tends to stabilise the projection of the yield curve and more accurately estimates future forward rates (and hence future spot rates). Further, we believe that the curve as at 31 December 2013 (and shown in Appendix B) is a reasonable ongoing choice, as at the time of writing.

The beta values vary most by projection time (delta) rather than term (t). The approximate beta values for some key delta values are shown below.

Delta (years)	Beta guide
0.25	0.9
0.50	0.8
0.75	0.7
1.00	0.6

Table 9: Broad beta indications for typical projection delays (delta)

However it should be noted that these are indications only for the practitioner. A more detailed split by delta and term is given in Appendix B (or can be approximated by the beta formula given above).

If discount rates in regulated pricing contexts are set based on the expectations hypothesis, then allowance should be made in insurance pricing for the cost for insurers to hedge this risk and lock in those rates (whether or not they actually choose to hedge this risk).

Other Findings

Other notable findings made in the course of preparing this paper are:

- Using spot rates rather than a single discount rate may affect ROE estimates in IRR models, particularly when yield curves are steep. Users of these models should be alert to this sensitivity.
- Changing from an investment strategy that involves some modest risk (e.g. some high quality credit exposure and modest mismatching) to a replicating portfolio will reduce capital requirements for a long tail class of business. But it will reduce the projected ROE by up to 1% per annum.

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Appendix A – IRR Model Details

The following tables show our CTP and Motor assumptions and results

Sensitivity	Base value	Alternative values	ROE (IRR)	Profit Margin	Net loss ratio (inf. & disc.)	Gross expense ratio	Capital Base to GWP	Net Accg Loss Ratio	ROE (NPAT / Net Assets)	Insurance Margin
Base Scenario			9.4%	12.6%	75%	11.5%	134%	92%	9.4%	12.7%
Net claim cost per $100 gross premium	75	67	16.0%	20.0%	67%	11.2%	121%	83%	16.2%	20.9%
		82	5.4%	5.2%	82%	11.9%	147%	101%	4.5%	4.4%
Gross expense rate	12%	10%	10.6%	14.6%	75%	9.5%	134%	92%	10.8%	14.9%
		14%	8.3%	10.6%	75%	13.5%	134%	92%	8.1%	10.5%
Net reinsurance cost	2%	0%	10.1%	14.4%	74%	11.7%	139%	91%	10.3%	13.8%
		4%	8.1%	10.5%	77%	11.6%	137%	95%	7.9%	10.4%
Inflation	7%	6%	12.0%	16.0%	71%	11.4%	129%	88%	12.5%	16.9%
		8%	7.2%	9.0%	78%	11.7%	139%	97%	6.6%	8.2%
Discount rate	3.1%	2.6%	7.9%	11.0%	76%	11.6%	136%	93%	7.7%	10.6%
		3.6%	10.9%	14.1%	73%	11.5%	132%	92%	11.2%	14.7%
Excess investment return	0.5%	0.0%	8.2%	12.6%	75%	11.5%	134%	92%	8.0%	10.8%
		1.0%	10.6%	12.6%	75%	11.5%	134%	92%	10.8%	14.5%
Equities share for investments backing technical provisions	0%	20%	9.7%	12.6%	75%	11.5%	154%	92%	9.8%	15.7%
Equities share for investments backing shareholders' funds	0%	50%	9.8%	12.6%	75%	11.5%	151%	92%	10.0%	12.7%
Outstanding claims risk margin (APRA)	10%	6%	9.9%	12.6%	75%	11.5%	131%	92%	10.1%	12.7%
		14%	8.9%	12.6%	75%	11.5%	137%	92%	8.9%	12.7%
Premium liability risk margin (APRA)	14%	10%	9.6%	12.6%	75%	11.5%	133%	92%	9.7%	12.7%
		18%	9.2%	12.6%	75%	11.5%	135%	92%	9.3%	12.7%
Outstanding claims risk margin (accounts)	20%	10%	9.3%	12.6%	75%	11.5%	134%	90%	9.3%	14.1%
		30%	9.5%	12.6%	75%	11.5%	134%	95%	9.7%	11.3%
Target capital ratio (Capital Base to PCA)	200%	175%	10.4%	12.6%	75%	11.5%	117%	92%	10.7%	12.7%
		225%	8.6%	12.6%	75%	11.5%	151%	92%	8.5%	12.7%
APRA average asset risk charge - cash and fixed interest	2.8%	1.8%	9.6%	12.6%	75%	11.5%	130%	92%	9.7%	12.7%
		3.8%	9.2%	12.6%	75%	11.5%	139%	92%	9.2%	12.7%
APRA average asset risk charge - equities	26.6%	20%	10.1%	12.6%	75%	11.5%	145%	92%	10.2%	12.7%
		30%	9.7%	12.6%	75%	11.5%	155%	92%	9.8%	12.7%
R/I counterparty default charge	4%	2%	9.4%	12.6%	75%	11.5%	134%	92%	9.5%	12.7%
		6%	9.4%	12.6%	75%	11.5%	134%	92%	9.4%	12.7%
ICRC per $100 gross premium	0	10	8.5%	12.6%	75%	11.5%	154%	92%	8.4%	12.7%
Payment pattern (undisc. mean term)	4.1 yrs	3.6 yrs	10.9%	13.8%	73%	11.5%	120%	89%	11.2%	14.1%
		4.6 yrs	8.2%	11.4%	76%	11.6%	148%	96%	8.1%	11.2%

