Securitization of Mortality Risks in Life Annuities
Outline

• Introduce mortality-based securities
• Swiss Re mortality bond (December 2003)
• Other side of the “mortality tail”--longevity risk.
• Mortality risk bond
• Uncertainty in mortality forecasts.
• Potential expansion of the annuity market.
Insurance Securitizations

- Really asset securitizations.
- Mortality securitizations – pure hedge of mortality risk.
Securitization: Insurance linked bond

\[ B_t + D_t = 1,000C \]
Bond Coupons and Insurance Benefits

Mortality Bond Coupon

Insurance
Floating for fixed swaps

- Floating cash flow swapped for fixed.

- Swap dealer replaces SPC.
Equivalent swap structure

\[ B_t + D_t = x + y \]
Swiss Re’s Mortality Bond

• Issued December 2003, matures January 1, 2007
• No coupons at risk
• Priced to sell at par with coupon LIBOR + 1.35%
• Principal at risk during four calendar years
Swiss Re’s Mortality Bond

• Weighted average mortality rate for US, UK, France, Italy Switzerland

\[ q = \max(q_1, q_2, q_3, q_4) \text{ where } q_i = 2002 + i \text{ Index} \]

\[ q_0 = \text{2002 Index} \]

Maturity value = \[
\begin{cases} 
400,000,000 & \text{if } q \leq q_0 \\
400,000,000 \frac{1.5q_0 - q}{0.2q_0} & \text{if } q_0 < q \leq 1.5q_0 \\
0 & \text{if } q > 1.5q_0
\end{cases}
\]
Wang Transform Pricing

Mortality distribution implied by annuity prices

\[ F(t) = \Pr(T \leq t) \]

\( \Phi(u) \) is the standard normal cdf

\[ F^*(t) = \Phi[\Phi^{-1}(F(t)) - \lambda] \]

\( \lambda \) is the market price of risk
Market Price of Mortality Risk

- Realistic distribution 1996 IAM
- Observed male age 65 market prices

\[ F(t) = t q_{65,m} \]

\[ F^*(t) = \Phi[\Phi^{-1}(t q_{65,m}) - \lambda] \]

\[ \lambda = \lambda_{65,m} \]
Transform Pricing

• Annuity market: solve for MPR

Annuity price \((1 - e) = a_x^{(12)}\)

\[
= \frac{1}{12} \sum_{i=1}^{\infty} p_x^* d(0, i/12)
\]

\[
t p_x^* = 1 - \Phi[\Phi^{-1}(t q_x) - \lambda_x]
\]

• Bond market price

\[
V = F d(0, T) + \sum_{t=1}^{T} E^*[D_t] d(0, t)
\]
Female, age 65

![Graph showing mortality rates for females aged 65, with two lines representing 1995 US Buck Experience and 1995 Market Mortality based on the Wang Transform.]
Investor Coupon

\[
\frac{D_t}{1,000} = \begin{cases} 
C & \text{if } \ell_{x+t} \leq X_t \\
C + X_t - \ell_{x+t} & \text{if } X_t < \ell_{x+t} \leq C + X_t \\
0 & \text{if } \ell_{x+t} > C + X_t 
\end{cases}
\]

\[
= \text{Max}(C + X_t - \ell_{x+t}, 0) - \text{Max}(X_t - \ell_{x+t}, 0)
\]
Coupon Market Value

\[ E^* [D_t] d(0,t) = 1,000E^* \left[ \max(C + X_t - \ell_{x+t}, 0) \right] d(0,t) \]
\[ -1,000E^* \left[ \max(X_t - \ell_{x+t}, 0) \right] d(0,t) \]

Binomial distribution with parameters \( \ell_x, t^* P_x \)
Insurance Benefit

\[ B_t = 1,000C - D_t \]

\[ = 1,000 \begin{cases} 0 & \text{if } \ell_{x+t} \leq X_t \\ \ell_x - X_t & \text{if } X_t < \ell_{x+t} \leq C + X_t \\ C & \text{if } \ell_{x+t} > C + X_t \end{cases} \]

\[ E^* [B_t] = 1,000C - E^* [D_t] \]
## Strike levels

<table>
<thead>
<tr>
<th>Age Range</th>
<th>Change of Force of Mortality</th>
</tr>
</thead>
<tbody>
<tr>
<td>65–74</td>
<td>-0.0070</td>
</tr>
<tr>
<td>75–84</td>
<td>-0.0093</td>
</tr>
<tr>
<td>85–94</td>
<td>-0.0103</td>
</tr>
</tbody>
</table>

1996 IAM is the reference table

\[
X_t = \begin{cases} 
\ell_x t p_x e^{0.0070t} & \text{for } t = 1, \ldots, 10 \\
\ell_x t p_x e^{0.07} e^{0.0093(t-10)} & \text{for } t = 11, \ldots, 20 \\
\ell_x t p_x e^{0.163} e^{0.0103(t-20)} & \text{for } t = 21, \ldots, 30 
\end{cases}
\]
<table>
<thead>
<tr>
<th>Description</th>
<th>Male (65)</th>
<th>Female (65)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Market price of risk ($\lambda$)</td>
<td>0.1792</td>
<td>0.2312</td>
</tr>
<tr>
<td>Number of annuitants</td>
<td>10,000</td>
<td>10,000</td>
</tr>
<tr>
<td>Annuity annual payout per person</td>
<td>1,000</td>
<td>1,000</td>
</tr>
<tr>
<td>Total premium from annuitants</td>
<td>99,650,768</td>
<td>107,232,089</td>
</tr>
<tr>
<td>Improvement level age 65 - 74</td>
<td>-0.0070</td>
<td>-0.0070</td>
</tr>
<tr>
<td>Improvement level age 75 - 84</td>
<td>-0.0093</td>
<td>-0.0093</td>
</tr>
<tr>
<td>Improvement level age 85 - 94</td>
<td>-0.0103</td>
<td>-0.0103</td>
</tr>
<tr>
<td>Face value of straight bond</td>
<td>10,000,000</td>
<td>10,000,000</td>
</tr>
<tr>
<td>Face value of mortality bond</td>
<td>10,000,000</td>
<td>10,000,000</td>
</tr>
<tr>
<td>Coupon rate of straight bond and mortality bond</td>
<td>0.07</td>
<td>0.07</td>
</tr>
<tr>
<td>Annual aggregate cash flow out of SPC ($1000\text{C}'$)</td>
<td>700,000</td>
<td>700,000</td>
</tr>
<tr>
<td>Straight bond price</td>
<td>10,000,000</td>
<td>10,000,000</td>
</tr>
<tr>
<td>Mortality bond price</td>
<td>9,988,507</td>
<td>9,955,663</td>
</tr>
<tr>
<td>Reinsurance premium</td>
<td>11,493</td>
<td>44,337</td>
</tr>
</tbody>
</table>
Mortality projections

- Trend is improvement (lower q)
- Optimistic: Life expectancy will increase to 150 or 200 years.
- Pessimistic: Projecting trends is dangerous.
- Data situation in the US: inadequate, but improving.
Demand for longevity hedge

- Longevity risk in corporate benefit plans shifting to individuals
- Proposed reforms in social security shift longevity risk to individuals
- Life insurers have not responded to the opportunity
Final comments

- Securitization - a tool for managing longevity risks through mortality linked bonds or swaps.
- A market for mortality-based securities - could develop.
- Needed - Better collection and dissemination of data and research in mortality dynamics.