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Fresh Insights – Australian/NZ Home Lending Default Risk

T Gorst*

Abstract

By using Basel 2 (‘B2’) risk and capital data published by the four largest Australian banks since 2008, this paper analyses Australian/NZ home lending credit default risk by comparing recent default experience against an implied long-term ‘through the cycle’ default probability.

Adapted and updated for the AAJ from the original paper ‘APS330 Home Lending Data–Application and Insights’ presented to the Institute of Actuaries of Australia, 5th Financial Services Forum, 13–14 May 2010.

Keywords: APS330, Banking, Home Lending, Default Risk, Capital

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Introduction

The B2 framework provides a comprehensive standard for the measurement of, and minimum requirements for, capital adequacy across the international banking industry. It seeks to improve on existing rules by aligning regulatory capital requirements more closely to the underlying risks that banks face. In addition, the B2 framework promotes a more forward-looking approach to capital supervision, one that encourages banks to identify the risks they may face, today and in the future, and to develop or improve their ability to manage those risks.

Detailed B2 risk and capital data has been published by the four largest Australian banks since 2008. Whilst these reports have since been used to assist in bank performance analysis, particularly by investment banking research divisions, there has been very little in the way of research on the data published in these reports, particularly in the context of the Australian banking system. The purpose of this paper is to provide a contribution on this emerging body of research by focusing on the topical issue of Australian/NZ home lending system credit default risks. Some other research relevant to this topic is acknowledged below.

Written in the lead up to the US housing crash, Elul (2006) (Residential Mortgage Default—Elul, Ronel Federal Reserve Bank of Philadelphia Business Review, 3rd Quarter 2006, pp. 21–30 AN: 1132527) discusses the models that economists have developed to help us understand the default risk inherent in home mortgages and how default risk and house prices are related. He also applies these models to show how falling house prices would affect mortgage default rates today and explores the impact that rising default rates would have on financial institutions and other participants in the mortgage market.

On the importance of approach around developing credit rating systems and the setting of default probabilities, Waagepetersen presents an efficient method for extracting expert knowledge when building a credit risk rating system (A Statistical Modeling Approach to Building an Expert Credit Risk Rating System—Waagepetersen, Rasmus; Journal of Credit Risk, Summer 2010, v. 6, iss. 2, pp. 81–94 AN: 1125660). (Waagepetersen 2010)
On pro-cyclicality, Kruger and Festic (Determinants of Capital Adequacy and Pro-cyclicality - Kruger, Ulrich; Festic, Mejra; Bancni Vestnik, November 2008, v. 57, iss. 11, pp. 59-67 AN: 1014950) discuss the level and cyclicality of minimum required capital according to the Basel II Framework.

2 Sizing the Home Lending System (Australia/NZ)

At September 2010, the total Australian/NZ credit system stands at approximately $2.2 trillion AUD. $1.3 trillion, or around 60%, of total credit relates to borrowing secured by housing with around 70% finance for owner-occupied housing, and the remaining 30% for investment housing. Around $1.0 trillion (77%) of the combined Australian/NZ home lending volumes are being managed through the four major Australian banks and their NZ-owned franchises on an advanced internal ratings based (AIRB) credit management approach. It is on this part of the lending system that the richest credit information is disclosed through regular risk and capital (APS330) reporting. The AIRB home lending system has therefore become a good sample from which to better understand overall system home lending credit risk issues.

Over the last 20 years credit for housing has grown at around 14% pa, far outstripping the growth in business credit of around 6% pa over the same period (Figure 1). This trend has been particularly prominent over the last couple of years, with businesses actively seeking to reduce the gearing in their balance sheets. At the same time low interest rates, strong employment, recent government first home buyer stimulus and a resilient housing market have led to continued strong demand for housing credit.
3 The Model

3.1 Key Concepts

The following concepts are key to the analysis conducted in this paper:

Basel 2 (‘B2’)

Since 2008, the four large Australian banks are endeavouring to comply with all 3 Pillars of the B2 regulation. For these banks, the new B2 regime represents an upgrade to the far less dynamic B1 regime with which they previously complied. Pillar 1 ‘Minimum Capital Requirements’ outlines the regulatory capital ratio and risk weighted assets (RWA) to be calculated for credit, market and operational risk. Three options – foundation, standardised and advanced – are available to banks in the calculation of their RWAs. Pillar 2 ‘Supervisory Review’ ensures banks maintain adequate capital, including for risks that are not adequately covered under Pillar 1. Pillar 3 ‘Market Discipline’ covers the disclosures required around capital requirements, risk exposures and assessments (APS330 reporting).
Exposure at Default (EAD)

Represents the expected exposure of the bank in the event of default. Utilised EAD, often described as the lending balance, is that part of the loan that has been drawn down by the borrower, plus any un-drawn balances that the bank has committed to providing funds. The residual EAD is defined as being unutilised EAD. For example, when a borrower re-draws on their loan, they are increasing their utilised EAD, offset by an equivalent reduction in the unutilised EAD. Whilst the proportion of unutilised EAD does not impact on the capital requirement, it does impact on both leverage and product margins.

Default

A borrower is considered in default when (a) the bank considers the borrower unlikely to pay its credit obligations to the banking group in full, without recourse by the bank, and/or (b) the borrower is past due more than 90 days by a material amount on any credit obligation. For regulatory purposes, it is possible for a borrower to shift between being in default or non-default over the lifetime of their loan. A borrower can also be in default, even if the bank expects no loss.

Probability of Default (PD)

Defined in this paper as a long run ‘through the cycle’ probability that a non-defaulted loan moves to a default state in the next 12 months. It is therefore not a forecast, but rather a long-term best estimate assumption that is set to remain largely stable through the ups and downs of the economic cycle. The actual default percentage (D) refers to the proportion of EAD that is currently in default. Given the long-tailed nature of the credit loss distribution (illustrated in Figure 2), it should be more common to observe defaults below the long-term PD by virtue of the mode of the distribution falling somewhat behind the mean (PD).
The expected loss in the event the loan defaults is a key driver of credit risk capital required to be held against a home loan. The Australian Prudential Regulatory Authority (‘APRA’) require the Australian banks to calculate LGD assuming a downturn in the credit cycle, and further require the assumption be set at a minimum of 20% of the EAD. Notwithstanding slightly different applications of this floor between Australia and NZ, the internal LGD calculations of the major banks are currently sitting below the 20% floor.

Credit Provisions

Reserves that the bank chooses to hold in the balance sheet, and therefore take to their P&L, against future credit losses. Generally reserves set against identified loans where a credit loss is expected to be taken are defined as specific provisions. Reserves set against the possibility that non-defaulted loans are written off and incur a loss are known as collective provisions.
Credit Risk Weighted Assets are a measure defined by APRA in APS113 to assist banks in the calculation of unexpected credit loss capital ($C_{UL}$). This is the capital the regulator requires a bank to hold to ensure that 99.9% of the time it will have enough funds to cover any loan losses in excess of the long run expectation (illustrated in Figures 3 and 4). It is on this risk-adjusted measure of assets that APRA determines the credit risk capital required to be held.

### Tier 1 capital

The higher quality capital, such as retained earnings or shareholders’ equity, that is being held to cover banking capital requirements. APRA standards currently require this type of capital to be held at a minimum of 4% of total RWAs although higher minimums can be individually assessed by APRA (but not disclosed) for each ADI (the Prudential Capital Requirement or PCR). Additionally, a minimum of 8% applies to ‘total capital’, where the additional requirement can be met by lower quality capital, such as subordinated debt. At March 2011, the major Australian banks are collectively holding a Tier 1 ratio of almost 10%, well in excess of the current 4% minimum requirement, though both the quantity and quality of the minimum capital required to be held by banks will rise substantially as an outcome of the ongoing post-GFC review of global banking regulations conducted by the Basel Committee of Banking Supervisors (‘BCBS’), referred to as ‘Basel 3’ (‘B3’).

### Expected Credit Loss Capital ($C_{EL}$)

Capital the regulator requires to be held for expected levels of loan losses. In the event that this is higher than pre-tax credit provisions, this capital requirement will be held as a combination of credit provisions and/or other deductions from shareholder capital for the purpose of assessing capital adequacy. This is illustrated in Figure 3 below.
Regulatory Capital Requirement (C)

Represents the sum of both regulatory expected and unexpected loss capital. Regulatory capital resource represents those shareholder assets available to meet the regulatory capital requirement. The relationship between capital requirement and resource is illustrated in Figure 3 below. Pro-cyclicality, discussed further in section four of this paper, is described as the extent to which the capital requirement increases in the bad times (i.e., adverse pro-cyclicality), and/or reduces in the good times. Measures to reduce the future impact of adverse pro-cyclicality are being considered as part of the ongoing BCBS review of global banking regulation.

Figure 3: Capital Resource vs Requirement
(Home Lending – Illustrative Only, Ignores Tax)

Leverage (L)

Describes the maximum ratio of assets to equity permitted under the B2 framework. For example, leverage of 50 implies that to fund the assets required to support $50 of lending, only $1 of equity needs to be put in by the bank, with the residual able to be borrowed or raised through customer deposits. For the purpose of this paper we only consider the credit risk capital requirement in assessing lending leverage.
3.2 Model Construct

By combining the concepts above, data available from APS330 reporting, and the prescribed Pillar 1 (advanced) approach to capital calculation outlined in APS113, four simple relationships have been developed to represent the AIRB home lending system (Figure 4). These combine to provide a model of the Australian/NZ AIRB home lending system credit risk capital requirement. In this context, capital is defined notionally, acknowledging that in practice both the capital requirement, and the capital held to meet that requirement, can differ from bank to bank.

Appendix A outlines these equations in more detail. It also acknowledges the various limitations with this approach. These include the impact of regulatory overlays, sample size, lack of data granularity, statistical assumptions, bank methodology differences, AIRB implementation issues, inconsistent reporting dates and evolving B3 regulatory changes. Appendix B provides a sample of data taken from the latest major bank APS330 reporting that has been used to populate this model.

Figure 4: Key Inputs of Credit Capital Requirement for the AIRB Home Lending System

| Equation 1 - Unexpected Loss Capital Requirement (non default only) | $C_{UL} = LGD_{ND} \cdot \left( N \left( \frac{G(PD) + \sqrt{15\% \cdot G(0.999)}}{\sqrt{1-15\%}} \right) - PD \right) \cdot EAD_{ND} \cdot 106\% $ |
|---------------------------------------------------------------|
| Equation 2 – Expected Loss Capital Requirement                | $C_{EL} = \text{MAX} \left( \left( LGD_{ND} \cdot PD \cdot EAD_{ND} + LGD_{D} \cdot EAD_{D} \right) \cdot 106\% , \text{Provisions} \right) $ |
| Equation 3 – Derivation of Actual (D') and Expected (PD) Defaults | $D' = \frac{EAD_{D}}{EAD}$  
And where $PD$ is the value: $C_{UL} + C_{EL} = 8\% \cdot \text{RWA}_{CR} + \text{EL}_{CR}$ |
| Equation 4 – Leverage                                         | $L = \frac{EAD_{\text{UTILISED}}}{C}$ |
4 Application and Insights

4.1 Insight 1

Despite the recent deterioration in home lending default experience, the long-term PD has remained broadly stable.

It is clear from Figure 5 that our own $1.3 trillion domestic (Australian/NZ) home lending system has held up remarkably well against the credit risk catastrophe continuing to unfold in the US. The RBA has described the much stronger home lending asset quality in Australia relative to the US as stemming from historically tighter local credit standards, differences in consumer regulation, historically higher local interest rates, a proactive regulator, and stronger housing fundamentals (demand versus supply). These features of our local jurisdiction should continue to limit the impact of any future housing downturn scenario relative to that being experienced in the US as at March 2011.

Figure 5: Global non-performing housing loans (percentage of loans)
(Source–RBA Financial Stability Review March 2011)
Whilst the RBA has always reported on system-wide default experience (Figure 5), the ability to compare this against the long run default rates expected by the banks has only been publicly available since the banks’ advanced B2 accreditation in 2008. By applying the equations in Figure 4, a trend of the actual default rate versus an implied long run default probability for each major bank has been derived (Figure 6). Some high level headlines from this analysis include the following.

Since September 2008, overall default rates have gradually risen, indicating a trend towards credit default stress in the system, but still impressively low against the US, UK and Spain. Over the same period, the long run PD has remained relatively stable.

NAB has the highest default rate (both expected and observed), but appears to be slightly improving its default risk profile. NAB is currently required to hold the most credit capital, relative to exposure, of all the four majors.

CBA continues to show the most rapid deterioration in actual default rates over the past two to three years, with the long run PD only moving slightly upwards over this time. Representing around 30% of the AIRB system, this has had a material impact on the overall system default trend.

ANZ has shown a stable to slightly improving long-term outlook on default rates. Like CBA and WBC, the gap between its current default rates and PD assumption indicates a growing level of credit stress relative to long-term expectation.

WBC has the best long-term outlook on default rates, with the lowest observed default rates.
Whilst movement in observed default rates can be expected from changes in macroeconomic settings (for example the recent rise in interest rates), there are other reasons that both observed and expected (PD) default rates might move from period to period. An individual bank’s portfolio mix can shift to the extent that new ‘front book’ lending has different characteristics to its ‘back book’. Recent examples of this might be the 2008-09 ‘vintage’ of first home buyers and acquisition of minor banks and non-bank market share into the major banks. The amount of time (seasoning) that homes loans have been on the books can affect observed default rates from period to period. Specific one-off events (eg recent Queensland floods) can also result in a material shift overall observed default rates. The PD for a bank can also move as a result of both internal bank model changes and/or the application of regulatory overlays. Differences in internal bank PD model structure can result in further variation between the banks.

Without a more detailed disclosure around these various drivers of change, it can be difficult to draw too many conclusions from default rate trends observed in high level APS330 reporting. Also with the average internal LGD assumption for all banks currently sitting below the APRA 20% minimum, APS330 reporting provides little insight into the impact of home lending security on relative credit risk across the major banks.
4.2 Insight 2

Whilst the home lending capital requirement has been relatively stable over the recent past, the system appears to be susceptible to adverse procyclicality in the near future.

4.2.1 Analysis of Change in Notional Credit Capital Requirement

By applying the equations in Figure 4 (outlined in Appendix A), we can decompose the change in the notional credit capital requirement for the AIRB home lending system into volumes, changes in assumptions and experience capital impacts (Figure 7).

Figure 7: Analysis of change in total airb home lending capital 2H08-1H11 (A$ m)

Key observations on the analysis of change in notional credit capital since September 2008 are as follows.

Volume Growth

Explains almost all of the $6.2 billion increase in the AIRB home lending system capital requirement since September 2008. Volume growth includes such things as system growth, market share that has been taken by the major banks at the expense of the minor banks and non-bank lenders, and lending transitioned into an advanced (AIRB) B2 approach (most notably St George and Bankwest).
Stable PD and LGD

Overall long-term PDs have remained broadly unchanged from 0.58% at September 2008 to 0.57% at March 2011, reducing the capital requirement by around $323 million. As expected, the LGD assumption has also been stable, reducing the capital requirement slightly ($93 million) over the period.

Deterioration in Actual Default Rate

March 2011 actual percentage defaults are currently 0.19% above long-term PD, after being 0.04% below back at September 2008. This difference in actual over-expected PD explains an additional $465 million of capital requirement (pro-cyclicality impact) at March 2011, compared to ($60 million) at September 2008.

4.2.2 Drivers of Future Pro-Cyclicality

In the context of the model, I define adverse pro-cyclicality as any tendency for the credit capital requirement to move upwards with any deterioration in observed default rates. The capital requirement is particularly sensitive to any changes in PD or LGD (eg. either a 5 bps increase in PD, or a 105 basis point increase in the LGD, will increase notional credit capital required by around $1 billion). The real driver of adverse pro-cyclicality for the system is any ensuing upward movement in the PD and LGD assumption. With regards to the AIRB home lending system, I can see four possible drivers of adverse pro-cyclicality in the event of any future macroeconomic deterioration.

Shifting Portfolio Mix

Spurred on by government incentives, first home buyer appetite to borrow increased substantially over 2008-09. Furthermore substantial home lending market share shifted from the minor banks and non-bank lenders to the major banks. It may take a little time before the long-term default rate for this unique ‘vintage’ of 2008–09 borrowers can be empirically settled, but it is possible that they result in upward pressure on the average PD of the major banks.
Credit Cycles Redefined

It appears that internal bank home lending long-term PD models set up at the inception of the B2 framework are still largely relying on the last decade of relatively benign Australian/NZ default experience. It is also unclear the extent to which long-term PD assumptions being set by the banks have recognised the possibility of occasional extreme events like those currently being experienced in the US and UK. I expect a higher future long-term expected loss assumption for the system as banks begin to update their long-term PD models for both the higher level of defaults experienced over the last couple of years (refer Figure 6), and a new appreciation of the potential frequency and impact of risk in the tail of the home lending loss distribution.

