



Institute of Actuaries of Australia

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Integrating Financial and Demographic Longevity Risk Models: An Australian Model for Financial Applications

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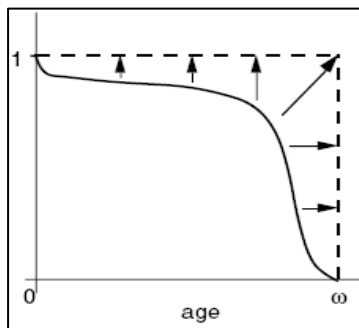
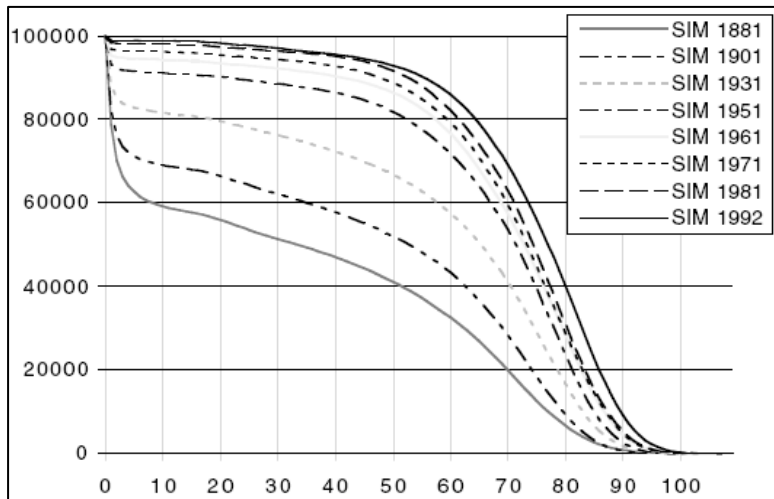
Outline

1. Longevity and Mortality Risk
2. Risk Management Strategies
3. Longevity Risk Securitisation
4. Models for Mortality
5. The Proposed Mortality Model
6. The Longevity Bond
7. The Pricing Model
8. Data and Assumptions
9. Results
10. Conclusion

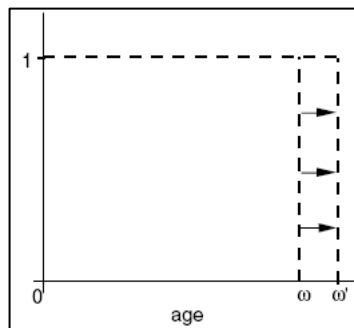


1. Longevity and Mortality Risk

Longevity improvement has seen the survival curve* shift in 2 ways:



Rectangularisation



Expansion

* Survival Functions for Italian Male Populations (1881-1992) (Pitacco, 1992)

There is increasing retail exposure to longevity risk..

- Longevity is improving with greater variability
- OECD Male 60-64 Labour Participation:
 - 60-90% (1970s) to 20-50% (today)
- Shift to DC Superannuation
- 3.4m Australians will suffer from insufficient income in retirement**

... and huge potential for investment in life annuities

- Australian Super Industry:
 - \$1,177b assets (Dec 2007)
 - 2/3 DC or Hybrids
- Australian Life Annuities:
 - \$3.9b assets (Dec 2007)

...though currently there are a number of constraints

- Supply/demand constraints (Purcal, 2006)
- Reinsurance:
 - Longevity is “toxic” (Wadsworth, 2005)

** AMP 2007 AMP Superannuation Adequacy Index Report, released January 2008



2. Risk Management Strategies

1. Avoidance

- Participating Annuities
- Reverse Mortgages

2. Retention

- **Capital Reserves**
- **Contingent Capital**

3. Transfer

- **Reinsurance**
- **Bulk Purchase Annuities**
- **Securitisation**

4. Hedging

- Natural Hedges
- Survivor Bonds
- Mortality Swaps
- Longevity Options and Futures



3. Longevity Risk Securitisation

Securitisation is a vehicle for risk transfer

- CDOs -late 1980s
- Insurance-Linked Securitization – USD 5.6b issued in 2006*
 - Insurance-Linked Bonds
 - Industry Loss Warranties
 - Sidecars
- Mortality Bond Issues (Vita I-III, Tartan, Osiris, 2003-2007)
- Survivor Bond Issues (BNP Paribas/EIB, 2004)

...with a number of benefits

- Improved capacity for risk transfer as tranching broadens appeal to investors
- Issue can be tailored to manage basis risk vs. moral hazard / info. asymmetry
- Diversification benefits for investors



4. Models for Mortality

a. Lee Carter (1992) Model and Extensions:

$$\ln[m(x,t)] = a_x + b_x k_t + \varepsilon_{x,t} \quad \text{where} \quad \sum_{\text{all } x} b_x = 1 \quad \text{and} \quad \sum_{\text{all } t} k_t = 0$$

- Single time-based index
- Assumes linear trend in k
- Difficult to incorporate risk-neutral pricing

b. Dahl (2004) Model and Extensions:

- Derived from finance theory (see Vasicek, 1977; Cox et al, 1985)

$$d\mu(t,x) = \alpha^\mu(t,x,\mu(t,x))dt + \sigma^\mu(t,x,\mu(t,x))dB_t$$

- Specific form based on Cox et al (1985):

$$d\mu(t,x+t) = (\beta^\mu(t,x) - \gamma^\mu(t,x)\mu(t,x+t))dt + \rho^\mu(t,x)\sqrt{\mu(t,x+t)}dB_t$$

- Readily adapted to risk neutral pricing
- Difficult to calibrate for pricing

c. Forward Rate Models:

- Model the dynamics of the forward mortality surface.
- Based on work by Heath, Jarrow and Morton (1992).