Table 10: CTP – IRR Model Inputs and Results, Base and Alternative Scenarios

Sensitivity	Base value	Alternative values	ROE (IRR)	Profit Margin	Net loss ratio (inf. & disc.)	Gross expense ratio	Capital Base to GWP	Net Accg Loss Ratio	ROE (NPAT / Net Assets)	Insurance Margin
Base Scenario			30.1%	7.3%	69%	18.8%	29%	73%	30.1%	9.8%
Net claim cost per $100 gross premium	68	62	75.5%	13.3%	62%	18.6%	27%	66%	73.6%	17.2%
		75	7.4%	1.3%	75%	19.1%	31%	80%	7.4%	2.4%
Gross expense rate	19%	17%	39.9%	9.3%	69%	16.8%	29%	73%	39.7%	12.3%
		21%	21.6%	5.3%	69%	20.8%	29%	73%	21.7%	7.3%
Net reinsurance cost	3%	0%	32.5%	9.6%	71%	19.5%	33%	76%	32.5%	10.3%
		6%	16.5%	4.1%	72%	19.0%	30%	77%	16.6%	5.9%
Inflation	3%	2%	33.8%	8.0%	68%	18.8%	29%	72%	33.7%	10.7%
		4%	26.6%	6.6%	69%	18.9%	29%	74%	26.7%	8.9%
Discount rate	2.5%	2.0%	29.8%	7.1%	69%	18.9%	29%	73%	29.8%	9.8%
		3.0%	30.3%	7.5%	68%	18.8%	29%	73%	30.3%	9.8%
Excess investment return	0.25%	0.0%	30.1%	7.3%	69%	18.8%	29%	73%	30.1%	9.8%
		0.5%	30.1%	7.3%	69%	18.8%	29%	73%	30.1%	9.8%
Equities share for investments backing technical provisions	0%	20%	24.5%	7.3%	69%	18.8%	34%	73%	24.4%	9.8%
Equities share for investments backing shareholders' funds	0%	50%	25.9%	7.3%	69%	18.8%	33%	73%	25.8%	9.8%
Outstanding claims risk margin (APRA)	6%	4%	30.2%	7.3%	69%	18.8%	29%	73%	30.2%	9.8%
		8%	30.0%	7.3%	69%	18.8%	29%	73%	30.0%	9.8%
Premium liability risk margin (APRA)	8%	4%	33.8%	7.3%	69%	18.8%	28%	73%	33.7%	9.8%
		12%	27.2%	7.3%	69%	18.8%	30%	73%	27.2%	9.8%
Outstanding claims risk margin (accounts)	12%	6%	30.1%	7.3%	69%	18.8%	29%	73%	29.8%	9.8%
		18%	30.1%	7.3%	69%	18.8%	29%	73%	30.3%	9.8%
Target capital ratio (Capital Base to PCA)	200%	175%	36.4%	7.3%	69%	18.8%	25%	73%	36.4%	9.8%
		225%	25.7%	7.3%	69%	18.8%	33%	73%	25.7%	9.8%
APRA average asset risk charge - cash and fixed interest	2.8%	1.8%	31.5%	7.3%	69%	18.8%	28%	73%	31.5%	9.8%
		3.8%	28.5%	7.3%	69%	18.8%	30%	73%	28.5%	9.8%
APRA average asset risk charge - equities	26.6%	20%	27.2%	7.3%	69%	18.8%	31%	73%	27.1%	9.8%
		30%	25.1%	7.3%	69%	18.8%	34%	73%	25.1%	9.8%
R/I counterparty default charge	4%	2%	30.6%	7.3%	69%	18.8%	29%	73%	30.6%	9.8%
		6%	29.5%	7.3%	69%	18.8%	29%	73%	29.5%	9.8%
Non R/I counterparty default charge	8%	6%	30.2%	7.3%	69%	18.8%	29%	73%	30.2%	9.8%
		10%	30.0%	7.3%	69%	18.8%	29%	73%	30.0%	9.8%
Non R/I recoveries (% of gross)	30%	25%	30.1%	7.3%	69%	18.8%	29%	73%	30.2%	9.8%
		35%	30.0%	7.3%	69%	18.8%	29%	73%	30.0%	9.8%
ICRC per $100 gross premium	1	10	16.8%	7.3%	69%	18.8%	48%	73%	16.8%	9.8%
Payment pattern (undisc. mean term)	0.58 yrs	0.5 yrs	31.9%	7.3%	69%	18.8%	28%	73%	31.9%	9.8%
		0.84 yrs	26.0%	7.4%	68%	18.8%	33%	74%	25.6%	10.0%