LGD ‘breach’

The APRA LGD floor appears to have succeeded as an important stabiliser in limiting the impact of any adverse pro-cyclicality over 2009 (ie by limiting the impact of volatility in the LGD assumption on capital requirement). It is however unclear what might be the signposts for the internal bank calculated LGD breaching the APRA floor, as even a relatively minor jump in LGD can have a material impact on the capital requirement. How close are these internal bank calculations currently to the 20% floor? At what point might a fall in house prices trigger an internal bank-calculated LGD in excess of 20%? Whilst it is possible to deduce an estimate of internal mortgage LGDs from capital ratio reconciliations (APRA to FSA reported in bank profit announcements), internal mortgage LGD calculations are not particularly transparent to the public.

Transition to New B3 Capital Levels

The recent deterioration in home lending default rates opens the possibility that the forthcoming introduction of the higher capital requirements under B3 may well coincide with a period of stress in the retail credit cycle. Combined with the three abovementioned factors, this has the potential to amplify any adverse pro-cyclicality that emerges over the next few years.
4.2.3 Sensitivity of Notional Capital to a Rise in the Long-Term Mortgage PD

Figure 8 shows the average impact of a hypothetical 10 bps increase in mortgage long run PD on major bank T1 capital ratios (as at March 2011) would be around 0.2%. It also shows that the major banks would collectively need to raise around $2.7 billion in order to offset the corresponding dilution in their T1 ratio. With the major bank T1 capital resource at March 2011 at nearly 10% of RWAs, such an adjustment would be well within the current balance sheet capacity of the banks. Nonetheless the experience of 2008–09 has shown that that in a stressed credit environment, raising capital can become more difficult and expensive than normal.

Figure 8: Sensitivity of major bank capital positions to changes in the mortgage PD assumption

4.3 Insight 3

Leverage to credit risk on home lending is around four times the leverage on other types of lending under the B2 framework.

Based on a simple measure of reported Assets to Equity, overall leverage for the large Australian banks reduced from around 21 times (September 2008) to 18 times (September 2010). This has been driven by capital raisings to respond to heightened investor and ratings agency scrutiny on balance sheet strength, the need for an internal buffer...
against potential adverse pro-cyclicality impacts on credit capital and losses, and advance preparation for the greater quantity and quality of capital likely to be required under emerging B3 global banking changes.

By applying equation 4 to APS330 data, we are able to calculate notional credit leverage across home and other lending (Figure 9). At September 2010, the AIRB home lending system had an implied credit leverage of around 50, four times that of the broader AIRB lending system. Whilst it is a fact that the high levels of home lending credit leverage are a valid function of the historically lower relative credit risk of this type of lending, it is important that the appropriateness of this leverage be examined against the forward-looking risks in this sector.

Figure 9: September 2010 AIRB home vs other lending (notional credit leverage, RWA/EAD, capital/balances)

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<thead>
<tr>
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<th>Credit Leverage</th>
<th>Credit RWA / EAD</th>
<th>Credit Capital (C) / Balance</th>
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</thead>
<tbody>
<tr>
<td>Home Lending</td>
<td>51%</td>
<td>51%</td>
<td>8.3%</td>
</tr>
<tr>
<td>Other Lending</td>
<td>18%</td>
<td>33%</td>
<td>4.5%</td>
</tr>
<tr>
<td>Total AIRB</td>
<td>22%</td>
<td>33%</td>
<td>4.5%</td>
</tr>
<tr>
<td>Total AIRB</td>
<td>50%</td>
<td>51%</td>
<td>8.3%</td>
</tr>
</tbody>
</table>

5 Conclusion

Within Australia/NZ, the ability for an outside party to assess expected long run defaults across the home lending system has only recently emerged as a result of enhanced B2 disclosures. This paper demonstrates an approach to undertaking such analysis, and shows how it can be used to develop some high level insights into the Australian/NZ AIRB home lending system.

I would like to thank Phillip Everett for his helpful input and review.
Appendix A – Model Derivation, Inputs and Outputs

Equation 1 – Unexpected Loss Capital Requirement (non-default only)

Equation 1 represents the calculation prescribed in APS113 for the purpose of calculating non-default credit RWAs for unexpected losses corresponding with a 99.9% confidence interval. This equation was originally derived by Oldrich Vasicek. It assumes the credit loss distribution is inherently long tailed, and has been adopted globally for use in the B2 framework.

\[
C_{UL} = \text{LGD}_{ND} \left[ N \left( \frac{G(PD) + \sqrt{15\% \cdot G(0.999)}}{\sqrt{1-15\%}} \right) - PD \right] \cdot \text{EAD}_{ND} \cdot 106\%
\]

Where:

- \(C_{UL}\) represents notional credit capital against unexpected losses in a home lending portfolio. It can also be used (by multiplying \(C_{UL}\) by 12.5) to proxy the B2 RWAs required to be held against non-defaulted loans.
- \(N(x) = \text{normsdist} (x) = \text{probability of a standard normal variable} \leq x\)
- \(G(n) = \text{normsinv}(n) = \text{the number} \ x \ \text{such that} \ N(x) = n\)
- 106\% represents an overall regulatory overlay applied to AIRB Credit RWAs
- 15\% represents the prescribed correlation for a normal home loan
- \(\text{EAD}_{ND}\) = assumed exposure at default for non-defaulted loans
- \(\text{LGD}_{ND}\) = loss given default assumption for non-defaulted loans (subject to 20\% LGD floor)
- \(PD\) = best estimate probability of default derived in equation 3
Equation 2 – Expected Loss Capital Requirement

Equation 2 is based upon the prescribed method in APS113 for the purpose of calculating regulatory expected losses. It implies no RWAs are held against defaulted loans, with the capital requirement instead being taken through provisions and as a capital deduction (50% Tier 1, 50% Tier 2) in respect of any excess of prudential reserve for credit losses over those provisions, an approach that differs across the major banks in practice.

\[ C_{EL} = \text{MAX} \left[ (LGD_{ND} \cdot PD \cdot EAD_{ND} + LGD_D \cdot EAD_D) \cdot 106\% , \text{Provisions} \right] \]

Where:

- \( C_{EL} \) = Notional capital required for expected losses on both defaulted loans and non-defaulted loans (ignoring tax effects)
- \( LGD_D \) = loss given default assumption for defaulted loans
- \( LGD_{ND} \) = loss given default assumption for non-defaulted loans
- \( EAD_D \) = assumed exposure at default for defaulted loans
- \( EAD_{ND} \) = assumed exposure at default for non-defaulted loans
- \( PD \) = best estimate probability of default derived in equation 3
- \( \text{Provisions} \) = total credit provisions created as a charge against the P&L (specific and collective)
- \( 106\% \) = an overall regulatory overlay applied to AIRB Credit RWAs

Equation 3 – Derivation of Actual (D’) and Expected (PD) Defaults

The ability to derive PD is fundamental to enable a comparison of actual vs expected long run defaults for the AIRB home lending system.

Actual defaults (D’) are defined as:

\[ D' = \frac{EAD_D}{EAD} \]
Expected long run defaults (PD) is defined as that number that calibrates modelled notional credit capital (as per equations 1 and 2) to published RWAs and EL:

\[ C_{UL} + C_{EL} = RWA_{CR} \times 8\% + EL_{CR} \]

Where:
- \( EAD_D \) = assumed exposure at default for defaulted loans
- \( EAD \) = overall exposure at default (\( EAD_D + EAD_{ND} \))
- \( RWA_{CR} \) = the RWAs published in APS330 reporting
- \( EL_{CR} \) = the EL published in APS330 reporting
- \( 8\% \) = minimum capital required as a percentage of RWAs

Total Capital can also be defined as \( C = C_{BE} + C_{EA} \)

Where:
- **Best Estimate Credit Loss Capital (\( C_{BE} \))** – represents the total capital requirement if the best estimate PD occurs in practice. \( C_{BE} \) is found by substituting \( EAD_{ND} \) for \( EAD \times (1 - PD) \), and \( EAD_D \) for \( EAD \times PD \) in equations 1 and 2.
- **Experience Capital Adjustment (\( C_{EA} \))** – represents the impact of actual defaults varying from the PD. It is simply the difference between \( C \) and \( C_{BE} \). If it is positive it implies actual defaults are currently running above the best estimate PD, and/or provisions are being held over and above the long-term EL requirement. If it is negative it implies actual defaults are currently running below.

\( C_{BE} \) and \( C_{EA} \) are useful for performing analysis of change in capital requirement as shown in Figure 7.

**Equation 4 – Leverage (L)**

Leverage is a measure for the maximum ratio of assets to equity permitted under the B2 framework. For example, where leverage of 50 implies that to fund the assets required to support $50 of lending, only $1 of equity needs to be put in by the bank, with the residual able to be borrowed or raised through customer deposits. For the purpose of this
paper we only consider credit risk capital. The higher the assumed PD or LGD, the lower the implied credit risk leverage under the B2 AIRB framework.

For the purpose of the analysis in this paper, leverage has been calculated as:

\[ L = \frac{\text{EAD}_{\text{UTILISED}}}{C} \]

Limitations

Key limitations with this model and approach include the following.

Distortion of Regulatory Overlays

We have discussed regulatory overlays such as the 20% LGD floor and the 6% factor being used to gross up AIRB RWA calculations. The use of the implied regulatory capital confidence interval of 99.9% is another prescribed overlay and most likely inconsistent with the confidence interval being used in the internal Economic Capital models used by the banks. The RBNZ has in some cases applied prescriptive PD assumptions for use in NZ-based AIRB models, resulting in the overall system PD being less responsive to changes in underlying risk.

Sample size

Whilst the AIRB home lending sample size is large it still excludes almost a quarter of the home lending system. For this reason a rough scale up of 5/4 might need to be applied to some of the outcomes of this model to calculate overall dollar home lending system impacts.

Lack of Data Granularity

APS330 reporting provides only limited ability to use the approach outlined in this paper to drill deeper into the lending portfolio (eg separate out across Aus/NZ geographies, states, new book vs back book analysis, etc.) The inability to split out NZ (15% of AIRB home lending system) and Australian data to the level required in this model is a constraint, which can be further distorted by movements in the AUD to NZD exchange rate.
Statistical Assumptions

The model approach assumes the shape of the loss distribution for the system is the same as the loss distribution being applied to each individual loan. This is probably not the case in practice, and may result in potential differences between the bottom-up (published) and top-down (modelled) derivation of EL and UL capital. Different banks can also adopt different approaches to both segment, and discriminate for risk, between loans for the purpose of assigning a long run PD. Such differences in approach can impact on comparison of relative capital outcomes between the banks.

Bank Methodology Differences

Differences in approach to classifying overall capital requirement between home lending UL and EL has also been observed across the major banks. There are possibly differences in the treatment of securitised home lending, though it should be noted the proportion of home loan volumes that have been securitised within Australia and NZ has declined over the last few years. Methodology differences have the potential to create differences between the bottom-up (published) and top-down (modelled) derivation of EL and UL capital.

AIRB Implementation Issues

The advanced B2 approach to credit risk capital management has only been recently adopted for use by the Australian banks. To the extent that banks deliver ongoing improvements to address any post-implementation issues identified in their models, both disclosed capital requirements and the results of the model will change.

Inconsistent Reporting Dates

CBA reports on a June year end, versus the other major banks on a September year end. The model has used June 2009 CBA data to compare with September 2009 for the other majors to define the end of 2009. Whilst each major also produces quarterly APS330 reporting, these abridged reports do not contain actual default EAD (D’) and are therefore not used for updating the model.
Foreshadowed B3 Capital Changes

Global changes to both the quantity and quality of banking capital have been foreshadowed. Whilst the implementation of such changes is still subject to local review, these relationships may require some recalibration once the specifics of these changes are settled.
Appendix B – Model Inputs and Outputs for the Four Major Australian Banks (1H11)

Key Model Inputs – AIRB Home Lending Capital Model ($m AUD)

<table>
<thead>
<tr>
<th>Input</th>
<th>ANZ</th>
<th>NAB</th>
<th>CBA</th>
<th>WBC</th>
<th>TOTAL 1H11</th>
</tr>
</thead>
<tbody>
<tr>
<td>EAD</td>
<td>226,659</td>
<td>239,040</td>
<td>336,647</td>
<td>362,009</td>
<td>1,164,355</td>
</tr>
<tr>
<td>EAD (Default)</td>
<td>224,768</td>
<td>237,082</td>
<td>333,865</td>
<td>359,802</td>
<td>1,155,517</td>
</tr>
<tr>
<td>EAD (Utilised)</td>
<td>1,891</td>
<td>2,782</td>
<td>2,070</td>
<td>2,067</td>
<td>8,838</td>
</tr>
<tr>
<td>LGD (Non Default)</td>
<td>203,815</td>
<td>205,396</td>
<td>283,579</td>
<td>311,533</td>
<td>1,004,323</td>
</tr>
<tr>
<td>LGD (Default)</td>
<td>21.3%</td>
<td>21.2%</td>
<td>20.3%</td>
<td>21.0%</td>
<td>20.9%</td>
</tr>
<tr>
<td>RWA (Credit)</td>
<td>40,126</td>
<td>51,389</td>
<td>59,797</td>
<td>55,952</td>
<td>207,264</td>
</tr>
<tr>
<td>EL (Credit)</td>
<td>587</td>
<td>823</td>
<td>1,326</td>
<td>889</td>
<td>3,625</td>
</tr>
</tbody>
</table>

Source: APS330 reporting

Key Model Outputs – AIRB Home Lending Capital Model ($m AUD)

<table>
<thead>
<tr>
<th>Modelled Output</th>
<th>ANZ</th>
<th>NAB</th>
<th>CBA</th>
<th>WBC</th>
<th>TOTAL 1H11</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capital (C)</td>
<td>3,797</td>
<td>4,934</td>
<td>6,110</td>
<td>5,365</td>
<td>20,206</td>
</tr>
<tr>
<td>RWA / EAD (%)</td>
<td>17.70%</td>
<td>21.50%</td>
<td>17.76%</td>
<td>15.46%</td>
<td>17.80%</td>
</tr>
<tr>
<td>Capital / EAD (%)</td>
<td>1.68%</td>
<td>2.06%</td>
<td>1.81%</td>
<td>1.48%</td>
<td>1.74%</td>
</tr>
<tr>
<td>Capital / Balances (%)</td>
<td>1.86%</td>
<td>2.40%</td>
<td>2.15%</td>
<td>1.72%</td>
<td>2.01%</td>
</tr>
<tr>
<td>Unutilised EAD</td>
<td>10.1%</td>
<td>14.1%</td>
<td>15.8%</td>
<td>13.9%</td>
<td>13.7%</td>
</tr>
<tr>
<td>Actual Defaults (D')</td>
<td>0.834%</td>
<td>0.819%</td>
<td>0.826%</td>
<td>0.610%</td>
<td>0.759%</td>
</tr>
<tr>
<td>PD</td>
<td>0.521%</td>
<td>0.737%</td>
<td>0.597%</td>
<td>0.482%</td>
<td>0.566%</td>
</tr>
<tr>
<td>Unexpected Loss Capital CUL</td>
<td>3,117</td>
<td>4,122</td>
<td>5,081</td>
<td>4,520</td>
<td>16,847</td>
</tr>
<tr>
<td>Expected Loss Capital CUL</td>
<td>680</td>
<td>812</td>
<td>1,028</td>
<td>845</td>
<td>3,359</td>
</tr>
<tr>
<td>Best Estimate Capital CEA</td>
<td>3,647</td>
<td>4,893</td>
<td>5,956</td>
<td>5,253</td>
<td>19,741</td>
</tr>
<tr>
<td>Experience Capital CEA</td>
<td>151</td>
<td>41</td>
<td>154</td>
<td>112</td>
<td>465</td>
</tr>
<tr>
<td>Leverage (L)</td>
<td>53.7</td>
<td>41.6</td>
<td>46.4</td>
<td>58.1</td>
<td>49.7</td>
</tr>
<tr>
<td>Notional Capital Costs</td>
<td>0.19%</td>
<td>0.24%</td>
<td>0.22%</td>
<td>0.17%</td>
<td>0.20%</td>
</tr>
</tbody>
</table>

Source: Home Lending Capital Model

Note – The total is being modelled as a separate model point, not as the sum of the four majors. In some cases this results in a small difference between the total and the sum of parts for modelled outputs.
References


RBA. March 2011, Financial Stability Review

Management of Closed Defined Benefit Superannuation Schemes – An Investigation Using Simulations

A Butt*

Abstract

This study investigates the actuarial monitoring of closed defined benefit superannuation schemes. This is done via a simulation approach of a model scheme, with economic and decrement factors varying stochastically. The desires of trustees and employer-sponsors are expressed numerically and the distribution of these desires analysed from the simulated output. In addition, a single objective function is developed, balancing the desires of all parties, in order to quantify the optimal use of investment and contribution strategies. In addition, adjustments to the model scheme are made to investigate their effect on the use of these strategies.

This paper provides some of the results of a recently completed PhD thesis undertaken by the author. Initial results were presented at the Institute of Actuaries 2009 Biennial Convention. All acknowledgements relevant to the PhD thesis are also relevant to this paper. Any errors or omissions remain the responsibility of the author.