- Less developed in literature than short rate models

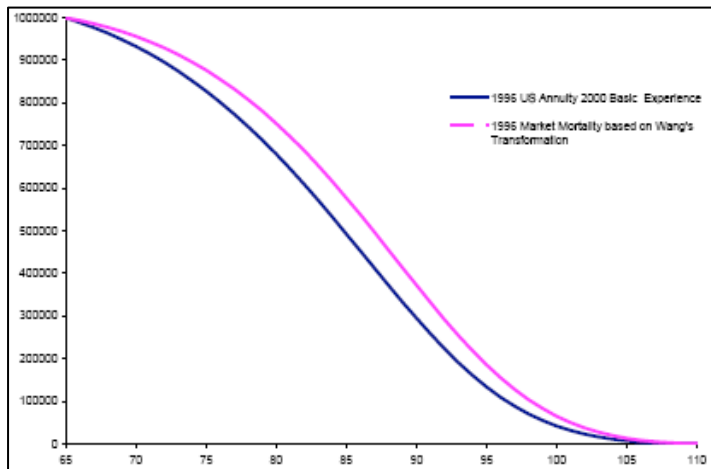


4. Models for Mortality

a. Lee Carter (1992) Model and Extensions:

- Pricing uses the Wang (1996, 2000, 2002) transform \rightarrow shifts the survival curve using fixed 'price of risk', λ :

$$F^*(t) = \Phi[\Phi^{-1}(F(t)) - \lambda]$$



Subject to criticism as it does not incorporate varying λ over age and time*

- Denuit, Devolder and Goderniaux (2007) using stochastic mortality, following Lin and Cox (2005) (deterministic).

* Cairns et al (2006), and Bauer and Russ (2006)



5. The Proposed Model

i) A Multivariate Mortality Process

- For lives at time t , initially aged x , the mortality rate $\mu(x,t)$ is given by:

$$d\mu(x,t) = (a(x+t) + b)\mu(x,t)dt + \sigma\mu(x,t)dW(x,t) \text{ for all } x.$$

- This falls within the Dahl (2004) family of models.
- To incorporate dependence, we introduce a M.V. random vector $\underline{dW}(t)$, length N :

$$\underline{dW}(t) = \Delta \underline{dZ}(t),$$

with each element

$$dW(x,t) = \sum_{i=1}^N \delta_{x,i} dZ_i(t) \text{ for all } x.$$

- Where $\underline{dZ}(t)$ is a random vector of independent B.M. of length N ; and Δ is a $N \times N$ matrix of constants, such that:

$$\begin{bmatrix} dW(x_1,t) \\ \vdots \\ dW(x_N,t) \end{bmatrix} = \begin{bmatrix} \delta_{11} & \dots & \delta_{1N} \\ \vdots & \ddots & \vdots \\ \delta_{N1} & \dots & \delta_{NN} \end{bmatrix} \begin{bmatrix} dZ_1(t) \\ \vdots \\ dZ_N(t) \end{bmatrix}$$

Note: the dimension of $\underline{dZ}(t)$ can be reduced using PCA.



5. The Proposed Model

i) A Multivariate Mortality Process

- The covariance matrix of $dW(t)$, Σ , has each element:

$$\begin{aligned} Cov(dW(x_n, t), dW(x_m, t)) &= \sum_{i=1}^N \delta_{ni} \delta_{im} Var(dZ_i(t)) \\ &= \sum_{i=1}^N \delta_{ni} \delta_{im} dt. \end{aligned}$$

such that

$$\Sigma = (\Delta \sqrt{dt}) (\Delta \sqrt{dt})'$$

- This gives the Cholesky decomposition of Σ

ii) Incorporating Age-Dependence

- Using PCA, decompose Σ into its eigenvectors (V), and eigenvalues (diagonal matrix T):

$$\begin{aligned} \Sigma &= VTV' \\ V\sqrt{T} &= \Delta\sqrt{dt} \end{aligned}$$

- Simulations of $dW(t)$ can be generated with the same dependence properties:

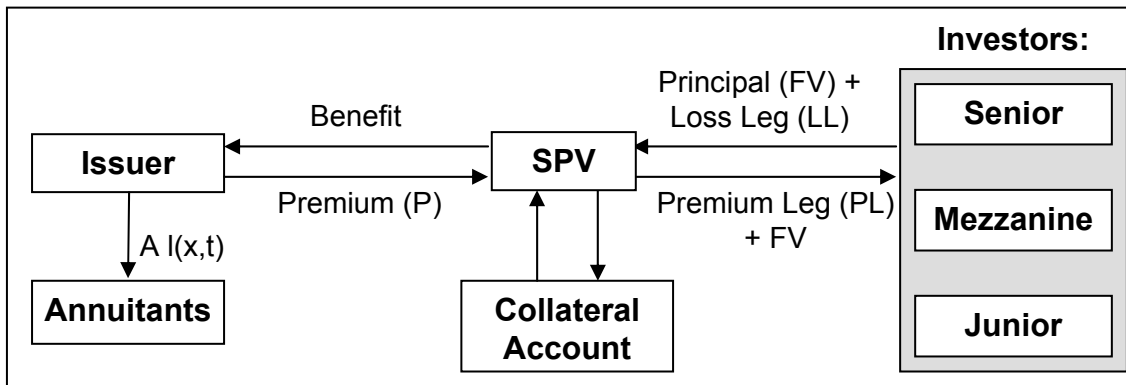
$$d\hat{W}(t) = V\sqrt{T}\underline{\eta}$$