Table 11: Motor– IRR Model Inputs and Results, Base and Alternative Scenarios

Other Assumptions

The other implicit assumptions underlying our IRR model are:

- All policies are for one year and incept on the same date.
- It is an annual model, with most cash flows occurring mid-year on average.
- A Deferred Reinsurance Expense (DRE) asset is established at time 0. This cost is expensed over the first projection year.
- CHE is not permitted for tax calculations, as per Australian tax law.
- GST related effects are ignored. All assumptions are implicitly net of GST and Input Tax Credits (ITC).
- There are no premiums due. All premiums are paid at time 0.
- The model's LAGIC capital charges are estimates, based on actual industry charges.
- Reinsurance and third party ("non-reinsurance") recoveries are assumed to be received at the same time as gross claims are paid. This simplification could be relaxed in actual implementations of the model.
- All policies are effectively extinguished after 20 years.
- The "Net Reinsurance Cost" is the reinsurance premium less expected reinsurance recoveries, expressed as a percentage of the gross premium
- The excess investment return is the constant margin over the risk free rate which we assume is earned across the insurer's cash and fixed interest investments. This could come from taking credit risk (e.g. investing in semi government and corporate bonds), or from duration positioning or trading.
- For CTP we assumed wage inflation of 3.5% per annum and superimposed inflation of 3.5% per annum.
- For Motor we assumed price inflation of 2.5% per annum and nil superimposed inflation.

Appendix B – Details on yield curve projection

We recommend a yield curve projection formula as follows:

$$f_t(s+\delta) = \beta_{t,\delta} f_{t+\delta}(s) + (1-\beta_{t,\delta})\bar{f_t}$$

Where the mean yield curve $\bar{f_t}$ has been slowly decreasing over time:

Term	Forward rate			Annual change
	Dec-07	Dec-10	Dec-13	
0	4.56	4.23	3.90	-0.11
0.5	4.54	4.18	3.82	-0.12
1	4.58	4.16	3.74	-0.14
1.5	4.71	4.23	3.75	-0.16
2	4.89	4.38	3.87	-0.17
2.5	5.07	4.56	4.05	-0.17
3	5.22	4.71	4.20	-0.17
3.5	5.33	4.82	4.31	-0.17
4	5.42	4.91	4.40	-0.17
4.5	5.49	4.98	4.47	-0.17
5	5.54	5.03	4.52	-0.17
5.5	5.59	5.08	4.57	-0.17
6	5.63	5.12	4.61	-0.17
6.5	5.68	5.17	4.66	-0.17
7	5.72	5.21	4.70	-0.17
7.5	5.77	5.26	4.75	-0.17
8	5.82	5.31	4.80	-0.17
8.5	5.87	5.36	4.85	-0.17
9	5.92	5.41	4.90	-0.17
9.5	5.97	5.46	4.95	-0.17
10 and later	5.97	5.51	5.00	-0.17

Table 12: Yield Curves

And the $\beta_{t,\delta}$ term equals

$$\beta_{t,\delta} = \frac{0.6143}{1+0.6364 \times \delta \times 0.4166^t} + \frac{1-0.6143}{1+1.0288 \times \delta \times 1.0653^t}$$

At the time of writing this paper we recommend adopting the December 2013 yield curve as the long term average, without further adjustment. We consider it inappropriate to allow for further reductions in this average (as observed in the trend over the last 20 years) from their current low levels.