Keywords: Defined Benefit Superannuation, Investment, Funding, Monte Carlo methods

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1 Introduction

Recent times have seen a strong trend away from defined benefit arrangements into other types of retirement benefit provisions, with this trend being stronger in Australia than most countries. In 1995, defined benefit assets made up 22% of all superannuation assets in Australia, whereas this figure had reduced to 7% by 2010 (Table 16, APRA, 2011 – note that if defined benefit assets in hybrid schemes are included this increases the 7% to 18%). At 30 June 2010, membership in defined benefit schemes made up only 2% of the over 32 million separate scheme memberships, with over 98% of the memberships in defined benefit schemes being from public-sector schemes (Table 15, APRA, 2011).

Campbell et al. (2006) note a number of reasons employers may wish to reduce defined benefit exposure. Some of the reasons are as follows:

- A desire to reduce total compensation paid to employees;
- Uncertain investment markets leading to unstable contribution requirements;
- Uncertain mortality improvement giving rise to potential asset shortfalls in future years;
- Regulatory concerns; and
- Effect of accounting standards, with scheme deficit and surplus required to be shown on the company’s balance sheet.

The above reasons pertain to the UK market – some additional reasons for the trend are:

- The prevalence of salary packaging and associated complications of defined benefits in this environment;
- The difficulty in allowing for switches between full-time, part-time and casual work in defined benefits;
• A general trend in employers becoming less paternalistic toward their employees and thus taking less responsibility over retirement benefit provision; and

• In Australia, the introduction of the Superannuation Guarantee giving all employees access to a minimum level of superannuation and reducing the relative attractiveness of defined benefit schemes in attracting and retaining staff.

One option available to employers in reducing defined benefit exposure is to close the defined benefit scheme to new entrants. The trend towards closing of defined benefit schemes has a clear impact on the demographics, and hence the appropriate actuarial management of these schemes. This paper presents an analysis of the stochastic simulation projections of a closed defined benefit superannuation scheme paying pension benefits, along with comparisons with schemes which are open to new entrants and schemes paying lump sums rather than pensions. The purpose of this analysis is to investigate optimal investment and contribution strategies and how they differ for these different types of schemes. As such, a holistic objective function is developed to perform this optimisation, combining all trustees and employer-sponsor objectives in relation to the defined benefit scheme. These objectives are considered in isolation from broader business objectives affecting the employer-sponsor in order to identify the scheme-specific impact on objectives. Some introductory discussion of the integration of superannuation and broader business issues is contained in the conclusions of the paper.

The structure of this paper is as follows. Section 2 introduces the objectives of trustees and employer-sponsors and the strategies to be considered in achieving these objectives in a closed defined benefit scheme. Section 3 provides the projection methodology and the measurement basis for the objectives (including the holistic objective function). Section 4 presents the results, and finally, Section 5 concludes and discusses potential future research. A number of appendices outline further information about the model scheme and the calculations undertaken.
2 Decision making in schemes

2.1 Trustees and employer-sponsor objectives

The clearest objective of a trustee is to ensure that the benefits under the rules of the scheme are able to be paid out to all members when they fall due. Trustees are therefore particularly concerned that the funding level (assets divided by actuarial liabilities) is maintained at a minimum of 100%. In Australia, trustees are also concerned about the level of assets relative to vested benefits and minimum requisite benefits; however these measures are not considered in this paper.

The objectives of employer-sponsors can be seen in the reasons that defined benefit schemes are being phased out, as discussed in Section 1. These are summarised as follows:

- Low contributions;
- Predictable contributions; and
- Minimal balance sheet effect.

In some cases these objectives are consistent with the objectives of the trustees. For example, accounting standards require deficits and surpluses to be recognised on the sponsoring employer’s balance sheet. The Australian standard AASB 119, which is based on the international standard, IAS 19, allows but does not require immediate recognition of the full deficit or surplus, although IAS 19 has recently been revised to require immediate recognition, so presumably AASB 119 will be similarly revised. This requirement means employers may be driven by the desire to avoid large deficits appearing on their balance sheet, which is consistent with the objective of the trustees to ensure an adequate level of assets. In this study these objectives are assumed to be identical for trustees and employer-sponsors. As such, all liabilities and contributions are calculated using the assumptions that would apply under an accounting valuation (ie. best estimate assumptions from the stochastic models, discounting liabilities at a ‘high-quality’ corporate bond rate – see Appendix B). Although trustees may view the funding question from the perspective of an actuarial valuation that may
discount liabilities at the expected return on scheme assets (which is typically higher than the corporate bond rate), ultimately the financial success of a scheme is reliant on the covenant with the employer-sponsor, whose risk is assumed to be primarily based on the accounting valuation. For calculation of funding levels, assets are assumed to be at market value.

However, the objective for low contributions may not be consistent with the trustees’ objective to secure benefits, particularly if the scheme is in deficit or the employer-sponsor is in financial difficulty. In addition, the overall employer-sponsor objectives for low, predictable contributions with minimal effect on the balance sheet may not be internally consistent. The lowest contributions are generated by seeking the maximum possible return on the investment of scheme assets, leading to investment in risky assets that may jeopardise the stability of the funding level and thus predictability of the contributions.

2.2 Strategies to be considered

This section considers the background of the strategies to be considered in meeting trustees’ and employer-sponsor objectives. In most cases the actuary does not have the power to apply investment and contribution strategies directly, but makes recommendations to be considered by the trustees and employer-sponsor.

2.2.1 Investment

There is a wide variety of literature discussing the most appropriate investment strategy for schemes, with much debate about the role of various asset classes in these strategies. This debate can generally be simplified to one of two views. The first is that the assets backing liabilities should be matched as closely as possible to the liabilities, using either a duration-matching strategy with appropriate bonds or investing in assets that provide income that matches projected liability cash flows exactly (see for example the financial economics arguments of Day, 2003). The second view is that the additional return expected to be generated by equities in comparison with bonds over the long-term of the liabilities is a significant enough incentive to invest in equities,
especially considering the difficulty in hedging liabilities affected by so many economic and demographic variables. In this paper, a cash flow matching strategy is not investigated (although it is considered in a forthcoming paper, Butt, 2011), with investments assumed to be undertaken among a range of growth and defensive asset classes. As such, the question of interest is determining what level of growth asset investment is appropriate.

Prior research provides conflicting evidence on appropriate levels of risky asset holdings, due to different objectives being measured. Boulier et al. (1995) find that schemes should invest a lower proportion in risky assets as the funding level increases, due to the diminishing use of additional investment returns in minimising future contributions as the funding level increases. Similar results are found by Cairns (1995) and Haberman et al. (2003), who aim to minimise the probability of insolvency. The opposite result is found by Taylor (2002) due to trying to maximise the average solvency ratios rather than minimise probability of insolvency. Peskin (1997) discusses the need to consider how closely the liabilities are linked to bond returns. High allocations to bonds are most attractive when the membership is decreasing, when the employer-sponsor has little access to surplus apart from reducing contributions and when asset levels are similar to liability levels.

In the UK, the Pensions Regulator and Pension Protection Fund (Chart 7.4, 2009) find that more mature schemes tend to invest a lower proportion of their assets in equities; a result also found by Amir and Benartzi (1999), using US data. Attempting to measure the asset allocation by schemes relative to funding level is not particularly useful, as the funding level of a scheme is directly related to recent investment performance, which is dependent on asset allocation.

2.2.2 Contributions

Traditionally, there are two components to determining a contribution rate for a scheme. The first is to calculate the normal contribution rate required to fund the future liabilities of the scheme. The second is to compare the level of assets with the current liabilities and adjust the normal contribution rate to take into account any surplus


or deficit. The speed at which any surplus or deficit is removed is known as the amortisation or spread period. In this paper a spread approach is used instead of an amortisation approach (see Owadally and Haberman, 1999, who show the spread approach leads to less variability in contributions and funding levels).

Owadally and Haberman (2004) use a simulation model to estimate that surpluses and deficits should be spread over a period of 7–13 years to minimise funding level and contribution volatility; this is a longer period than is typically allowed by APRA in Australia. Cairns (1994) finds that more frequent valuations and quicker implementation of contribution changes lowers the variability of contributions for short amortisation periods, but increases the variability for longer amortisation periods. Positive autocorrelation in investment returns reduces the optimal amortisation period and vice-versa.

3 Methodology

3.1 Projection of scheme assets and liabilities

One thousand simulations are performed of the future assets and liabilities of an initially fully funded model scheme for a period of 30 years using stochastic economic and demographic assumptions. A graphic description of the projection process and the relevant assumptions is shown in Figure 3.1. All modelling is performed on a discrete annual basis, with cash flows assumed to occur mid-year and investment earnings received at half the rate for that year. A uniform random variable is generated each year for each individual in the scheme to project any movement in membership status.

The approach used is broadly similar to that presented by the Stochastic Valuation Working Party of the UK Pensions Board (see Haberman et al., 2003), although in the simulations presented in that paper the contribution rate was considered to be a free variable to be investigated. In this study, normal contributions are calculated on a traditional actuarial basis (see Appendix A), with the initial allocation to growth assets and spread period allowed to vary for each scenario in order to determine optimal strategies. As described in Section 2.1,
liability and contribution calculations use a corporate bond discount rate. In the base scenario, actuarial valuations occur annually on a projected unit credit basis and contribution recommendations are implemented with no time lag. Allocation to growth assets is fixed and rebalanced each year. Surplus is assumed to be unrecoverable and unusable by the employer-sponsor, except to reduce contributions required to fund future liabilities. Further information on the base scenario inputs for the model scheme is provided in Appendix A.

The Wilkie (1995) model is used as the starting point for the economic model, which is described in Appendix B. Decrement models are described in Appendix C.

### 3.2 Measurement of objectives

In order to compare investment and contribution strategies, it is necessary to define a measurement basis for the trustees and employer-sponsor objectives described in Section 2.1. The aim of these measures is to turn the annual observations from each simulation into a single figure.
from that simulation, giving each scenario a distribution of 1,000 single observations, the average of which can then be analysed for each strategy.

3.2.1 Low contributions

In each simulation the present value of the contributions, \( PV(C) \), is calculated as follows:

\[
PV(C) = \sum_{t=1}^{30} \left( C(t) \times \exp[DF(t-1) - 0.5ac(t)] \right) + \max \left[ L(30) - N(30), 0 \right] / 0.85 \times \exp[DF(30)]
\]

where \( C(t) \) is the contribution paid during time \( t - 1 \) to time \( t \) and \( DF(t) \) is the discount factor that applies for \( t \) years (with \( DF(0) = 0 \)) and is calculated as follows:

\[
DF(t) = -\sum_{a=1}^{t} ac(a);
\]

and \( ac(a) \) is the continuously compounding return on cash from the economic model (see Appendix B). The term outside the summation in \( PV(C) \) is the contribution required for any final deficit of liabilities \( L(30) \) compared with assets \( N(30) \), allowing for contributions tax. Other contributions are assumed to be paid mid-year.

Since the contributions for each simulation are dependent on the membership development of that simulation, it is appropriate to scale the contributions by the present value of salaries paid during that simulation, \( PV(S) \), giving an average contribution rate, \( \bar{c} \) as follows:

\[
\bar{c} = \frac{PV(C)}{PV(S)};
\]

where \( S(t) \) is the total salaries paid during time \( t - 1 \) to time \( t \) giving:

\[
PV(S) = \sum_{t=1}^{30} \left( S(t) \times \exp[DF(t-1) - 0.5ac(t)] \right)
\]
This approach is similar to that of Haberman (1997), although in Haberman (1997) the real valuation rate of interest was used to discount contributions, rather than the cash rate used above. Using a cash rate takes into account the different economic circumstances of each simulation. In addition, Haberman (1997) did not express contributions as a rate of salary, since the liability factors were deterministic in nature and thus constant.

3.2.2 Predictable contributions

Generally an employer-sponsor is not particularly concerned if contributions drop below expectations, but is concerned if the opposite is the case. Therefore, a single-sided measure of the predictability of contributions is appropriate, as opposed to an overall volatility approach. In order to keep consistency between contribution measures, a broadly similar approach is used to the previous section. The average excess contribution rate $c_{exc}$ is calculated as follows:

$$c_{exc} = PV(C_{exc}) / PV(S);$$

where

$$PV(C_{exc}) = \sum_{t=1}^{30} \left( \max \left[ (C(t) - E[C(t)], 0 \right] \times \exp \left[ DF(t-1) - 0.5 \times ac(t) \right] \right) + \max\left[ L(30) - N(30), 0 \right]/ 0.85 \times \exp \left[ DF(30) \right]$$

and $E[C(t)]$ is the expected contributions from time $t - 1$ to time $t$, calculated using the normal contribution rate without any allowance for spreading of surplus or deficit.

The approach used is broadly similar to that used by Haberman et al. (2003), although in Haberman et al. (2003) $E[C(t)]$ is not calculated actuarially but is allowed to vary for testing against objectives. Note that this approach does not explicitly measure year to year contribution volatility – just the total of all contributions above expectations.
3.2.3 Funding level (balance sheet effect)

A consistent measure to those described above is used for funding level, calculating the mean deficit of assets to liabilities, treating surpluses as zero deficits. This approach was also used by Haberman et al. (2003). One difference between the Haberman et al. (2003) measure and that used in this study, is that Haberman et al. (2003) measures the mean deficit at a given time across all simulations, whilst this study takes the mean deficit of the whole simulation to generate 1,000 mean deficits for a given scenario. For a single simulation, with assets $N(t)$ and current liabilities $L(t)$ at time $t$, the mean funding deficit $Df_{ct}$ over the 30 year projection period is calculated as follows:

$$
Df_{ct} = \frac{\sum_{t=1}^{30} \max \left[ \left( L(t) - N(t) \right), 0 \right] \times \exp \left[ DF(t) \right]}{30 \times N(0)};
$$

where $N(0)$ is used to scale the deficits as a percentage of the initial asset level. As above, this approach does not measure year to year funding level volatility, but the total of deficits over the simulation.

3.2.4 A holistic approach

Whilst the measures above allow individual objectives to be analysed, they do not allow decision making across the inconsistent objectives. For example, an objective to reduce deficits would lead to the shortest spread period possible, whilst an objective to have predictable contributions would lead to the longest spread period possible. Therefore, in order to make an overall decision on asset allocation and spread periods taking into account all objectives, it is necessary to weight the objectives in some way. A weighting approach similar to that used by Taylor (2002) is used, although in Taylor (2002) the individual objectives are in a different form to that described above. This leads to the following single objective function, $V$:

$$
V = \bar{c} + \alpha \times \bar{c}_{\text{as}} + \beta \times Df_{ct}
$$
It is assumed that contributions above expectations cause double the ‘loss’ as contributions up to expectations, therefore giving Error! Objects cannot be created from editing field codes. It is also assumed that the contribution effects Error! Objects cannot be created from editing field codes. and Error! Objects cannot be created from editing field codes. make up two-thirds of the total objective, since contributions represent the actual cost to the employer-sponsor of the scheme. Thus Error! Objects cannot be created from editing field codes. makes up one third of $V$ for the base asset allocation (60% growth assets) and spread period (three years) in the base scenario, giving Error! Objects cannot be created from editing field codes. The value of Error! Objects cannot be created from editing field codes. and Error! Objects cannot be created from editing field codes. are fixed to allow for comparisons between scenarios, except where noted in Section 4.

4 Results

4.1 Base scenario

The first analysis performed is to calculate and plot the median and 95% confidence interval for the contribution rate (as a percentage of salaries) and funding level for each future year of projection. The 70% and 90% percentile observations for contribution rates are also included for additional information, due to the tendency for contributions to move towards zero for many simulations. This analysis, similar to that performed by Wright (1998, pp879–889), provides a broad overview of the movement in contribution rates and funding levels over time, but does not help to specifically analyse the trustees and employer-sponsor objectives. The analysis is performed for the base 60% growth asset allocation and three-year spread period only and is presented in Figure 4.1.

The funding level percentiles show a clear upward trend, with the median funding level increasing exponentially over time. The median funding level moves from 100% to over 300% in 30 years. Even the 2.5 percentile funding level increases from a minimum of 82% after three years to 123% after 30 years. This appears initially to be a strange result, given surplus and deficits are both spread over a period of three
years. However there are two biases affecting the results. The first is that the expected return on assets is larger than the corporate bond liability discount rate (although all other assumptions are equal to expectations).

**Figure 4.1: Percentiles for base (by year)**

The second is that any deficit must be removed by additional contributions, but there is no corresponding option to refund the surplus not needed to fund future benefits. Thus all deficits tend to be removed over time, but surplus can grow exponentially. Due to differences between economic and decrement experience and assumptions, most simulations experience enough years of beneficial experience to lead to a surplus position that is greater than the reducing level of future liabilities in a closed scheme. This causes an exponential increase in surplus as investment earnings continue to accrue. The lack of use for surplus is what Boulier et al. (1995) refers to when describing the diminishing upside gain as the funding level increases.