# of Eigenvectors	% of Observed Variation
1	29.3%
5	69.8%
10	85.1%
15	92.4%
20	96.5%
25	97.1%
30	99.1%
31	99.5%
32	100.0%



6. The Longevity Bond

- The proposed longevity bond has the following structure:



- Both the PL and the LL are based on the percentage cumulative losses incurred on an underlying annuity portfolio:

$$CL(t) = \frac{\sum_{s=1}^t L(s)}{FV}$$

- Where the loss on the portfolio in each period is:

$$L(t) = \left(A \sum_{\text{all } x} l(x, t) - E \left[A \sum_{\text{all } x} l(x, t) \right] \right)^+ \\ \approx \left(A \sum_{\text{all } x} l(x, 0) {}_t p_x - A \sum_{\text{all } x} l(x, 0) {}_t \bar{p}_x \right)^+$$

Differs from existing models as:

- Based on multi-age portfolio
- Allows for variability in ${}_t p_x$
- Provides detailed analysis of longevity bond tranches



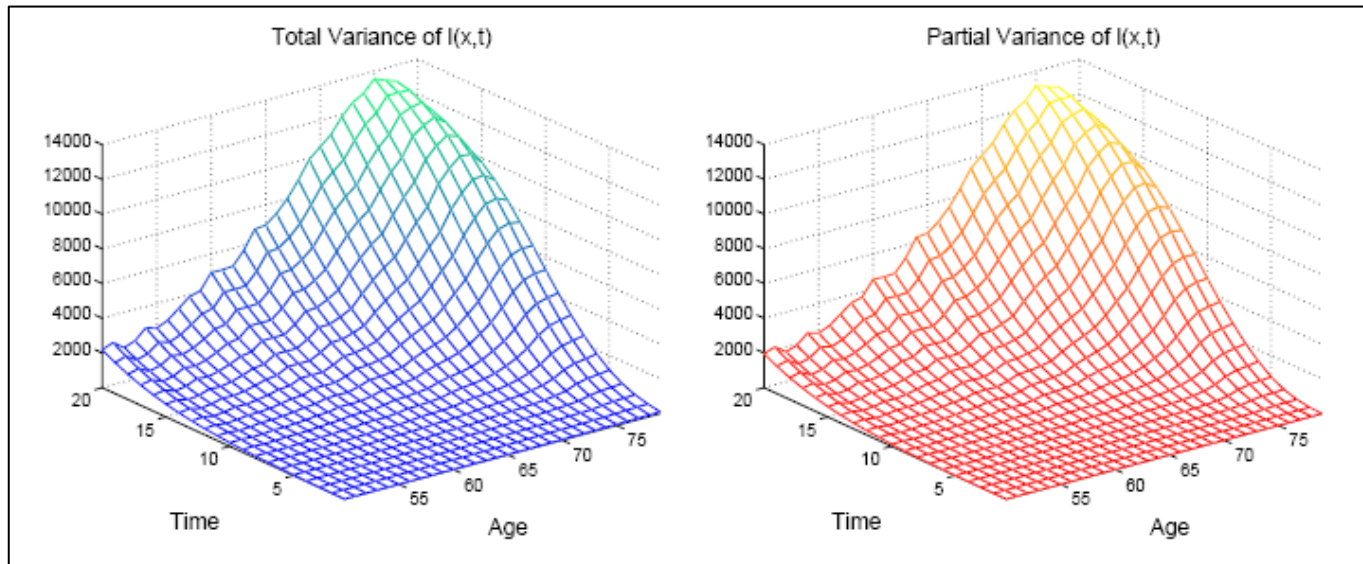
6. The Longevity Bond

- The total variance of the number of lives alive at time t , initially aged x is given by:

$$\text{Var}[l(x, t)] = E[\text{Var}[l(x, t) | {}_t p_x]] + \text{Var}[E[l(x, t) | {}_t p_x]].$$

- The first term gives the binomial variability in the portfolio given a fixed ${}_t p_x$ (the focus of Lin and Cox, 2005).
- The second is the variability due to changes in the mortality rate, which accounts for almost all of the portfolio variance:

Variability in ${}_t p_x$ accounts for almost all the variability in $l(x, t)$.





6. The Longevity Bond - Tranching

- Tranche losses are allocated by the cumulative loss on the portfolio. From this we can find the cumulative tranche loss:

$$CL_j(t) = \begin{cases} 0 & \text{if } L(t) < K_{A,j}; \\ CL(t) - K_{A,j} & \text{if } K_{A,j} \leq L(t) < K_{D,j}; \\ K_{D,j} - K_{A,j} & \text{if } L(t) \geq K_{D,j}, \end{cases}$$