The table below gives the empirical $\beta_{t,\delta}$ found in our analysis, and their relative confidence. We have used these to estimate the formula above.

	Term (t)																													
Projection time (δ)	**0.3**	**0.5**	**0.8**	**1**	**1.3**	**1.5**	**1.8**	**2**	**2.3**	**2.5**	**2.8**	**3**	**3.3**	**3.5**	**3.8**	**4**	**4.3**	**4.5**	**4.8**	**5**	**5.3**	**5.5**	**5.8**	**6**	**6.3**	**6.5**	**6.8**	**7**	**7.3**	**7.5**
0.25	90	90	93	94	91	88	87	85	86	87	87	88	90	90	91	91	92	92	93	92	93	93	93	92	92	92	91	92	91	91
0.5	81	80	82	82	80	78	77	76	75	75	76	78	79	80	80	82	82	83	83	83	83	84	83	83	82	83	83	83	82	82
0.75	75	73	71	70	69	68	67	68	68	68	71	72	73	75	75	75	77	77	77	75	77	77	77	76	76	76	75	75	75	74
1	68	59	55	53	54	54	57	55	57	58	62	64	65	65	67	67	69	71	68	70	69	68	70	69	67	66	68	67	66	66
1.25	57	49	47	48	49	52	53	56	57	60	61	64	64	67	67	68	69	69	69	69	69	71	70	68	69	69	68	68	67	67
1.5	44	35	37	43	47	52	56	60	61	65	67	68	68	70	71	71	70	71	71	70	70	70	70	71	69	70	69	69	69	68
1.75	28	28	32	39	45	51	56	60	64	68	70	72	74	75	76	74	75	76	75	74	75	76	76	75	74	74	74	76	75	75
2	22	21	28	36	45	52	57	65	70	74	76	78	81	83	81	83	84	85	85	84	85	85	84	87	86	87	86	88	87	88
2.25	12	20	25	38	45	51	60	64	70	75	78	78	82	82	82	83	79	80	80	78	78	78	77	77	76	76	74	73	73	74
2.5	13	22	33	41	49	56	63	67	73	75	80	80	79	80	78	78	75	76	72	73	71	69	69	66	66	65	63	62	61	57
2.75	10	31	45	55	62	65	70	75	76	79	79	78	80	77	74	71	69	68	64	62	61	58	58	57	53	52	50	48	43	41
3	17	39	48	55	57	62	61	61	62	60	62	60	59	57	54	54	52	50	49	47	45	45	43	40	38	36	34	30	29	

Key for colour scale – width of confidence interval:

15%	30%	45%	60%	75%	90%	105%	120%	135%	145%

Table 13: Estimated β for various Term and projection time regressions using Australian bond data. Numbers are percentages. Colour indicates width of confidence interval

Methodology for Beta estimation

We applied the following approach to estimating the β:

- Data used was Australian instantaneous forward rates from July 1996 through to June 2014, as described in Finlay and Chambers (2008). We had one record for each end of month in that period.
- For each combination of δ and t in the table above, we performed a simple linear regression of the forward rates $f_t(s+\delta)$ as a function of $f_{t+\delta}(s)$ (plus intercept). The estimated parameter for $f_{t+\delta}(s)$ from this regression of is then the central estimate of $\beta_{t,\delta}$
- To calculate the confidence intervals, we performed bootstrap replications of the regression, randomly selecting blocks of 12 months from the dataset (sampling with repetition). The regression is then performed again and a distribution around $\beta_{t,\delta}$ derived, from which it is possible to determine confidence intervals. The decision to sample blocks rather than individual months gives a better estimate of uncertainty as it recognises the auto-correlation between successive months.