The contribution rate results are consistent with the funding level trend. The median contribution rate moves to zero in five years, with the 70, 90 and 97.5 percentile lines moving to zero in nine, 15 and 27 years respectively. The increasingly volatile rates at the upper tail of the distribution in later years are due mainly to the decreasing salary base across which contributions are paid as the scheme matures.
The second plot presented in Figure 4.2 is of contours of the mean of the single figure objectives described in Section 3.2, allowing both asset allocation and spread period to vary. Two smoothed lines are incorporated into the contour plot to show the growth asset allocation required for the minimum mean objective across each spread period, and vice versa. The contour plots are similar to plots presented in Haberman et al. (2003), although in this case the y-axis is the spread period rather than the normal contribution rate.

The average contribution rate plot indicates that the contribution rate can be reduced by increasing both the proportion allocated to growth assets and the spread period from the base levels of 60% and three years. The ideal growth asset allocation increases to 100% when the spread period passes seven years. The minimum contribution rate is obtained with 100% growth assets and a spread period of nine years, giving a contribution rate of 13.24%, compared with that of 14.29% for the base growth asset allocation and spread period. The greatest reduction in contribution rate is achieved by moving away from a very low growth asset allocation.

As expected, the minimum excess contribution rate is obtained by using the longest possible spread period for all asset allocations. However, if alternative spread periods are used, the ideal growth asset allocation to minimise excess contributions reduces as the spread period increases, which is opposite to the result for minimising average contributions. Therefore using a longer spread period creates a greater mismatch between the ideal asset allocations to minimise average contributions and excess contributions. The minimum excess contribution rate is obtained with 30% growth assets and a spread period of 20 years, giving an excess contribution rate of 1.58%, compared with that of 4.77% for the base growth asset allocation and spread period. The inconsistency between the desire for low contributions and predictable contributions is shown here, with low contributions achieved through an increase in growth assets from the base level and predictable contributions achieved through a reduction in growth assets from the base level. The greatest reduction in excess contribution rate is achieved by introducing some growth asset
investment to very low growth asset allocations (as expected asset returns are less than the liability discount rate when the growth asset allocation is less than 20%) or moving away from a very short spread period.

**Figure 4.2:** Mean for base (by asset allocation and spread)
Figure 4.2: Mean for base (by asset allocation and spread) (continued)

The mean funding deficit plot shows similar results to the excess contributions plot in terms of the choice of asset allocation; however, as expected, the minimum deficit is obtained by using the shortest spread period rather than the longest spread period. The minimum funding deficit is obtained with 50% growth assets and a spread period of one
year, giving a mean funding deficit of 0.56% per annum of initial assets, compared with that of 0.88% for the base growth asset allocation and spread period. The greatest reduction in funding deficit is achieved by introducing some growth asset investment to very low growth asset allocations, for the same reasons described for excess contributions.

The objective function plot shows that the optimal growth asset allocation and spread period combination, given the coefficients described in Section 3.2.4, is a 60% allocation to growth assets and a three-year spread period, giving a value of 0.2859 (signified by the small triangle at this point in this and all future objective function plots in this paper). Using a lower allocation to growth assets increases the average contribution rate by too much to compensate for the lower excess contributions and funding deficits. Using a shorter spread period increases the excess contribution rate by too much to compensate for the lower funding deficits. Increasing growth asset allocation and spread period does not have as large an impact on the objective function, but increases excess contributions and funding deficits by too much to compensate for the lower average contributions.

It should be noted that the form of an objective function can exert significant influence on the recommended outcomes, thus the above result is only appropriate given the single objective function described in Section 3.2.4. The effect of different objective functions is outside the scope of this paper, and thus only a function of the form described in Section 3.2.4 is considered.

4.2 Alternative fixed inputs

4.2.1 Lump sum benefits only (LS)

This scenario pays all benefits as a lump sum according to the rules of the leaving service benefit. There are no deferred or pensioner members at any time as the payment of the lump sum upon exit from the scheme completely discharges the liability of the scheme. This form of benefit structure is common for a number of schemes in Australia.
Since a scheme paying only lump sum benefits has no deferred or pensioner members, the future service component is a larger proportion of all liabilities and the initial assets for full funding are smaller than the base scenario. For this reason, the trend towards surplus for all percentiles in Figure 4.3 is not as pronounced as for the base scenario, as the surplus has more use in funding future liabilities. This is consistent with the longer time for the percentiles of contribution rate to move to zero. The lower contribution rates for the higher percentiles reflects the fact that, like future liabilities, salaries paid to members are a higher proportion of all liabilities and deficits and thus the payment of any contributions to fund deficits is across a larger relative salary base.

The spread period has less of an impact on the average contribution objective than in the base scenario, with the 100% growth asset allocation being optimal for all spread periods in Figure 4.4. This is because the deficits for LS do not require as large an injection of additional contributions relative to salaries, making the higher volatility of the 100% growth asset allocation a less significant risk. Average contribution rates are broadly similar for both scenarios, as the lump sum benefit has a broadly similar actuarial value to the pension benefit.
Figure 4.4: Mean for lump sum (LS) (by asset allocation and spread)

![Graph showing mean for lump sum (LS) by asset allocation and spread with lines indicating median, 2.5/97.5 percentiles, and 70/90 percentiles for LS and Base scenarios.]

**Average Cont Rate**

**Average Excess Cont Rate**
Figure 4.4: Mean for lump sum (LS) (by asset allocation and spread) (continued)

Excess contributions and the funding deficit are minimised by a smaller allocation to equities than the base scenario, due to the shorter term of the lump sum liabilities. Interestingly, the funding deficit is not reduced by the smallest spread period in these cases, indicating that slow
removal of surplus at low growth asset allocations is more significant than fast removal of deficits in reducing funding deficits.

These offsetting factors lead to the optimal growth asset allocation and spread period under the objective function being very similar for LS compared with the base scenario, with an unchanged 60% growth asset allocation and an increased spread period from three to five years. The slightly lower values in this plot are reflective of the lower deficit levels relative to salaries, requiring a lower level of excess contributions when paying lump sums. However, this difference is immaterial and thus the coefficient $\beta$ of the objective function is not adjusted.

4.2.2 New entrants allowed (NE)

In this scenario each member who leaves active membership of the scheme is replaced by a new entrant. The gender and seniority of the new entrant is assumed to be identical to the leaving member, with age reduced to keep the demographic structure of the scheme consistent. Salary is also reduced to take into account promotional salary increases not yet experienced.

Figure 4.5: Percentiles for new entrants (NE) (by year)

![Percentiles for new entrants (NE) (by year)](image)

A scheme accepting new entrants has a constant regeneration of future service liabilities – therefore the trend towards surplus is not
nearly as significant as for the base scenario, as can be seen in Figure 4.5. This is because the regeneration of future liabilities brings increased use for the surplus in funding future liabilities. However a trend towards surplus does exist due to the expected return on assets being larger than the discount rate for liabilities; for upper percentiles of the funding level the trend is exponential when the investment earnings on surplus become greater than the contributions needed to fund future liabilities for current and new members. Similar to LS, the slower surplus trend is consistent with the longer time for the higher percentiles of contributions to move to zero. The lower contribution rates for the higher percentiles are reflective of the fact that, with new entrants being accepted, there is a constant regenerating source of future salaries against which to pay off deficits.

The contribution rate results in Figure 4.6 are broadly similar to LS, with the asset allocation being far more significant than the spread period in affecting average contribution rates. Like LS, a 100% allocation to growth assets is recommended to minimise average contribution rates, as deficits are smaller as a proportion of regenerating future salaries than the base scenario. However, salaries remain a relatively constant proportion of liabilities in NE, which is not the case for LS or the base scenario. For this reason, the average contribution rate is far lower for NE than the base scenario as the level of salaries does not decrease after surplus is achieved and contributions move to zero.

Conversely, the mean deficit is higher under NE than the base scenario, due to the introduction of new liabilities to the scheme making future deficits a larger proportion of initial assets. This lengthening of the time horizon of the liabilities leads to an increase in the allocation to growth assets to minimise the mean deficit. Average excess contributions are smaller under NE than the base scenario due to the larger salary base over which deficits are funded, although the effect on the growth asset allocation is again almost identical to that for mean deficits.
Figure 4.6: Mean for new entrant (NE) (by asset allocation and spread)

<table>
<thead>
<tr>
<th></th>
<th>Median</th>
<th>2.5 / 97.5 percentiles</th>
<th>70 / 90 percentiles</th>
</tr>
</thead>
<tbody>
<tr>
<td>NE</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Base</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Average Cont Rate

Average Excess Cont Rate

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This lengthening time horizon leads to the optimal growth asset allocation under the objective function being increased from 60% to 90% under NE. However, the spread period is reduced from three years to two years as the excess contributions required are far lower as a proportion of salaries than under the base scenario. The significantly
lower values in this plot are based on an update to the $\beta$ coefficient of
the objective function to 4.948 in order to ensure the contribution and
deficit factors are weighted in the same way as the discussions of Section
3.2.4.

4.3 Alternative discretionary inputs

The following scenarios are tested:

AL1. Each year the proportion allocated to growth assets is
reduced by 1% from the standard asset allocation for every
1% that the funding level is greater than 100% at the start of
the year (as recommended by Boulier et al., 1995, Cairns,
1995, and Haberman et al., 2003). This scenario tests if it is
beneficial to reduce investment risk as the funding level
exceeds 100%. Due to the trend towards surplus described in
Section 4.1, this scenario would be very similar to one where
the growth asset allocation is reduced with scheme maturity
(as recommended by Peskin, 1997). Hence a scheme
maturity scenario is not tested.

AL2. Each year the proportion allocated to growth assets is
increased by 1% from the standard asset allocation for every
1% that the funding level is greater than 100% at the start of
the year (as recommended by Taylor, 2002). This scenario
tests if it is beneficial to increase investment risk as the
funding level exceeds 100%.

AL3. Contributions are calculated triennially under an attained
age basis. This scenario is chosen as this is the maximum
period over which actuarial valuations can be undertaken in
Australia.

AL4. Application of contribution calculations is delayed until
12 months after the valuation date. This scenario is chosen to
allow for the fact that there is often a significant delay
between the effective date of actuarial valuations and the
application of contribution calculations.
AL5. Expected future investment returns are allowed for in contribution calculations. In this case the contribution is calculated in the same way as the expense under AASB119, which allows the interest cost on the liability to be more than offset by the expected return on assets. In this case, the removal of the bias of differences between the liability discount rate and the expected return on assets is expected to reduce or eliminate the trend towards surplus.

Reducing allocation to growth assets when in surplus (AL1) significantly reduces surpluses in Figure 4.7, whilst leaving deficit levels largely unchanged. The upper confidence levels for contribution rates indicate a very slight reduction in contributions due to this reduction in investment risk. Increasing the allocation to growth assets when in surplus (AL2) has the opposite effect.

Performing valuation calculations triennially (AL3) increases contributions and the funding level, mainly due to fact the initial contributions are paid for three years rather than one year. Similar results are found for when a one-year lag is introduced between valuation calculations and contribution application (AL4). Allowing for expected investment returns (AL5) reduces the initial contribution rate from 23.5% to 12.9%, with a corresponding reduction in funding levels (including larger deficits) and an increase in the contribution rate at the upper percentiles of the distribution. The median contribution rate moves to zero in four years rather than the five-years for the base scenario. However the funding level still shows a significant exponential increase, despite the contribution rates allowing for expected investment returns. The bias of unusable surplus compared with deficits being required to be funded ensures this trend is evident even when contributions allow for the expected return on assets and all other assumptions match expectations.
Figure 4.7: Percentiles for alternative discretionary inputs (by year)

<table>
<thead>
<tr>
<th>AL</th>
<th>Median</th>
<th>2.5 / 97.5 percentiles</th>
<th>70 / 90 percentiles</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base</td>
<td>Median</td>
<td>2.5 / 97.5 percentiles</td>
<td>70 / 90 percentiles</td>
</tr>
</tbody>
</table>

Contribution Rate (AL1)  
Funding Level (AL1)

Contribution Rate (AL2)  
Funding Level (AL2)
Figure 4.7: Percentiles for alternative discretionary inputs (by year) (continued)
Figure 4.8: Mean for alternative discretionary inputs (by asset allocation and spread)

Objective Function (AL1)

Objective Function (AL2)
Figure 4.8: Mean for alternative discretionary inputs (by asset allocation and spread) (continued)
The percentile plot results are confirmed by looking at the minimum values of the objective functions in Figure 4.8, with the minimum objective function reducing from 0.2859 for the base scenario to 0.2757 for AL1. The optimal initial growth asset allocation is increased from 60% to 70% due to the reduction in growth asset allocation when in surplus, whilst the optimal spread period is maintained at three years. The minimum objective function for AL2 of 0.3115 is larger than that of the base scenario. Therefore it can be concluded that, under the objective function described in Section 3.2.4, it is optimal to reduce the allocation to growth assets when in surplus for a scheme closed to new entrants, rather than maintain a constant asset allocation. This is consistent with the results described by Boulier et al. (1995), Cairns (1995) and Haberman et al. (2003) for schemes open to new entrants.

The significant increase in initial contributions required under triennial funding (AL3) and lagged contribution application (AL4) result in a deterioration of the minimum objective function to 0.3034 and 0.3047 respectively. This is consistent with the Cairns (1994) results when short amortisation periods are used. In reaction to this problem longer spread periods (five-years) and lower growth asset allocations
(50%) are also found for AL3 and AL4. Being less conservative in allowing for expected returns in contribution calculations (AL5) gives a higher minimum objective function than the base scenario with the reduction in average contributions being more than offset by the increase in deficit (hence the reduction in spread period from three to two years) to give an objection function of 0.3528. The objective function value at lower growth asset allocations is not as high under AL5 compared with the base scenario due to the normal contribution rate allowing for the lower returns.

4.4 Sensitivity analysis

The most important assumption inherent in the economic model is the 5.4% per annum risk premium of equity returns over long-term interest rates (see Table B.4). Whilst this figure is relatively consistent with longer term historical results in Australia (see Brailsford et al., 2008), many financial analysts debate the existence and size of a premium for returns on equities; Mehra (2003) provides a detailed review. Thus a brief sensitivity analysis is presented here. Since the driving force behind equity returns in the Wilkie model is dividend increases, a downward adjustment to $\mu_d$ from Table B.2 is considered. The current value of 0.0401 implies dividend increases of 4.01% greater than price inflation on average, a number that is very high considering Wilkie (1995) recommends a value of zero. An alternative value for $\mu_d$ of 2% (EQ) is tested, which effectively reduces the equity risk premium by 2% per annum.
Figure 4.9: Percentiles for EQ (by year)

<table>
<thead>
<tr>
<th>EQ</th>
<th>Base</th>
</tr>
</thead>
<tbody>
<tr>
<td>Median</td>
<td>Median</td>
</tr>
<tr>
<td>2.5 / 97.5 percentiles</td>
<td>2.5 / 97.5 percentiles</td>
</tr>
<tr>
<td>70 / 90 percentiles</td>
<td>70 / 90 percentiles</td>
</tr>
</tbody>
</table>

Contribution Rate

Funding Level

Figure 4.10: Mean for EQ (by asset allocation and spread)

As expected, the reduction in equity returns reduces the trend towards surplus and increases the contributions required in Figure 4.9. What is of most interest, however, is how the reduction affects the optimal asset allocation and spread period based on the objective function (with $\beta$ updated to 9.739 to reflect the different mix between contributions and deficits as per the process described in Section 3.2.4),
as shown in Figure 4.10. When $\mu_d$ is reduced to 2%, the optimal growth asset allocation declines from 60% to 50%, with the spread period increasing from three to five-years (although there is very little difference in the EQ2 results between three- and five-year spread periods).

5 Conclusions and future research

This paper has investigated the investment and contribution strategies in managing trustees and employer-sponsor objectives relating to a defined benefit superannuation scheme that is closed to new entrants. This is done via the development of stochastic economic and demographic models that are used to simulate the financial outcomes of a model scheme which pays pensions.

It is found that, given the model(s) described in Section 3.1, Appendix B and Appendix C, the objective function described in Section 3.2.4, and the base inputs described in Appendix A, it is optimal for the scheme to invest 60% of its assets in equities, with the remainder in fixed interest and bonds. It is optimal for surpluses and deficits to be spread over a three-year period. A similar recommendation (with slightly longer spread period) is made for schemes that pay benefits as lump sums only. The choice of asset allocation and spread period is found to differ for schemes that are open to new entrants, where an increased allocation to equities and a reduced spread period is recommended. The approximately 30% reduction in equity investment allocation for closed schemes is particularly relevant in Australia, where most schemes are closed to new entrants.

A significant trend towards surplus is observed, due to both the liability discount rate being lower than the expected return on assets and the need for employer-sponsors to make additional contributions to fund deficits without a corresponding ability to recover surplus. The trend towards surplus may cause some industrial relations concerns for schemes which are closed to new entrants in terms of debates about ownership of the surplus. It is optimal to reduce the allocation to equities as the funding level increases above 100%, due to the diminishing upside gain of the surplus. Additionally, a significant
deterioration in the objective function is observed when contributions are updated triennially rather than annually and when there is a delay of one year between the effective date of contribution calculations and their application. These results suggest that reviewing contribution strategy regularly and quickly is vital in achieving optimal outcomes.