where

$$CL(t) = \sum_{j=1}^J CL_j(t).$$

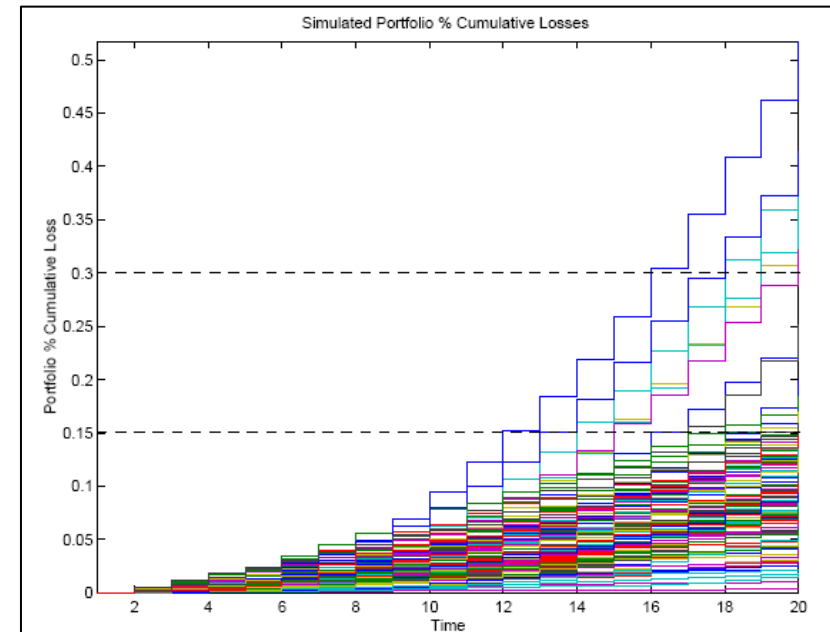
- The tranche loss as a percentage of its prescribed principal is given by:

$$TCL_j(t) = \frac{E[CL_j(t)]}{K_{D,j} - K_{A,j}}.$$

- The assumed tranche thresholds are:

Tranche j	$K_{A,j}$	$K_{D,j}$
1	0%	15%
2	15%	30%
3	30%	100%

Portfolio cumulative loss simulations





7. The Pricing Model

- The premium on tranche j , P_j^* , is set to equate the cashflows on the premium leg (PL_j), and the loss leg (LL_j):

$$PL_j = \sum_{t=1}^T P_j B(0, t-1) [1 - TCL_j(t-1)]$$
$$LL_j = \sum_{t=1}^T B(0, t) [TCL_j(t) - TCL_j(t-1)]$$

such that $PL_j(P_j^*) - LL_j(P_j^*) = 0$.

where:

- $B(0, t)$ is the price of a ZCB.

- $TCL_j(t)$ is the tranche % cum. loss at time t .

- Premiums need to be set under a risk-adjusted \mathbb{Q} mortality measure. Using the Cameron-Martin-Girsanov Theorem:

$$dW^{\mathbb{Q}}(x, t) = \sum_{i=1}^N \delta_{xi} (dZ_i(t) + \lambda_i(t)dt)$$
$$= dW(x, t) + \sum_{i=1}^N \delta_{xi} \lambda_i(t)dt.$$

and for all ages: $dW^{\mathbb{Q}}(t) = dW(t) + \Delta\lambda(t)dt$

where $\Delta\lambda(t)$ is a 'risk adjustment' that can differ for each age and time.

and the risk adjusted mortality process is:

$$d\mu^{\mathbb{Q}}(x, t) = \left(a(x+t) + b + \sum_{i=1}^N \delta_{xi} \lambda_i(t) \right) \mu^{\mathbb{Q}}(x, t)dt + \sigma \mu^{\mathbb{Q}}(x, t) dW(x, t)$$

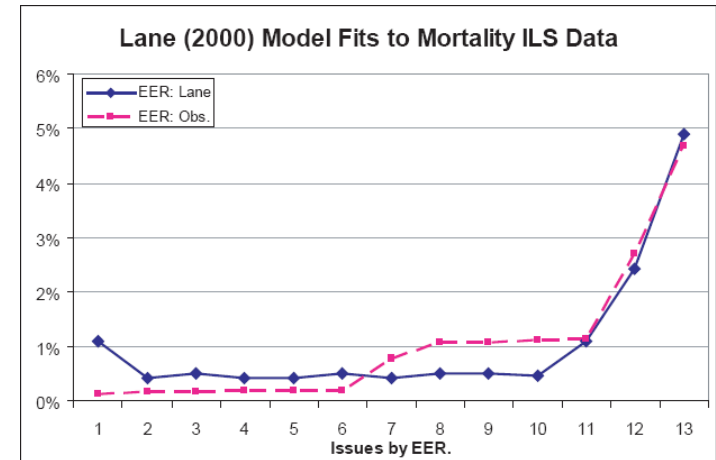


7. The Pricing Model

- However, the choice of \mathbf{Q} , and thus $\Delta\lambda(t)$ is not unique (like IR derivatives). It thus needs to be calibrated to market prices.

- These are approximated using an empirical model proposed by Lane (2000), fit to the price of 2007 mortality bond issues using non-linear least squares:

$\hat{P}_j^L = EL_j + EER_j$	
$EER_j = \gamma(PFL_j)^\alpha (CEL_j)^\beta$	
Parameter	2006-07 Mortality Bonds
γ	0.9980
α	0.8965
β	0.5034
X^2	0.04
χ^2_9 at 99%	2.09



- To facilitate calibration with limited data, simplifying assumptions are made on the risk adjustment:

$$\Delta\lambda(t) = \lambda^* \text{ where } \lambda^* = [\lambda^*, \dots, \lambda^*]'$$

So that for each x and t :

$$d\mu^{\mathbf{Q}}(x, t) = \left(a(x + t) + b - \sigma\lambda^* \right) \mu^{\mathbf{Q}}(x, t) dt + \sigma \mu^{\mathbf{Q}}(x, t) dW(x, t).$$

- λ^* is chosen so that:

$$P_j^{\lambda^*} = P_j^L$$

- As a result,

$$\lambda_j^* = f(P\hat{F}L_j, C\hat{E}L_j, \gamma, \alpha, \beta)$$



8. Data and Assumptions

Data

- Australian Population Mortality Data, ages 50-99, 1971-2004. Human Mortality Database (www.mortality.org)
- Australian Gov't Treasury Bill and Note rates: maturity 1-12 years. Bloomberg 24/09/2007.
- Market insurance-linked security data: 2007 issues. Drawn from Lane and Beckwith (2007).

Assumptions

Parameter	MLE: Male	MLE: Female
\hat{a}	-9.4398E-04	2.6993E-04
\hat{b}	0.1347	0.0608
$\hat{\sigma}$	0.0906	0.0873

Mortality process parameter estimates.

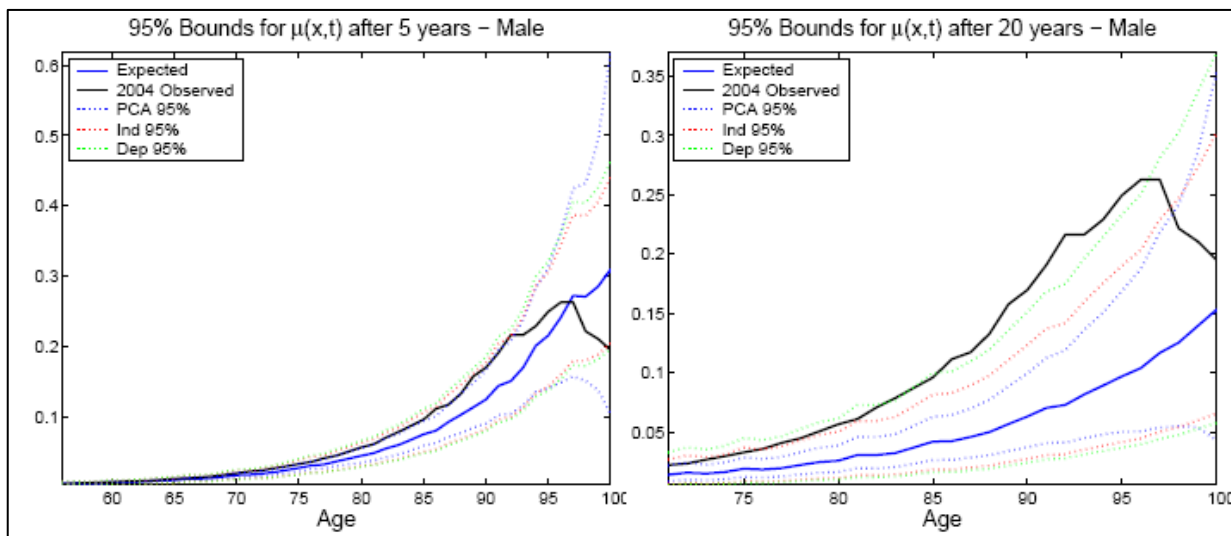
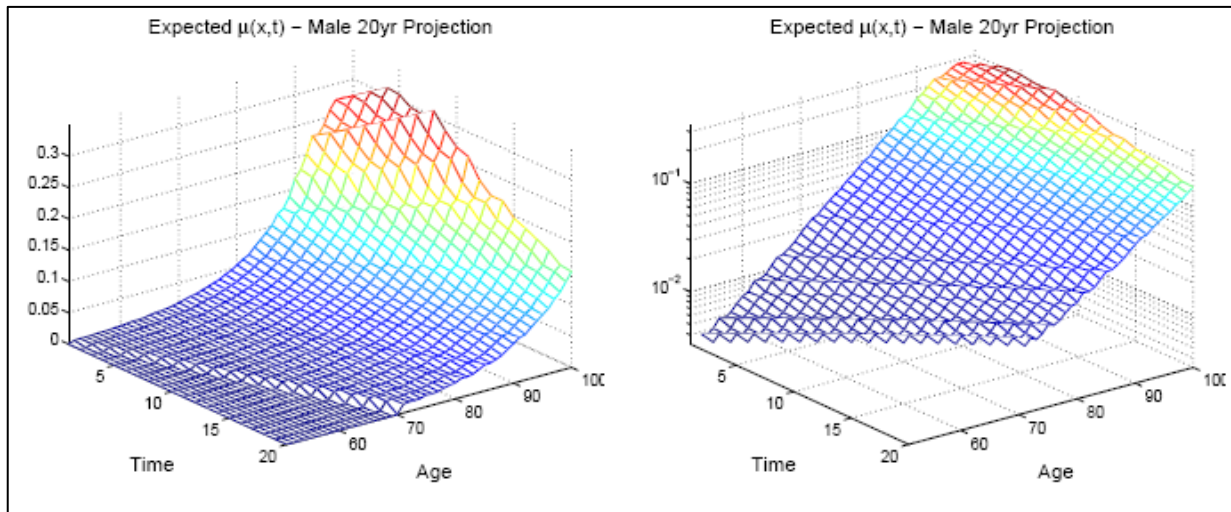
- $dW(t)$ is modeled under 3 assumptions of age dependence:

1. Perfect age independence.
2. Observed age dependence using PCA.
3. Perfect age dependence.

Proposed Longevity Bond Assumptions	
Bond Face Value:	$FV = \$750,000,000.$
Term to Maturity:	$T = 20$ years.
Payment Frequency:	Annually, for both premium and loss payments.
Number of Tranches:	$J = 3.$
Initial Age of Annuitants:	$x = 50, \dots, 79.$
Initial No. of Annuitants:	$n(x, 0) = 60,000.$ We assume this is evenly distributed between the 30 ages, with $l(x, 0) = 2,000 \forall x.$
Annuity Payments:	$A = \$50,000$ paid at the end of each year to each living annuitant.