It might also be expected that alternative coefficients or structures of the objective function will influence the results. Future research could investigate how the structure and weighting of different desires influences optimal investment and contribution strategies. More complicated and dynamic investment and contribution strategies could also be investigated.

Finally, the results of this study have been separated from the performance of the employer-sponsor. In reality, any objectives an employer-sponsor might have for a scheme will be related to the business’s overall objectives. Some discussion in the literature does exist on this issue. For example Ralfe et al. (2004) maintain that scheme liabilities are a debt of the employer-sponsor; therefore investing scheme assets in equities is effectively the same as issuing a long-term bond and investing the proceeds in equities. Gold (2005) argues that scheme investment in equities may hurt shareholders who are unaware of the volatility underlying the investment. Sharpe (1976) states that incorporating pension liabilities and assets into corporate balance sheets gives rise to call and put options due to the mismatch between asset and liability values. Some of these issues are discussed further in Chapter 3 of Blake (2006). Future research could incorporate the modelling done in this study into a broader framework considering overall employer-sponsor desires.
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Appendix A: Model scheme base scenario

Benefit design

The scheme has one category of defined benefit membership and was closed to new entrants at the commencement of projections. The benefit design of the scheme is as follows.

<table>
<thead>
<tr>
<th>Member Contributions</th>
<th>Nil.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Leaving service benefit</td>
<td>Annual pension of 1/60 of final salary per year of membership payable until death, commencing in the year turning age 65. For members who leave before completing seven years of service a lump sum benefit is payable, which is equal to the annual pension multiplied by 15 x $1.015^\text{ (age at withdrawal -65)}$.</td>
</tr>
<tr>
<td>Pension increases</td>
<td>Pensions are increased on an annual basis at the price inflation rate of the previous year, subject to a maximum increase of 10% and a minimum increase of 0%. This applies to both pensions in payment and those that are deferred until age 65.</td>
</tr>
<tr>
<td>Death benefit (active)</td>
<td>A lump sum benefit equal to 15 multiplied by the annual pension that would have been paid had the member remained in the scheme until age 65 with an unchanged salary.</td>
</tr>
<tr>
<td>Death benefit (deferred)</td>
<td>A lump sum benefit equal to the annual pension multiplied by 15.</td>
</tr>
<tr>
<td>Death benefit (pensioner)</td>
<td>For pensioners who die before age 75, a lump sum benefit equal to the annual pension multiplied by 1.5 x (75 – age at death).</td>
</tr>
<tr>
<td>Insured part of death benefit (active only)</td>
<td>$1/60 \times \text{salary} \times 15 \times (65 - \text{age at death})$</td>
</tr>
</tbody>
</table>
Member data

A summary of the membership of the scheme at the start of the projections is provided in Table A.1:

<table>
<thead>
<tr>
<th>Member Status</th>
<th>Active</th>
<th>Deferred</th>
<th>Pensioner</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Members</td>
<td>5,000</td>
<td>1,680</td>
<td>1,920</td>
</tr>
<tr>
<td>Average Age</td>
<td>38.0</td>
<td>51.8</td>
<td>75.6</td>
</tr>
<tr>
<td>Average Membership</td>
<td>4.6</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>Total Accrued Pension</td>
<td>$25,463,937</td>
<td>$20,428,937</td>
<td>$36,905,060</td>
</tr>
<tr>
<td>Average Accrued Pension</td>
<td>$5,096</td>
<td>$12,160</td>
<td>$19,221</td>
</tr>
<tr>
<td>Total Salary</td>
<td>$292,974,700</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>Average Salary</td>
<td>$58,595</td>
<td>N/A</td>
<td>N/A</td>
</tr>
</tbody>
</table>

Initial asset value

The scheme is exactly 100% funded at the start of the projection period.

Legislative and other environment

The main legislative factor affecting projections in the scheme is the tax basis. The tax on contributions and investment earnings is assumed to be the current rate of 15%, with two exceptions:

- Tax on Australian equity prices is 10%, reflecting capital gains tax on assets held for more than one year; and

- Tax on Australian equity dividends is -12%, reflecting an imputation credit on dividends (Australian Taxation Office, 2008, statistics reveal that approximately 75% of dividends have been franked over the past seven years to 30 June 2008).
Investment income tax on assets backing pensions in payment is nil, with a full franking credit of 32% paid on Australian equity dividends. A tax of 15% is also applied to the liability discount rate for pre-pensioner liabilities.

No other legislative factors are assumed to affect the scheme. Additionally, it is assumed that the covenant between the employer-sponsor and the scheme to provide benefits is strong enough to assume that the employer-sponsor is always willing and able to pay the contributions required.

**Investment strategy**

The investment strategy of the scheme is split between ‘growth’ (equities) and ‘defensive’ (all other) asset classes. Within these classifications the split between the assets classes is as follows:

**Growth assets**

- Australian equities 58.33%
- International equities 41.66%

**Defensive assets**

- Australian bonds 37.5%
- International bonds 25%
- Inflation-linked bonds 25%
- Australian cash 12.5%

Therefore, for a 60/40 split between growth and defensive assets, the asset allocation of the scheme is as follows:

- Australian equities 35%
- International equities 25%
- Australian bonds 15%
- International bonds 10%
- Inflation-linked bonds 10%
- Australian cash 5%
This is the base asset allocation of the scheme, with the effect of the split between growth and defensive assets tested in 10% increments across the spectrum 0% to 100% in the results. The asset allocation used to model the overall investment return is rebalanced at the end of each year, meaning that the asset allocation remains unchanged from year to year.

**Contribution strategy**

Contributions are calculated annually on a projected unit credit basis, with contributions applied immediately from the date of the valuation calculation. Past and future liabilities are discounted at a rate equivalent to that required under accounting standards (see Section 2.1 and Appendix B) with other assumptions based on expectations from the economic model (see Appendix B) and decrement models (see Appendix C). Under the base scenario, surpluses and deficits are spread over a period of three years, although the effect of the range of spread periods from one to 20 years is tested in the results. For a closed scheme, the diminishing numbers of members still accruing benefits makes it more appropriate to spread surplus and deficit on a fixed dollar per annum basis rather than as a percentage of salary.

**Surplus management**

Surplus is retained in the scheme at all times during the projection period. Upon the completion of projections, any surplus is paid to members as additional benefits. Should the scheme be in deficit at the completion of projections, the employer-sponsor immediately contributes the amount required to fund all liabilities (with an allowance for contributions tax).
Appendix B: Economic model

Wilkie (1995) is an update of Wilkie (1986), which considered only price inflation, share dividends, share dividend yields and long-term interest rates. Modelling share dividends and share dividend yields implicitly models share price returns, as seen in Table B.2. Wilkie (1995) extends Wilkie (1986) to incorporate wage inflation, short-term interest rates, property rentals, property yields, yields on inflation-linked bonds and currency markets. Wilkie (1995) states that the purpose of the model is not to provide the best statistical fit of past or future data in the short or medium term, but to produce relationships between the economic variables that hold over the longer term. Since the projections in this paper are undertaken over a 30-year period, it is felt that the long-term objectives of the Wilkie model are appropriate for this study.

Property returns are not considered in this study, due to the low allocation to property of most schemes, and the paucity and unreliability of Australian data available to fit such models. All allocation to risky assets is assumed to be made through domestic or international equities.

A flowchart of the relationships between the economic variables of interest in the Wilkie model is given in Figure B.1. An arrow from Variable X to Variable Y indicates that the calculation of Variable Y is dependent on Variable X, but not vice-versa. The returns on Australian bonds, international equities, international bonds and inflation-linked bonds are not part of the Wilkie model and therefore it is necessary to introduce additional relationships to model these variables. The return on Australian bonds is determined by levels of long- and short-term interest rates in both the current and previous periods, in order to take into account the effect that movements in interest rates have on the capital value of bonds. A similar approach is used for inflation-linked bonds, including an allowance for price inflation. International returns are correlated with their Australian counterparts, with expected returns set equal for equivalent Australian and international investments to ensure no bias between Australian and international investment. Currency movements for international equities are assumed to be unhedged, whilst for international bonds are assumed to be hedged.
Figure B.1: The Wilkie model cascade structure

* Not calculated directly by the Wilkie model, but can be obtained indirectly via Australian equity dividends and dividend yield.

^ Not part of the Wilkie model but included as an addendum to the original structure.

The Wilkie model parameters are estimated based on historical data. Table B.1 gives the indexes for the variables fitted in the economic model. Data from 30 June 1983 to 30 June 2009 is used, with the exception of International Bonds Total Return, Inflation-Linked Bond Yield and Inflation-Linked Bonds Total Return, for which the indexes are available only from 31 December 1984, 31 July 1986 and 31 March 1991 respectively. The reasons for this start date are as follows:
Both 30 June 1983 and 30 June 2009 reflect a recent resurgence in equity prices after a period of severe downturn;

Both dates had recently experienced significant reductions in interest rates;

All data except for that mentioned above is available over the whole period; and

The 26-year period between 30 June 1983 and 30 June 2009 is similar to the 30-year projection period being used in this study.

### Table B.1: Indexes used

<table>
<thead>
<tr>
<th>Variable</th>
<th>Index</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price Inflation</td>
<td>Consumer Price Index (all groups)</td>
</tr>
<tr>
<td>Salary Inflation</td>
<td>Average Weekly Ordinary Time Earnings (all employees)</td>
</tr>
<tr>
<td>Australian Equity Dividend Yield</td>
<td>S&amp;P/ASX200 Accumulation and Price Index</td>
</tr>
<tr>
<td>Australian Equity Dividends</td>
<td>S&amp;P/ASX200 Accumulation and Price Index</td>
</tr>
<tr>
<td>Australian Equity Prices</td>
<td>S&amp;P/ASX200 Price Index</td>
</tr>
<tr>
<td>Short-Term Interest Rate</td>
<td>90-Day Bank Accepted Bills</td>
</tr>
<tr>
<td>Long-Term Interest Rate</td>
<td>10-Year Australian Government Bond Yield</td>
</tr>
<tr>
<td>International Equities Total Return</td>
<td>MSCI World ex-Australia Net Index converted to A$ using the A$/US$ exchange rate</td>
</tr>
<tr>
<td>Return (Unhedged)</td>
<td></td>
</tr>
<tr>
<td>Australian Bonds Total Return</td>
<td>UBS Warburg Australia Composite Bond Accumulation Index</td>
</tr>
<tr>
<td>Return (Hedged into A$)</td>
<td></td>
</tr>
<tr>
<td>International Bonds Total Return</td>
<td>Citigroup World Government Bond Index – Hedged into A$</td>
</tr>
<tr>
<td>Return (Hedged into A$)</td>
<td></td>
</tr>
<tr>
<td>Inflation-Linked Bond Yield</td>
<td>10-Year Australian Government Inflation-Linked Bond Real Yield</td>
</tr>
<tr>
<td>Inflation-Linked Bonds Total Return</td>
<td>UBS Warburg Australian Inflation-Linked Government Bond All Maturities Total Return Index</td>
</tr>
</tbody>
</table>

However, there is a fundamental difference in the use of monetary policy in Australia between 1983 and 2009. In 1983, the Federal Government Treasurer maintained control of short-term interest rates and used them to target a certain monetary level in the economy. A major change occurred in the late 1980s and early 1990s to a system
where the Reserve Bank of Australia now controls monetary policy with an objective to keep price inflation between 2% and 3% per annum. This has reduced inflation significantly over the period 1983–2009, such that the estimated parameters generated by fitting a model over the period 1983–2009 may not adequately reflect current monetary policy. In addition, household debt levels have increased dramatically with decreases in interest rates over the past 26 years. This means small movements in interest rates have a much more significant impact than they did during the 1980s and therefore a return to the high interest rates of the 1980s is highly unlikely. Thus the interest rates projected by a model fit over the period 1983–2009 may be higher than likely in future.

Thus model parameters may not accurately represent expectations of future economic values, although this does not necessarily mean the parameters should automatically be adjusted to reflect this. The modelling performed in this study is measuring changes in assets and liabilities of a superannuation scheme. The economic factors most affecting the liabilities of a superannuation scheme are price inflation, salary inflation and long-term interest rates, whilst investment returns most affect the value of assets. The cascade structure used ensures price inflation flows through to other economic variables; therefore no adjustment is made to the estimated parameter values to reflect the issues discussed above. Although future expectations of the economic factors may be lower than those generated by the economic model, this is expected to have equal effect on the assets and liabilities of the scheme and thus not bias the comparison of the two.

The model is fit using annual data with all interest rates and returns being modelled on a continuously compounding basis. The model equations and fitted parameters are found in Tables B.2 and B.3. All error terms $\varepsilon(t)$ are independently and identically distributed normally with mean zero and variance $\sigma^2$. Summary statistics for the output of the model over 30 years of projections and 1,000 simulations are provided in Table B.4; average returns are geometric.
### Table B.2: Wilkie model – summary of equations used

<table>
<thead>
<tr>
<th>Variable</th>
<th>Notation</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price Inflation</td>
<td>$q(t)$</td>
<td>$q(t) = \mu_q (1 - \phi_q) + \phi_q q(t - 1) + \varepsilon_q (t)$</td>
</tr>
<tr>
<td>Salary Inflation</td>
<td>$w(t)$</td>
<td>$w(t) = \psi_{w,t} q(t - 1) + \mu_w + \varepsilon_w (t)$</td>
</tr>
<tr>
<td>Australian Equity Dividend Yield</td>
<td>$y(t)$</td>
<td>$\ln [y(t)] = \ln \mu_y + X_y (t)$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$X_y (t) = \phi_y X_y (t - 1) + \varepsilon_y (t)$</td>
</tr>
<tr>
<td>Australian Equity Dividends</td>
<td>$d(t)$</td>
<td>$d(t) = q(t) + \mu_d + \tau_{d,1} \varepsilon_y (t) + \tau_{d,2} \varepsilon_y (t - 1)$ + $\varepsilon_d (t) + \theta_d \varepsilon_d (t - 1)$</td>
</tr>
<tr>
<td>Australian Equities Price Return</td>
<td>$p(t)$</td>
<td>$p(t) = \ln \left( \frac{D(t)}{\ln (1 + y(t))} \right) - \ln (P(t - 1))$</td>
</tr>
<tr>
<td>Australian Equities Total Return</td>
<td>$ae(t)$</td>
<td>$ae(t) = p(t) + \ln \left( \frac{1 + (\exp (y(t)) - 1)}{\exp (p(t))} \right)^{0.5}$</td>
</tr>
<tr>
<td>Long-Term Interest Rate</td>
<td>$il(t)$</td>
<td>$il(t) = \psi_{il} M_{il} (t) + \mu_{il} + X_{il} (t)$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$M_{il} (t) = \rho_{il} q(t) + (1 - \rho_{il}) M_{il} (t - 1)$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$X_{il} (t) = \phi_{il,1} X_{il} (t - 1) + \phi_{il,2} X_{il} (t - 2) + \phi_{il,3} X_{il} (t - 3) + \varepsilon_{il} (t)$</td>
</tr>
<tr>
<td>Short-Term Interest Rate</td>
<td>$is(t)$</td>
<td>$is(t) = \ln [i_s (t)] = \ln \left[ i_l (t) \right] - X_{is} (t)$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$X_{is} (t) = \mu_{is} (1 - \phi_{is}) + \phi_{is} X_{is} (t - 1) + \varepsilon_{is} (t)$</td>
</tr>
<tr>
<td>Australian Cash</td>
<td>$ac(t)$</td>
<td>$ac(t) = (is(t) + is(t - 1)) / 2$</td>
</tr>
<tr>
<td>International Equities Total Return</td>
<td>$ie(t)$</td>
<td>$ie(t) = \mu_{ie} + \psi_{ie} ae(t) + \varepsilon_{ie} (t)$</td>
</tr>
<tr>
<td>Australian Bonds Total Return</td>
<td>$ab(t)$</td>
<td>$ab(t) = \psi_{ab,1} il(t) + \psi_{ab,2} il(t - 1) + \psi_{ab,3} is(t) + \psi_{ab,4} is(t - 1) + \varepsilon_{ab} (t)$</td>
</tr>
<tr>
<td>International Bonds Total Return</td>
<td>$ib(t)$</td>
<td>$ib(t) = \mu_{ib} + \psi_{ib} ab(t) + \tau_{ib} \varepsilon_q (t) + \varepsilon_{ib} (t)$</td>
</tr>
<tr>
<td>Inflation-Linked Bond Yield</td>
<td>$qy(t)$</td>
<td>$qy(t) - (\tilde{\psi}<em>{il} - 1) M</em>{il} (t) = X_{qy} (t)$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$X_{qy} (t) = \mu_{qy} (1 - \phi_{qy}) + \phi_{qy} X_{qy} (t - 1) + \tau_{qy} \varepsilon_q (t) + \varepsilon_{qy} (t)$</td>
</tr>
<tr>
<td>Inflation-Linked Bond Total Return</td>
<td>$qb(t)$</td>
<td>$qb(t) = q(t) + \psi_{qb,1} qy(t) + \psi_{qb,2} qy(t - 1) + \varepsilon_{qb} (t)$</td>
</tr>
</tbody>
</table>