9. Results – The Mortality Model



- Mortality expected to continue improving over the next 20 years (except ages 95-100)

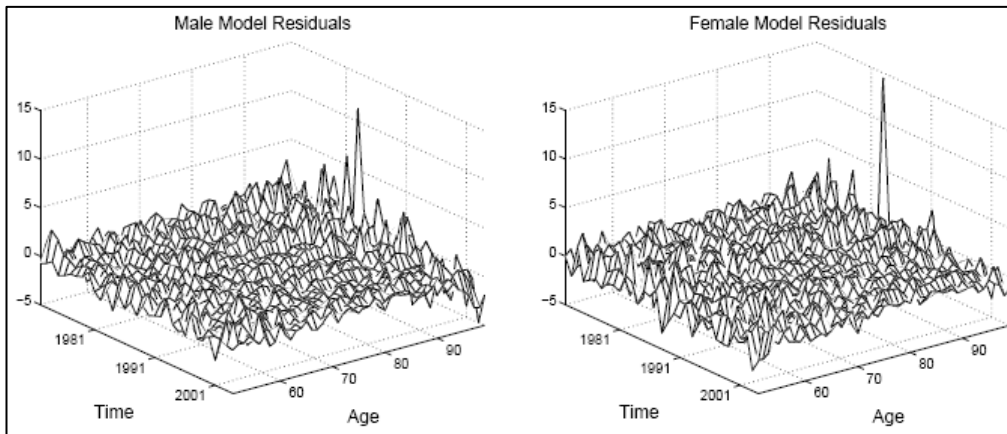
- Passage of cohort through time can be noted

- Volatility highest under perfect dependence, except at the oldest ages



9. Results – The Mortality Model

Analysis of fit shows the model accurately fits observed data



- Fitted residuals are normally distributed, without trend across age or time

	Male	Female
Mean	-3.12E-03	4.84E-08
Standard Error	0.024	0.025
Standard Deviation	0.972	1.000
Minimum	-6.320	-4.396
Maximum	6.277	14.459
Confidence Level(95.0%)	0.047	0.049

- Residuals are distributed with mean 0 and std dev 1

	Male			Female		
	<i>a</i>	<i>b</i>	σ	<i>a</i>	<i>b</i>	σ
<i>a</i>	5.53E-13	4.24E-11	-	5.13E-13	3.94E-11	-
<i>b</i>	4.24E-11	3.14E-09	-	3.94E-11	2.91E-09	-
σ	5.01E-11	2.84E-07	1.61E-09	-4.48E-11	-2.54E-07	1.50E-09

- Low asymptotic var/covar values suggest high confidence in each parameter estimate

X^2 Male	X^2 Female	χ^2_{388} at 99%
71.08	23.16	326.15

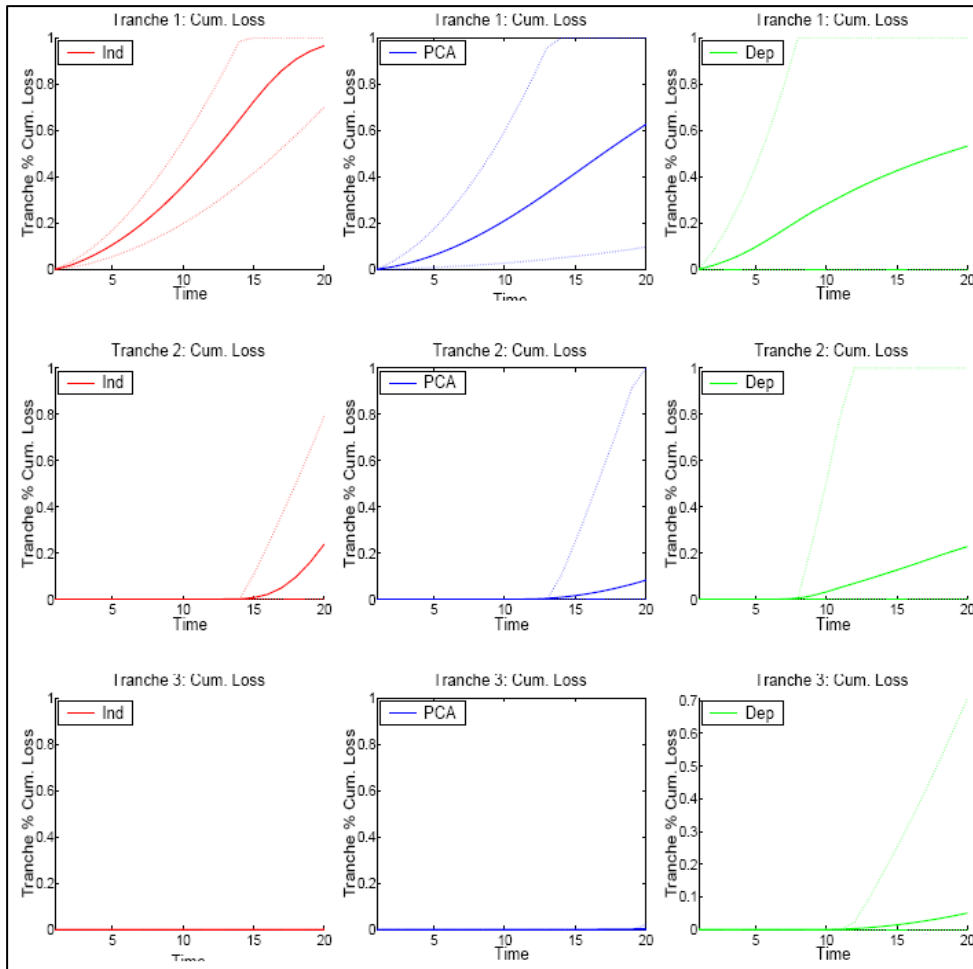
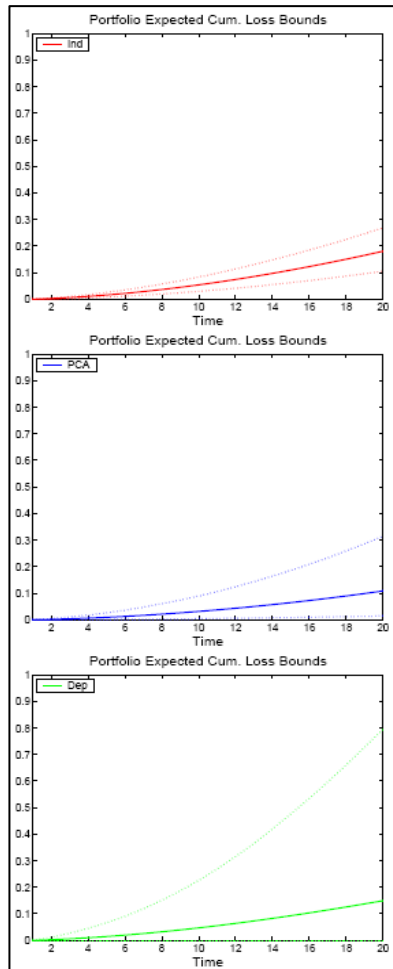
- Pearson's chi-square shows that the model fits the observed data very well



9. Results – The Longevity Bond

Portfolio expected cum. loss and 95% bounds

Tranche expected cum. loss and 95% bounds under 3 age-dependence assumptions.



- Variability of portfolio loss increases with age dependence

- Expected loss higher under dep., due to option-like payoff

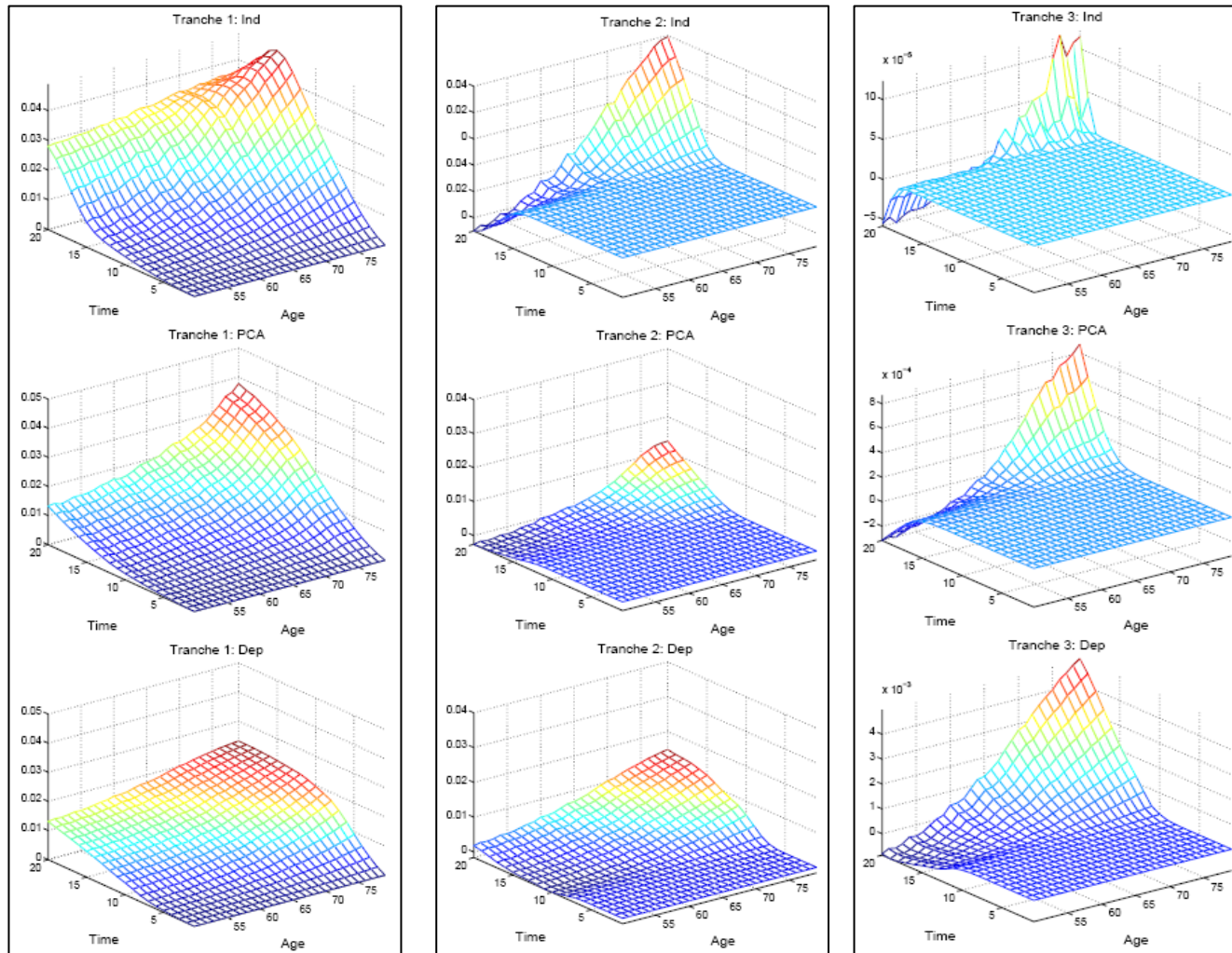
- Tranches losses are over/under-estimated due to dependence