^ Notation in capitals [eg. $D(i) = D(t - 1) \exp (d(t))$] refers to the index value of that variable.
Table B.3:  Wilkie model – fitted parameter values and standard errors

<table>
<thead>
<tr>
<th>Notation</th>
<th>Standard error residuals</th>
<th>Parameter</th>
<th>Fitted value</th>
<th>Standard error parameter estimate</th>
</tr>
</thead>
<tbody>
<tr>
<td>q(t)</td>
<td>0.0164</td>
<td>( \mu_q )</td>
<td>0.0349</td>
<td>0.0104</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \phi_q )</td>
<td>0.7218</td>
<td>0.1276</td>
</tr>
<tr>
<td>w(t)</td>
<td>0.0103</td>
<td>( \psi_{w,2} )</td>
<td>0.4896</td>
<td>0.0769</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \mu_w )</td>
<td>0.0298</td>
<td>0.0037</td>
</tr>
<tr>
<td>y(t)</td>
<td>0.1654</td>
<td>ln(( \mu_y ))</td>
<td>-3.2457</td>
<td>0.0610</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \phi_y )</td>
<td>0.4900</td>
<td>0.1774</td>
</tr>
<tr>
<td>d(t)</td>
<td>0.0925</td>
<td>( \mu_d )</td>
<td>0.0401</td>
<td>0.0269</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \tau_{d,1} )</td>
<td>0.3801</td>
<td>0.1195</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \tau_{d,2} )</td>
<td>-0.2601</td>
<td>0.1236</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \theta_d )</td>
<td>0.5014</td>
<td>0.1664</td>
</tr>
<tr>
<td>il(t)</td>
<td>0.0057</td>
<td>( \psi_{il} )</td>
<td>1.2268</td>
<td>0.0032</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \mu_{il} )</td>
<td>0.0221</td>
<td>0.0595</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \rho_{il} )</td>
<td>0.1720</td>
<td>N/A</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \phi_{il,1} )</td>
<td>0.1105</td>
<td>0.1859</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \phi_{il,2} )</td>
<td>-0.2994</td>
<td>0.1641</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \phi_{il,3} )</td>
<td>-0.3369</td>
<td>0.1620</td>
</tr>
<tr>
<td>is(t)</td>
<td>0.1939</td>
<td>( \mu_{is} )</td>
<td>0.0937</td>
<td>0.0644</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \phi_{is} )</td>
<td>0.4324</td>
<td>0.1900</td>
</tr>
<tr>
<td>ie(t)</td>
<td>0.1378</td>
<td>( \mu_{ie} )</td>
<td>0.0074</td>
<td>N/A</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \psi_{ie} )</td>
<td>0.9341</td>
<td>0.1860</td>
</tr>
<tr>
<td>ab(t)</td>
<td>0.0075</td>
<td>( \psi_{ab,1} )</td>
<td>-3.5495</td>
<td>0.2248</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \psi_{ab,2} )</td>
<td>4.4186</td>
<td>0.2047</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \psi_{ab,3} )</td>
<td>-0.2434</td>
<td>0.1139</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \psi_{ab,4} )</td>
<td>0.3789</td>
<td>0.1152</td>
</tr>
<tr>
<td>ib(t)</td>
<td>0.0297</td>
<td>( \mu_{ib} )</td>
<td>0.0225</td>
<td>N/A</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \psi_{ib} )</td>
<td>0.7788</td>
<td>0.1175</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \tau_{ib} )</td>
<td>1.2243</td>
<td>0.4134</td>
</tr>
<tr>
<td>qy(t)</td>
<td>0.0041</td>
<td>( \phi_{qy} )</td>
<td>0.7594</td>
<td>0.1321</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \tau_{qy} )</td>
<td>0.4757</td>
<td>0.1590</td>
</tr>
<tr>
<td>qb(t)</td>
<td>0.0108</td>
<td>( \psi_{qb,1} )</td>
<td>-8.4877</td>
<td>0.5163</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \psi_{qb,2} )</td>
<td>9.5807</td>
<td>0.4960</td>
</tr>
</tbody>
</table>
### Table B.4: Results generated by the Wilkie model

<table>
<thead>
<tr>
<th>Factor</th>
<th>Average return % (p.a.)</th>
<th>Standard deviation % (p.a.)</th>
<th>Yearly autocorrelation % (average)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price Inflation</td>
<td>3.6</td>
<td>2.4</td>
<td>71</td>
</tr>
<tr>
<td>Salary Inflation</td>
<td>4.8</td>
<td>1.6</td>
<td>39</td>
</tr>
<tr>
<td>Long-Term Interest Rate</td>
<td>6.8</td>
<td>1.8</td>
<td>85</td>
</tr>
<tr>
<td>Australian Equities Total Return</td>
<td>12.2</td>
<td>18.6</td>
<td>15</td>
</tr>
<tr>
<td>International Equities Total Return</td>
<td>12.3</td>
<td>23.7</td>
<td>9</td>
</tr>
<tr>
<td>Australian Bonds Total Return</td>
<td>6.8</td>
<td>4.6</td>
<td>6</td>
</tr>
<tr>
<td>International Bonds Total Return</td>
<td>6.8</td>
<td>4.7</td>
<td>0</td>
</tr>
<tr>
<td>Australian Cash</td>
<td>6.3</td>
<td>2.0</td>
<td>85</td>
</tr>
<tr>
<td>Inflation-Linked Bond Yield</td>
<td>3.1</td>
<td>0.8</td>
<td>78</td>
</tr>
<tr>
<td>Inflation-Linked Bonds Total Return</td>
<td>7.1</td>
<td>5.6</td>
<td>4</td>
</tr>
<tr>
<td>Base (non-pension assets)</td>
<td>9.6</td>
<td>10.3</td>
<td>14</td>
</tr>
<tr>
<td>Base (pension assets)</td>
<td>11.1</td>
<td>11.8</td>
<td>14</td>
</tr>
</tbody>
</table>

A promotional salary inflation scale is also used. The scale can be found in Appendix C.3 and is based on the Employee Earnings, Benefits and Trade Union Membership report (Australian Bureau of Statistics (ABS), various issues). These values were tested across a variety of years from 1999 to 2007 and found to be relatively consistent across this time period and gender. Thus a single promotional salary scale is used for males and females and is assumed to be consistent over the life of the projections. The total salary increase for an individual is equal to the salary inflation calculated in the economic model plus the age-based promotional salary inflation found in Table C.3.

The discount rate, which is based on accounting standards (see Section 2.1), is not considered as part of the rest of the economic model due to a lack of data available to parameterise such a model. AASB 119 requires a discount rate to be ‘determined by reference to market yields at the end of the reporting period on high quality corporate bonds. In countries where there is no deep market in such bonds, the market yields (at the end of the reporting period) on government bonds shall be used’. Typically, AA-rated bonds are used as reference for the discount
rate required under AASB 119. In Australia, many actuaries use a
government bond discount rate due to the ‘deep market’ requirement.
Exposure draft ED/2009/10 from IASB has indicated a desire to remove
from IAS 19 (on which AASB 119 is based) the ‘deep market’
requirement for AA-rated bonds to be used rather than a risk-free rate
and require estimation techniques to be used where a ‘deep market’
does not exist, although this project has been deferred. Therefore it is
decided to base the discount rate on AA-rated bonds without analysing
if a deep market is thought to exist.

In determining discount rates to be used it should first be noted that,
for simplicity, the long-term interest rate from the economic model
(which is based on a maturity period of 10 years) is assumed to be
representative of yields for all maturities on government bonds, despite
scheme liabilities mostly being far longer in maturity than 10 years. At
30 June 2009, less than 50 AA-rated (by Moody) corporate bonds
existed in Australia, with none having a term to maturity greater than 10
years. Thus a measure of credit spread between government and AA-
rated bonds over a 10 year maturity period is not possible. Anecdotal
evidence suggests that actuaries use a range of techniques in estimating
spreads for this purpose, from extrapolating short-term corporate bond
spreads, to using spreads on swap rates (for which longer duration rates
are available). All solutions are characterised by a lack of data.

AA-rated spreads over shorter terms have been relatively consistent
over the past 10 years, until the commencement of the global financial
crisis in late 2007. The model ensures the discount rate, \( dr(t) \), moves in
line with movements in risk-free rates, with a constant spread of 0.5%
(continuously compounding), except when risk-free rates are much
lower than expected, in which case the spread increases (ie. the discount
rate stabilises):

\[
dr(t) = il(t) + 0.5\% + \max \left( 0, -0.8\% - X_{il}(t) \right)
\]

No error is allowed for in the model, as it is not based on actual data.
When developing random economic values for simulation, neutral starting conditions are used, which means all economic variables needed for $t \leq 0$ are assumed to be equal to zero or the long-term mean, whichever appropriate. Price inflation expectations for actuarial valuation calculations are based on $M_p(t)$ from Table B.2, whilst salary inflation expectations are generated from price inflation expectations applied to the salary inflation model.
Appendix C: Decrement models

C.1 Withdrawal rates

In order to determine underlying withdrawal rates, analysis has been performed on the Labour Force Survey and Labour Mobility Survey Confidentialised Unit Record File (2008) (CURF) produced by the Australian Bureau of Statistics (ABS). Age and job tenure are both found to be negatively correlated with the probability of leaving a job, with the job tenure effect being relatively consistent between genders. Females are more likely to leave a job during child-bearing years (until age 40), with rates being fairly consistent between males and females after this date.

There is much debate in the literature regarding the effect and significance of economic factors on labour turnover. There are two established arguments linking labour turnover to economic activity. The ‘chilling’ hypothesis (Osberg, 1991), states that in times of economic downturn and high unemployment workers will not seek alternative employment. The ‘structural adjustment’ hypothesis (Lillen, 1982), states that differences in hiring rates between industries in an economic downturn encourages labour turnover between industries.

Wooden (1999) presents aggregate information on the percentage of Australians who voluntarily or involuntarily left a job for biennial periods from February 1988 to February 1998. The data source for this table is the ABS Labour Mobility report (which is based upon the CURF described above); Table C.1 presents the information from Wooden (1999) extended to February 2008. The periods in Table C.1 are characterised by strong economic growth at all times except the early 1990s. The voluntary and involuntary columns show a clear opposing trend, with individuals more likely to leave a job voluntarily during times of boom, supporting the ‘chilling’ hypothesis, but more likely to lose a job involuntarily in recession, supporting the ‘structural adjustment’ hypothesis. However, the trend is not so clear in total, with the voluntary/involuntary trends offsetting each other.
Table C.1: Job ceasing statistics 1988 - 2008

<table>
<thead>
<tr>
<th>12 months ended February</th>
<th>Voluntary (%)</th>
<th>Involuntary (%)</th>
<th>Total (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1988</td>
<td>12.3</td>
<td>8.2</td>
<td>20.5</td>
</tr>
<tr>
<td>1990</td>
<td>13.8</td>
<td>8.0</td>
<td>21.8</td>
</tr>
<tr>
<td>1992</td>
<td>8.5</td>
<td>9.5</td>
<td>18.0</td>
</tr>
<tr>
<td>1994</td>
<td>9.9</td>
<td>8.8</td>
<td>18.7</td>
</tr>
<tr>
<td>1996</td>
<td>11.5</td>
<td>8.0</td>
<td>19.5</td>
</tr>
<tr>
<td>1998</td>
<td>10.5</td>
<td>7.6</td>
<td>18.1</td>
</tr>
<tr>
<td>2000</td>
<td>11.7</td>
<td>7.2</td>
<td>18.9</td>
</tr>
<tr>
<td>2002</td>
<td>13.3</td>
<td>8.1</td>
<td>21.4</td>
</tr>
<tr>
<td>2004</td>
<td>14.3</td>
<td>6.5</td>
<td>20.8</td>
</tr>
<tr>
<td>2006</td>
<td>12.6</td>
<td>6.0</td>
<td>18.6</td>
</tr>
<tr>
<td>2008</td>
<td>12.7</td>
<td>5.4</td>
<td>18.1</td>
</tr>
</tbody>
</table>

Analysis has also been performed on the Labour Mobility Survey (1994) CURF to investigate the effect of different economic conditions on age, gender and job tenure influences of labour turnover. When compared with 2008, 1994 was a slower economic period, with Australia having recently come out of a recession and experiencing high unemployment and decreasing interest rates. Results are relatively consistent with those of 2008, thus no adjustment is made to withdrawal rates to allow for economic factors.

Underlying withdrawal rates for each age, gender and job tenure can be found in Appendix C.3. It is assumed that no industry or employer-specific factors affect withdrawal rates and thus only binomial variability through individual withdrawal decisions is observed.

C.2 Mortality rates

The starting point in generating mortality rate assumptions is the most recent Australian Life Tables (ALT) produced by the Australian Government Actuary (2009). These relate to the three-year period 2005–07 and are split by age and gender. ALT 2005–07 also provides information on mortality improvement from 1891 to 1900 until 2005–07.
For the purposes of this study a life reaching age 110 is assumed to die in the next year.

Mortality improvements are allowed for in a similar fashion to that described in Section 3 of the ALT 2005–07 report, although smoothed reduction factors are used to ensure a consistent shape in mortality rates for the period of projection. See Appendix C.3 for underlying mortality rates \( q(x) \) and improvement factors \( rf(x) \).

The literature on the link between economic factors and mortality is inconclusive. Ruhm (2004) provides a summary of recent literature and its contradictions. As such, no allowance is made for economic effects on mortality.

It now remains to consider variance of underlying mortality rates. This is incorporated by modelling the mortality rate \( q(x,t) \) for an individual aged \( x \), at \( t \) years from the commencement of projections, as follows:

\[
q(x,t) = q(x) \times (rf(x))^{k(t)}
\]

where \( k(t) = k(t - 1) + 1 + \epsilon(t) \), \( k(0) = 0 \) and \( \epsilon(t) \) is distributed normally with mean zero and variance \( \sigma^2 \) and uncorrelated for \( t \). Taking the natural logarithm of both sides of the equation gives:

\[
\ln[q(x,t)] = \ln[q(x)] + k(t) \times \ln[rf(x)]
\]

This model is a simple form of the Lee and Carter (1992) model. The error term \( \epsilon(t) \) represents unexpected shocks to underlying mortality, in addition to the mortality improvement already assumed.

In estimating \( \sigma^2 \) the actual mortality rates from ALT 1990–92 to ALT 2005–07 are compared with the expected mortality rates using the previous table 25-year average improvement factors, obtaining the observed \( \epsilon^2 \) for each gender and age as per the model above. It is noted that this process estimates mortality rates five-years away, whereas in this study mortality rates are estimated from year to year. Table C.2
gives the average observed $\varepsilon^2$ for a variety of combinations of age and gender from this approach:

Table C.2: Mortality variability average $\varepsilon^2$

<table>
<thead>
<tr>
<th>Years</th>
<th>Male (17–100)</th>
<th>Male (65–90)</th>
<th>Female (17–100)</th>
<th>Female (65–90)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2005-07</td>
<td>450.67</td>
<td>5.53</td>
<td>58.18</td>
<td>6.66</td>
</tr>
<tr>
<td>2000-02</td>
<td>67.37</td>
<td>29.67</td>
<td>23.07</td>
<td>13.09</td>
</tr>
<tr>
<td>1995-97</td>
<td>16.22</td>
<td>5.41</td>
<td>7.15</td>
<td>3.84</td>
</tr>
<tr>
<td>1990-92</td>
<td>412.24</td>
<td>30.99</td>
<td>128.94</td>
<td>2.50</td>
</tr>
</tbody>
</table>

The reason for having a separate column for ages 65–90 is that the mortality rates at these ages have the largest influence on the liabilities of a pension-paying scheme. The lower numbers for the 65–90 age groups reflect the greater ability to predict mortality improvement at these ages compared with younger ages. In general, male mortality improvement has been more difficult to predict than females’, due to the increasing improvement of male mortality in recent decades. Based on the results in Table C.2, a value of $\sigma^2 = 2.5$ is selected based on an average 65–90 error squared term of approximately 12.5, then divided by five for an annual basis. This implies that $k(t)$ is increased each year by an average of one year, with a standard deviation of 1.6 years.

The most important characteristic of the above model is that mortality shocks are carried forward through the non-stationary autoregressive structure of $k(t)$. These shocks are assumed to be identified and flow directly through to actuarial assumptions used for liability calculations each year, in that mortality assumptions are based on the up to date $k(t)$ and future mortality improvement with no $\varepsilon(t)$.

The insurance premium for an individual is equal to the sum insured multiplied by the mortality rate multiplied by 1.5, which indicates a 33% loading on the risk premium for profits, expenses and commissions.
C.3 Age-based decrement and promotional salary inflation tables

The promotional salary inflation and decrements that affect the scheme each year are outlined in the tables below. The withdrawal decrement is based on a new entrant to the scheme. The withdrawal rates are reduced for membership effects using a formula that reflects the reduction in withdrawal rates with job tenure, with an increasing effect in early job tenure for those over age 35. The withdrawal decrement reduction (per 1,000 members) for an individual aged x with membership of m years is calculated as $20m + 0.5 \times \max(0, x - 35) \times \min(m, 5)$. The overall withdrawal decrement is subject to a minimum of 30 per 1,000 members.