- Dependence has a strong impact on the size of tranche expected losses



9. Results – The Longevity Bond

Tranche cumulative losses, disaggregated by age.

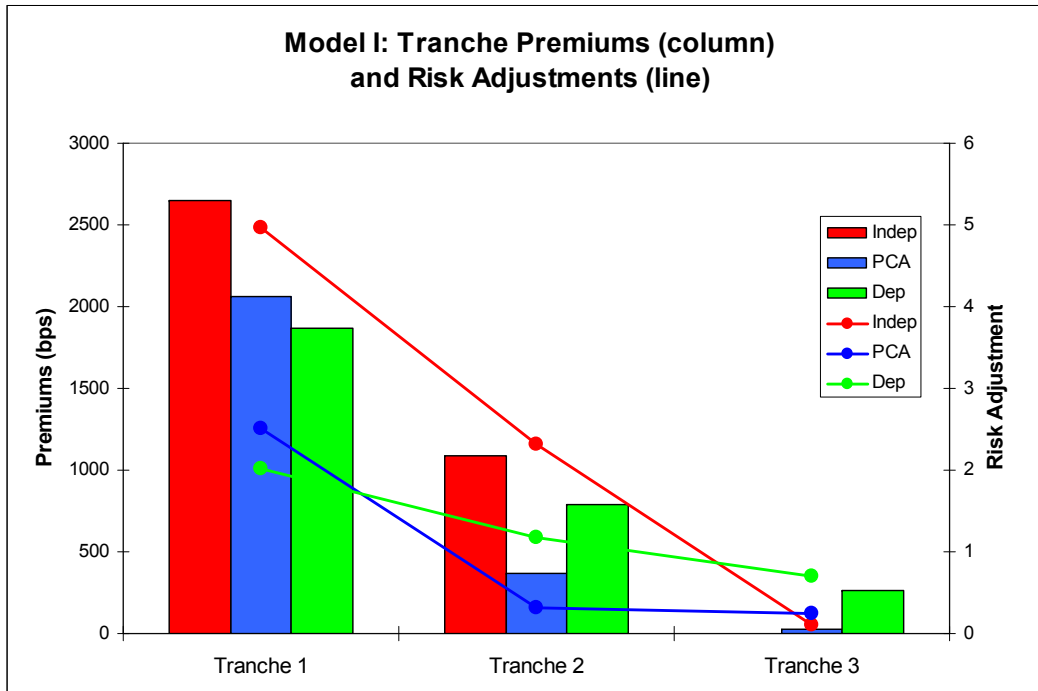


- Tranche losses not equally incurred across all ages

- Lower losses in young cohorts offset high losses in old cohorts



9. Results – The Pricing Model



- Calibrated tranche premiums and associated 'prices of risk' λ are consistent with risk averse investors

- λ sensitivities* show the model is very sensitive to the choice of data and the fit of the Lane (2000) model

λ sensitivities:

(observed dependence)

$$\lambda_j^* = f(P\hat{F}L_j, C\hat{E}L_j, \gamma, \alpha, \beta)$$

	Premium	λ_j^*	Sensitivities				
			$P\hat{F}L_j$	$C\hat{E}L_j$	γ	α	β
Tranche 1	2058	2.52	-	1.39	2.04	-	-3.52
Tranche 2	371	0.31	1.24	0.76	1.21	-1.76	-2.33
Tranche 3	31	0.25	0.29	0.17	0.31	-0.95	-0.79

*In the absence of a closed form



9. Results – The Pricing Model

Implications of Results

- Mortality can effectively be modelled as a dynamic, multi-age process.
- Tranched longevity bonds provide an effective vehicle for managing longevity risk.
- Dynamic mortality models are well suited to pricing longevity-linked securities.

Further Research

- Calibration of the risk-adjusted mortality process.
- Application of the proposed mortality model to a broader range of ages
- Alternative definitions for portfolio loss, eg. change in future annuity obligations (Sherris and Wills, 2007).



10. Conclusion

The Mortality Model

- Fit Dahl (2004) framework successfully to changes in mortality by age and time simultaneously
- Verified age-dependence as crucial
- Facilitated modelling of mortality-linked securities on multi-age portfolios

The Longevity Bond

- Investigated longevity-linked security on multiple ages
- Performed detailed analysis of the impact of tranching, under a range of age dependence assumptions

The Pricing Model

- Calibrated price of risk was consistent with risk averse investor with non-linear risk/return tradeoff
- 'Price of risk' able to vary by age and time, to incorporate range of investor sentiments



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Questions and Comments