<table>
<thead>
<tr>
<th>Age (x)</th>
<th>Promotional salary inflation</th>
<th>Withdrawal^ (Male)</th>
<th>Withdrawal^ (Female)</th>
<th>Death^ (q(x)) (Male)</th>
<th>Death^ (q(x)) (Female)</th>
<th>Death (rf(x)) (Male)</th>
<th>Death (rf(x)) (Female)</th>
</tr>
</thead>
<tbody>
<tr>
<td>17</td>
<td>9.5%</td>
<td>300</td>
<td>325</td>
<td>0.54</td>
<td>0.26</td>
<td>0.973</td>
<td>0.975</td>
</tr>
<tr>
<td>18</td>
<td>8.5%</td>
<td>300</td>
<td>327</td>
<td>0.70</td>
<td>0.28</td>
<td>0.973</td>
<td>0.975</td>
</tr>
<tr>
<td>19</td>
<td>7.5%</td>
<td>300</td>
<td>329</td>
<td>0.73</td>
<td>0.28</td>
<td>0.973</td>
<td>0.975</td>
</tr>
<tr>
<td>20</td>
<td>6.5%</td>
<td>300</td>
<td>331</td>
<td>0.74</td>
<td>0.28</td>
<td>0.973</td>
<td>0.975</td>
</tr>
<tr>
<td>21</td>
<td>5.5%</td>
<td>300</td>
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<td>Withdrawal(^\wedge)</td>
<td>Death(^\wedge) ((q(x)))</td>
<td>Death ((rf(x)))</td>
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<td>10.85</td>
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</table>

~ Add an additional 4% for senior employees under age 40 and an additional 12% for executive employees under age 40.

\(^\wedge\) Per 1,000 members. The decrements above are assumed to be dependent.

* All remaining members who do not experience mortality.
Table C.4: Post-retirement base mortality $q(x)$*

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* Per 1,000 members.
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Estimating Value-at-Risk for Portfolios: Skewed-EWMA Forecasting via Copula

Z Lu, S Li*

Abstract

With the increasing complexity of risks, how to estimate the risk of portfolios with complex dependencies is challenging. Recently, Lu and Huang (2007) proposed a skewed-EWMA procedure to calculate value-at-risk (VaR) for individual financial assets, which is derived from an asymmetric Laplace distribution and takes into account both skewness and heavy tails of the return distribution that are adaptive to the time-varying nature in practice by adjusting shape parameter in the distribution. In this paper, we extend the skewed-EWMA procedure to estimating the risk of complex portfolios with dependencies modelled via copula. Monte Carlo simulation procedure that combines copula techniques with skewed-EWMA forecasting is developed. The empirical backtesting evaluation of the VaR forecasting demonstrates that the proposed method can be a useful tool in estimating extreme risks of some complex portfolios.

Keywords: Value-at-risk (VaR) for complex portfolio, copula, skewed-EWMA forecasting, Monte Carlo simulation, back testing

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1 Introduction

Value at Risk (VaR) is probably one of the most popular risk measures and plays a central role in risk management. In the past decades, it has greatly extended its range of applications in risk management, from purely market risk, to credit and operational risks, etc., and from the measurement of financial risk to the measurement of insurance risk. For example, early in 1993 the Group of Thirty (G-30) advised to value positions using market prices and to assess financial risks with VaR, and in 1996, the Basel Committee on Banking Supervision endorsed the use of VaR, contingent on important qualitative and quantitative standards. As Jorion (2000, preface) pointed out, risk management had truly experienced a revolution in the last few years, and this was started by VaR, a new method to measure financial market risk that was developed in response to the financial/derivative disasters of the early 1990s. The recent Global Great Financial Crisis (GFC) since 2007 has once again called into question financial risk management (FRM) methods and practice. The measurement of financial risk has also become one of the main preoccupations of actuaries and insurance practitioners. Comprehensive reviews of VaR and its developments from financial perspectives can be found in Duffie and Pan (1997), Dowd (1998), Jorion (1997, 2001), Dempster (2002), Allen (2003), Holton (2003), among others, and from an insurance perspective, the reader is referred to a recent survey by Dowd and Blake (2006).

In the literature, different methods have been developed to estimate the VaR for individual financial assets or simple portfolios that consist of constant weight investment components over a time period concerned. The popular methods include parametric models, historical simulation and Monte Carlo simulation, as well as extreme value theory. Often ARCH/GARCH models established by Engle (1982) and Bollerslev (1986) are utilised to estimate time-varying volatility, which is an important component in estimating VaR. A significant driving force behind the popularity of VaR may be the release to the public of JP Morgan’s (1996) RiskMetrics Technical Document, where an exponentially weighted moving average (EWMA) estimator, a kind of
integrated GARCH model, was developed to forecast the conditional volatility of asset returns in terms of conditional variance. This standard-EWMA estimator is appropriate for financial assets when a return series has an (approximate) conditional Gaussian distribution. Recently, Guermat and Harris (2001) established a robust EWMA estimator using an alternative procedure based on the Laplace distribution. Their empirical research showed that a robust EWMA estimator expressed in terms of the absolute value, rather than squared value, of the return series offers an improvement over the standard EWMA estimator. This estimator takes into account the heavy tails of the return distributions. However, a Laplace distribution is a symmetric distribution with a constant kurtosis, which may not always be suited to real return series. Lu and Huang (2007) therefore proposed a skewed-EWMA procedure to estimate VaR for individual financial assets. The skewed EWMA, derived from an asymmetric Laplace distribution, is an extended version of, and takes as a special case, the robust-EWMA of Guermat and Harris (2000). This extended version is more adaptive and flexible, allowing the shape parameter to vary with time, which controls the skewness and kurtosis of the return distributions. The backtesting results provided in Lu and Huang (2007) have demonstrated the flexibility of the skewed-EWMA over the robust-EWMA as well as the standard EWMA for individual assets or simple portfolios.

With the increasing complexity of risks, the problem of estimating risks for the portfolios with complex dependencies is a challenging one. In the literature the methodology of calculating the VaR for univariate assets has been quite mature (c.f., the references in the above). However, the calculation of VaR for multivariate assets becomes more difficult, partly because it is often hard to describe the interdependence in the joint distribution of a multivariate random vector. Recently, a copula function has been found useful to model the dependence structure in multivariate distributions with applications to risk management problems in finance (Nelson 1999). It links univariate marginal distributions to their full multivariate distribution by using transformed marginal variates with a uniform distribution. For example, Li (2000) studied the problem of default correlation using a copula function for credit risk; Cherubini and Luciano (2001) estimated VaR by
using the Archimedean copula family; Georges et al. (2001) used Gaussian copulas to model options and derivative pricing. These publications have illustrated a wide range of applications of copula functions in the area of quantitative finance.

In this paper, our main objective is to extend the skewed-EWMA procedure of Lu and Huang (2007) by combining with copulas, to estimate the risks for the portfolios with complex dependencies and/or varying-weight investing strategies. An important advantage of using copulas lies in that we can separate the marginal distributions from the complex dependence structure, which makes it possible and simpler to deal with multivariate joint distributions. For example, in practical risk management, the investing components of a portfolio may often need to be adjusted with time. In such a context, we cannot take for granted treating the component-adjusted or varying-weight portfolio like an individual financial asset in the estimation of its VaR. In this paper, the methodology developed will allow for a complex portfolio with its components adjusted or changing with time. Note that the unconditional distribution of such a portfolio return series may vary with time owing to the adjustment of the portfolio weights, and hence the robust – or skewed – EWMA method may not be simply applicable as Guermat and Harris (2001) and Lu and Huang (2007) did for simple portfolios. We therefore suggest building up the joint distribution for a complex portfolio by using the copulas to combine the marginal distributions, where the individual marginal distributions can be modelled by utilising the skewed-EWMA of Lu and Huang (2007). Monte Carlo simulation procedures that combine the copula techniques with the skewed-EWMA procedure will be developed. Backtesting evaluation of the proposed VaR forecasting procedure for portfolios will then be examined empirically.

The structure of this paper is as follows. Section 2 introduces the proposed methodology for estimating the VaR for complex portfolios by combining a copula function with the skewed-EWMA procedure of Lu and Huang (2007). Section 3 presents empirical applications of our proposed method to the constant-weight and varying-weight portfolios consisting of Chinese stock market indexes, respectively. The results
show that our new method provides reasonably accurate outcomes in predicting the VaR for complex portfolios, where the skewed-EWMA may not simply apply. Section 4 concludes with remarks.

2 Combining copula with skewed-EWMA for estimating portfolio VaR

2.1 VaR of a portfolio

The VaR of a portfolio at time \( t \), at significance level \( \alpha \), where \( \alpha \in (0,1) \), can be defined as:

\[
VaR_t(\alpha) = \inf \{ s : F_{r_{p,t}}(s) \geq \alpha \}, \tag{2.1}
\]

where \( F_{r_{p,t}} \) is the conditional distribution function of the one-period portfolio return \( r_{p,t} \) at time \( t \) given the information up to time \( t-1 \). This means that we are confident at level \( 100(1-\alpha)\% \) that the loss over the given period will not be larger than \( VaR_t(\alpha) \).

How to estimate the VaR of a portfolio? One key point is how to get \( F_{r_{p,t}} \). For this, two important factors need to be considered carefully, that is:

- the marginal distributions; and
- the joint distribution

of the returns on the individual assets that compose the portfolio. Many methods can be used to estimate the VaR for an individual financial asset (see the references listed in Section 1). In order to catch the time-varying features of skewness and heavy tails in the individual return distributions, we will adopt an asymmetric Laplace distribution with a time-varying shape parameter for the marginal distributions as Lu and Huang (2007) proposed, where an approach named skewed-EWMA is developed to forecast the changing volatility. The basic idea for the marginal distributions will be introduced in Subsection 2.2. For the joint distribution, multivariate normal distributions are traditionally often assumed on the returns of assets. This assumption is simple for calculating
VaR, but suffers from the violation of real financial observations, leading to a lower estimate of the risk in the event of extremes. To get across this difficulty, we suggest using the technique of the copulas to construct the joint distribution. The basic idea on how to use copulas to construct the joint distribution will be outlined in Subsection 2.3.

2.2 Skewed-EWMA forecasting for marginal distributions

A skewed-EWMA method, motivated from an asymmetric Laplace distribution (ALD), which is an extended Laplace distribution (c.f., Stockute and Paul Johnson, 2006), takes into account both the skewness and heavy tails of the financial distributions that are adaptively of a time-varying nature through the shape parameter adjusted by another EWMA procedure. Here the ALD is defined, in an alternative form to that of Kozubowski and Podgorski (1999, 2000, 2001), Kotz et al. (2002) and Yu and Zhang (2005), as follows.

**Definition 1:** A random variable $X$ has an asymmetric Laplace distribution (ALD), denoted by $X \sim AL(\mu, \sigma, p)$, if it has the following probability density function

$$f(x | \mu, \sigma, p) = \frac{\xi}{\sigma} \exp\left\{ -\left( \frac{1}{1-p} I_{[x>\mu]} + \frac{1}{p} I_{[x<\mu]} \right) \frac{x}{\sigma} \right\} |x - \mu|, \quad (2.2)$$

where $\mu$ is the location parameter, $\sigma$ is the standard deviation of $X$, and $p$ is the shape parameter taking on values between 0 and 1, with $k = k_p = \sqrt{p^2 + (1-p)^2}$.

The shape parameter $p$ controls the skewness and kurtosis of the asymmetric Laplace distribution. Its value determines the positive or negative skewness. If $p < 0.5$, the density function is positively skewed to the right; if $p = 0.5$, ALD reduces to a traditional Laplace distribution which is symmetric; and if $p > 0.5$, the density function is skewed to the left with a negative skewness.
Let \( r_t \) denote the return on an individual asset at time \( t \), whose marginal distribution is modelled by ALD. It is a common practice in financial risk modelling that the location value of the return is approximately taken as zero. In the following, we will set \( \mu = 0 \).

For the volatility \( \sigma_t \), a skewed-EWMA model proposed by Lu and Huang (2007) is defined as

\[
\sigma_{t+1} = \lambda \sigma_t + (1 - \lambda) \left\{ k_{t+1} \frac{1}{1 - p_{t+1}} I_{\left[ r_t > 0 \right]} + \frac{k_{t+1}}{p_{t+1}} I_{\left[ r_t < 0 \right]} \right\} r_t, \tag{2.3}
\]

where \( \sigma_t \) stands for the (conditional) volatility at time \( t \),

\( k_{t+1} = \sqrt{p_{t+1}^2 + (1 - p_{t+1})^2} \),

\( p_t \) denotes for the shape parameter at time \( t \),

\( I_{\left[ \cdot \right]} \) is an indicator function, and \( \lambda \) is a decaying factor in this EWMA. Note that if all \( p_t \)’s are equal to a constant of 0.5 in (2.3), then the skewed EWMA reduces to the robust EWMA. More flexibly, here \( p_t \) is modelled by

\[
p_{t+1} = 1 / \left( 1 + \sqrt{u_{t+1} / v_{t+1}} \right), \tag{2.4}
\]

where \( u_{t+1} = \beta u_t + (1 - \beta) |r_t| I_{\left[ r_t < 0 \right]} \),

\( v_{t+1} = \beta v_t + (1 - \beta) |r_t| I_{\left[ r_t > 0 \right]} \)

and \( \beta \) is another decaying factor. The determination of the decaying factors \( \lambda \) and \( \beta \) can be based on a maximum likelihood principle. For details, the reader is referred to Lu and Huang (2007).

2.3 Modelling joint distribution via copula

Since introduced in statistics by Sklar (1959), copula has been successfully applied in many areas, such as hydrology, actuarial science and finance. Using a copula to separate the marginal distributions from the complex dependence structure makes it simpler to deal with multivariate joint distribution. The copula function describes how the marginal distributions ‘come together’ to determine a multivariate distribution, while the marginal distributions only describe how an individual variable moves ‘on its own’. For simplicity, we introduce some basic ideas on the copula, focusing on two-dimensional case below.
**Definition 2:** A two-dimensional copula \( C \) is a real function defined on \( I^2 = I \times I \), where \( I = [0,1] \), such that:

(1) for every \((u,v)\) in \( I^2 \), \( C(u,0) = C(0,v) = 0 \),
\[ C(u,1) = u, \quad C(1,v) = v; \]

(2) for every rectangle \((u_1,u_2)\times(v_1,v_2)\) in \( I^2 \), where \( u_1 \leq u_2 \) and \( v_1 \leq v_2 \), we have: \( C(u_2,v_2) - C(u_2,v_1) - C(u_1,v_2) + C(u_1,v_1) \geq 0 \).

The copula function represents the joint distribution function of two standard uniform random variables \( U, V \) over \( I \):

\[ C(u,v) = \Pr\{U \leq u, V \leq v\}. \]

If \( F_1(x), F_2(y) \) are two univariate distribution functions, then \( C(F_1(x), F_2(y)) \) is a multivariate joint distribution function with margins \( F_1(x), F_2(y) \).

The most important result regarding the copula functions is the following:

**Theorem 1 (Sklar (1959))** Let \( F(x,y) \) be a two-dimensional joint distribution function with continuous marginal distributions \( F_1(x) \) and \( F_2(y) \). Then there exists a unique copula \( C \) such that

\[ F(x,y) = C(F_1(x), F_2(y)). \tag{2.5} \]

Conversely, if \( C \) is a copula and \( F_1(x), F_2(y) \) are continuous univariate distributions, then \( F(x,y) = C(F_1(x), F_2(y)) \) is a joint distribution function with marginal distributions \( F_1(x), F_2(y) \).

From Theorem 1, multivariate distribution can be decomposed into two parts: the dependence structure and the margin distributions. The dependence structure is represented by an appropriate copula function. In applications, among many classes of copulas, the class of Archimedean copulas is widely used for its ease of construction, estimation and manipulation. In particular, there exists a one-to-one relationship between the parameter of Archimedean copulas and the association index measured by Kendall’s tau coefficient, which makes the class of Archimedean copulas easily applicable.
2.3.1 Archimedean copulas

Archimedean copula is constructed through a function \( \phi : [0,1] \rightarrow [0,\infty] \) which is continuous, strictly decreasing and convex, such that \( \phi(1) = 0 \). Define a pseudo inverse of \( \phi \) as follows:

\[
\phi^{-1}(t) = \begin{cases} 
\phi^{-1}(t) & 0 \leq t \leq \phi(0) \\
0 & \phi(0) \leq t \leq \infty 
\end{cases} 
\]  

(2.6)

Note that if \( \phi(0) = \infty \), the pseudo inverse collapses into an ordinary inverse.

**Definition 3:** Let \( \phi \) and \( \phi^{-1}(t) \) be defined in the above. Then

\[
C(u,v) = \phi^{-1}(\phi(u) + \phi(v)) \tag{2.7}
\]

is called an Archimedean copula. Here \( \phi \) is a generator of the copula \( C \).

Different generator function \( \phi \) yields different families of Archimedean copulas. The following are the four families, listed in Table 1, that are of particular interest in applications.

<table>
<thead>
<tr>
<th>Family</th>
<th>Generator ( \varphi )</th>
<th>Range for ( \vartheta )</th>
<th>Copula ( C(u,v) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Product copula</td>
<td>( -\ln t )</td>
<td>Not applicable</td>
<td>( uv )</td>
</tr>
<tr>
<td>Clayton (1978)</td>
<td>( t^{-\vartheta} - 1 )</td>
<td>( \vartheta &gt; 1 )</td>
<td>( (u^{-\vartheta} + v^{-\vartheta} - 1)^{-1/\vartheta} )</td>
</tr>
<tr>
<td>Gumbel (1960)</td>
<td>( (-\ln t)^{\vartheta} )</td>
<td>( \vartheta \geq 1 )</td>
<td>( \exp \left{ (-\ln u)^{\vartheta} + (-\ln v)^{\vartheta} \right}^{1/\vartheta} )</td>
</tr>
<tr>
<td>Frank (1979)</td>
<td>( \ln \frac{e^{\vartheta t} - 1}{e^{\vartheta} - 1} )</td>
<td>( -\infty \leq \vartheta \leq \infty )</td>
<td>( -\frac{1}{\vartheta} \ln \left( 1 + \frac{(e^{\vartheta u} - 1)(e^{-\vartheta v} - 1)}{e^{\vartheta} - 1} \right) )</td>
</tr>
</tbody>
</table>
The parameters in Archimedean copulas are important, which control the degree of dependence between their components.

2.3.2 Selecting an optimal Archimedean copula

One of the important problems in applications is, among the Archimedean copulas in Table 1, which family should be applied. We will apply QQ-plot method introduced by Frees & Valdez (1998) to select an optimal Archimedean copula. Define a univariate function $K_c(t)$ by

$$K_c(t) = t - \frac{\phi'(t)}{\phi(t)}, \quad \text{for } t \in I$$

(2.8)

where $\phi'(t)$ is the first derivative of a supposed generator $\phi(t)$, and a nonparametric estimation of $K_c(t)$ by

$$\hat{K}(t) = \frac{1}{T} \sum_{i=1}^{T} I_{(\nu_i \leq t)}$$

(2.9)

where $\nu_i = \frac{1}{T-1} \sum_{j=1}^{T} I_{(X_j < X_i, Y_j < Y_i)}$, and $(X_i, Y_i), i = 1, \ldots, T$, are the historical observations of the returns on two financial assets. QQ-plot draws the values of the function $K_c(t)$ against the values of its non-parametrically estimated function on x and y axes, respectively. If the supposed copula is correct to the real data, then the QQ plot is approximately a diagonal line. In practice, the closer to the diagonal line the QQ-plot for $\hat{K}, K_c$, the better the supposed copula is, by which we can select an Archimedean copula that fits the data set best.

The optimal copula can be defined by minimising the distance, based on the $L^2$ norm, between (2.8) and (2.9):

$$d_2(\hat{K}, K_c) = \int_{0}^{1} [\hat{K}(t) - K_c(t)] dt.$$  

(2.10)
2.3.3 Parameter estimation in Archimedean copulas

For a given Archimedean generator \( \phi \), another important problem in applications is how to estimate the associated parameter \( \theta \) in the copula. This is done through the link between the copula parameter and the Kendall’s \( \tau \) coefficient. The Kendall’s \( \tau \) coefficient is a rank correlation, a useful measure of the association between two different financial assets, an estimator of which can be calculated as

\[
\hat{\tau} = \left( \frac{n}{2} \right)^{-1} \sum_{1 \leq i < j \leq n} \text{sign}(X_i - X_j)(Y_i - Y_j), \quad \text{where}\quad \text{sign}(x) = \begin{cases} 
1 & x > 0 \\
0 & x = 0 \\
-1 & x < 0 
\end{cases} \quad (2.11)
\]

The Kendall’s \( \tau \) coefficient is closely related to the parameter in each family of the Archimedean copulas by the following theorem.

**Theorem 2 (Nelsen (1999))**: If \((X,Y)\) are continuous random variables with an Archimedean copula \( C \), the following representation holds for \( \tau \):

\[
\tau = 1 + 4 \int_0^1 \frac{\phi(t)}{\phi'(t)} dt \quad (2.12)
\]

where \( \phi(t) \) is the generator of the Archimedean copula \( C \).

By applying Theorem 2, an analytic relationship between the parameter in each family of the Archimedean copulas and the Kendall’s \( \tau \) coefficient can be derived, which is listed in Table 2.

**Table 2**: The relationship between the parameter of Archimedean Copulas and the Kendall’s \( \tau \) coefficient

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Kendall’s ( \tau )</td>
<td>0</td>
<td>( 1 - \theta^{-1} )</td>
<td>( \theta / (\theta + 2) )</td>
<td>( 1 - \frac{\theta}{\varphi} [1 - D_i(-\theta)] )</td>
</tr>
</tbody>
</table>

where: \( D_i(x) = \frac{1}{x} \int_0^x t (e^t - 1)^{-1} dt \)
Through the estimation of the $\tau$ coefficient in (2.11), the one-to-one relationship in Table 2 between the parameter in each family of the Archimedean copulas and the Kendall’s $\tau$ coefficient provides a useful method to estimate the unknown parameter in these copula functions.

2.4 Forecasting the VaR of a portfolio by skewed-EWMA with copula

Given the information up to time $t$, how can we forecast the portfolio VaR at time $t+1$? We propose using a copula-based skewed-EWMA method. In this subsection, we outline a Monte Carlo simulation algorithm for the calculation with a general Archimedean copula, which is stated for two assets only for simplicity. We will examine two different significance levels, ie. $\alpha = 1\%$ and $\alpha = 5\%$ in the empirical application in Section 3.

Monte Carlo simulation algorithm for estimating $\text{VaR}_{t+1}(\alpha)$ at time $t$

Step 1: From the historical observations of the daily returns $(r_{1,j}, r_{2,j})$ on two financial assets up to time $t$, where $j = 1, 2, ..., t$, we can calculate the Kendall’s $\tau$ coefficient, estimate the parameter $\theta$ using the relationship between $\tau$ and $\theta$ in Table 2 in Subsection 2.3.3 for different copulas listed in Table 1 of Subsection 2.3.1, and select an optimal Archimedean copula with estimated parameter by QQ-plot in Section 2.3.2.

Step 2: Generate $(u,v)$ from the specific Archimedean copula function $C(u,v)$, with the parameter $\theta$ estimated in Step 1.

Step 3: Calculate the parameters $(\mu_{i,t+1}, \sigma_{i,t+1}, p_{i,t+1})$ in the distributions ALD of the returns on individual assets, at time $t+1$, for $i = 1, 2$ by an iterative procedure of the skewed-EWMA as follows:
\[
\sigma_{i,t+1} = \lambda_i \sigma_{i,t} + (1 - \lambda_i) \left\{ \frac{\hat{k}_{i,t+1}}{\hat{v}_{i,t+1}} I_{r_{i,t} > 0} + \frac{\hat{k}_{i,t+1}}{\hat{v}_{i,t+1}} I_{r_{i,t} < 0} \right\} r_{i,t},
\]

\[
p_{i,t+1} = 1 / \left( 1 + \sqrt{\hat{u}_{i,t+1} / \hat{v}_{i,t+1}} \right),
\]

as defined in Subsection 2.2, and taking all \( \mu_{i,t+1} = 0 \), as usually done in the volatility estimation. Then calculate \( r_{1,t+1} = F_1^{-1}(u) \), \( r_{2,t+1} = F_2^{-1}(v) \), where \( F_1^{-1} \) and \( F_2^{-1} \) are the inverse functions of the cumulative distributions \( F_1(x) \sim AL(\mu_{1,t+1}, \sigma_{1,t+1}, p_{1,t+1}) \) and \( F_2(y) \sim AL(\mu_{2,t+1}, \sigma_{2,t+1}, p_{2,t+1}) \), respectively.

Step 4: From Step 3, we can calculate a simulated value of the portfolio return at time \( t+1 \) by

\[ r_{p,t+1} = \omega r_{1,t+1} + (1 - \omega) r_{2,t+1}, \]

where \( \omega = \omega_{t+1} \) is the weight on asset 1 in the portfolio, and \( r_{p,t+1} \) is the return on the portfolio, at time \( t+1 \).

Step 5: Repeat Steps 2–4 M times (eg. \( M=10000 \)), and according to the M simulated values of the return on the portfolio at time \( t+1 \), we can easily calculate \( VaR_{t+1}(\alpha) \) by the quantile of the \( M \) simulated values of the return on the portfolio, at a given significance level \( \alpha \).

In Step 2 we need the simulation of \( (u,v) \) from a specific Archimedean copula function \( C(u,v) \), which can be done as that in Steps 1–3 of Wu, Valdez and Sherris (2007, pp1021) with the specific Archimedean copula selected based on the real data by applying the Q-Q principle (Section 2.3.2). For example, in our empirical application (see Section 3), the Clayton copula is chosen by the Q-Q principle, so we can simulate \( (u,v) \), in Step 2 of the above algorithm, from the Clayton copula function with the estimated parameter \( \theta \), which will be particularly stated in detail in Section 3.2 below.

2.5 Model backtesting

In order to evaluate the performance of our proposed copula-based portfolio VaR model in the above, we will apply the widely used backtesting methods of proportion failure test (ie. unconditional coverage test) by Kupiec (1995) and the conditional coverage test by
Christofferson (1998). These tests are based on likelihood ratios and the null hypothesis that the VaR should exhibit a conditional or unconditional coverage equal to the nominal significance level, \( \alpha \), including: test for ‘unconditional coverage’; test for ‘independence’ and test for ‘conditional coverage’. These tests – based on likelihood ratios – are the widely used measures for this type of empirical research; see, for example, Jorion (2000) and Guermat and Harris (2001) for more details. In the following, we will only sketch the likelihood ratio (LR) statistic for these tests.

For the test of ‘unconditional coverage’, Kupiec (1995) proposed the likelihood ratio test based on the fact that the number \( N \) of exceeding VaR among the testing sample of size \( T \) is binomially distributed. Hence the likelihood ratio (LR) statistic is

\[
LR_u = -2 \ln L_u - 2 \ln L_N \sim \chi^2(1) \tag{2.13}
\]

where \( L_u = (1-\alpha)^{N_\alpha} \alpha^N \), \( L_N = (1-N/T)^{T-N} (N/T)^N \), \( \alpha \) is the desired significance level, \( N \) is the number of times in the testing sample when the forecasted VaR is exceeded, and \( T \) is the number of the testing sample.

For the test of ‘independence’, Christofferson (1998) suggested the LR statistic

\[
LR_{in} = 2(\ln L_A - \ln L_0) - \chi^2(1) \tag{2.14}
\]

where \( L_A = \pi_{00}^T \pi_{10}^T \pi_{11}^T \), \( L_0 = (1-\pi)^{T_{00}} \pi_{10}^{T_{01}} \pi_{11}^{T_{11}} \), and the related quantities are defined as follow: first set an indicator function \( I_i = 1 \) if \( r_{p,t} < -VaR_t \) and 0 otherwise, then let \( T_{ij} = \) the number of \( I_i = i \) following \( I_{i-1} = j \) in the testing sample series, \( i, j = 0,1 \) and

\[
\pi_{ij} = P(I_i = i | I_{i-1} = j) = T_{ij} / (T_{00} + T_{10}) \quad i, j = 0,1
\]

\[
\pi = (T_{00} + T_{11}) / (T_{01} + T_{11} + T_{00} + T_{10}).
\]
For the test of ‘conditional coverage’, Christofferson (1998) gave the $LR$ statistic as follows:

$$LR_c = -2 \ln L_{\alpha} + 2 \ln L_{\alpha} \sim \chi^2(2),$$

(2.15)

which is a combination of the ‘unconditional coverage’ test and the ‘independence’ test.

3 Empirical application of the skewed-EWMA forecasting via copula

In this section, we demonstrate an empirical application of the proposed method of combining the skewed-EWMA forecasting with copulas to two portfolios composed of the Chinese stock market indices, with constant and varying weights, respectively. We will first estimate the parameters in each marginal distribution, and then compute the $VaR_t(\alpha)$ series by Monte Carlo simulation. We will finally evaluate the performance of our proposed model.

3.1 Data

We use the daily log-returns of the two Chinese stock market indices, that is the Composite Index of Shanghai Stock Exchange and the Component Index of Shenzhen Stock Exchange, simply denoted as Shanghai index (SHidx) and Shenzhen index (SZidx), respectively, from http://finance.yahoo.com/. The dataset contains 2568 observations of the daily closing prices of the two indices, from 1 June 2000 to 19 January 2011. The returns are defined by $r_t = 100(\ln P_t - \ln P_{t-1})\%$, where $P_t$ is the closing price on day $t$. Figure 1 plots the return series of both indices and Table 3 provides the preliminary sample statistics for each daily return series, including mean, standard deviation (S.D.), skewness (Sk) and kurtosis (Ku). We can easily see from Table 3 that these return series are skewed and have high kurtosis. In the following, we will use the first 1,000 daily return observations in determining the decaying parameters in (2.3) and (2.4) by maximum likelihood method as done in Lu and Huang (2007), and the remaining daily return observations of more than 1,000 as the testing sample for forecasting evaluation.
Table 3: Descriptive statistics of the daily returns of Shanghai and Shenzhen indices

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>S.D.</th>
<th>Sk</th>
<th>Ku</th>
</tr>
</thead>
<tbody>
<tr>
<td>SHidx</td>
<td>0.0145%</td>
<td>1.711%</td>
<td>-0.2202</td>
<td>7.3419</td>
</tr>
<tr>
<td>SZidx</td>
<td>0.0368%</td>
<td>1.869%</td>
<td>-0.2904</td>
<td>6.9131</td>
</tr>
</tbody>
</table>

Figure 1: Daily returns of the Shanghai index (SHidx) and Shenzhen index (SZidx)

3.2 Forecasting the $\text{VaR}_i(\alpha)$ with portfolios

In this subsection, we look at the performance of our proposed method in the VaR forecasting by examining risk forecasting for some given portfolios. For a constant-weighted portfolio, where the portfolio weights do not vary with time, we can make the VaR forecasting in two ways: (1) using the methodology proposed in this paper by combining copula with skewed-EWMA; (2) simply treating the portfolio like an individual asset with the skewed-EWMA applied as in Lu and Huang (2007). However, for a portfolio with the investing components adjusted with time, which is often the case in a long-term investment, we
obviously can still apply the methodology proposed in this paper, but in such a case, we cannot take for granted treating the component-adjusted or varying-weighted portfolio like an individual financial asset in estimating its VaR. We are to show this point by considering two portfolios that are composed of the Shanghai and the Shenzhen indices, detailed below:

1. Constant-weight portfolio: This portfolio keeps investment weights constant at an investment proportion 50%–50%; and

2. Varying-weight portfolio: This portfolio has a varying weight scheme. For ease of illustration of our point, we particularly suppose the portfolio weights are roughly adjusted once per month with the time-varying weight on Shanghai index to be specified as displayed in Figure 2.

We choose the three families of Archimedean copulas as candidates for copula function; they are Clayton copula function, Gumbel copula function and Frank copula function. In order to select an empirically optimal one to fit the dataset, the QQ-plots for the three families of Archimedean copulas are depicted in Figure 3.

It follows from Figure 3 that all the three families of Archimedean copulas may suit the dataset, but the Clayton copula function appears to fit the dataset best. Therefore, we will adopt Clayton copula function as the dependence structure, which joins the marginal distributions of the returns of the Shanghai and the Shenzhen indices. Based on this, we can simulate \((u, v)\), in Step 2 of the algorithm given in Section 2.4, from the chosen Clayton copula function with the estimated parameter \(\theta\), which is especially conducted as follows:

**Simulation algorithm with Clayton copula for predicting \(VaR_{t+1}(\alpha)\) at time \(t\)**

**Step 1**: From the historical observations of the daily returns \((r_{1,j}, r_{2,j})\) on the two financial indices up to time \(t\), where \(j = 1, 2, ..., t\), calculate the Kendall’s tau coefficient, 00 and estimate the parameter \(\theta\) in Clayton copula function, as introduced in Subsection 2.3.3, with \(\theta = 2\tau / (1 - \tau)\).
Step 2*: Generate \((u,v)\) from the Clayton copula function \(C(u,v)\), listed in Table 1 of Subsection 2.3.1, with the estimated parameter \(\theta\), detailed below:

a) Generate 2 independent random numbers \(x, y\) from exponential distribution with parameter \(\lambda = 1\). Here the cumulative exponential distribution has the form 
\[F(x; \lambda) = 1 - e^{-\lambda x}, x > 0.\]

b) Generate a random number \(z\) from a \(Gamma(1/\theta, 1)\) distribution, where \(z\) is independent of \(x, y\).

**Figure 2:** Varying-weight portfolio consisting of the two Chinese stock indexes: (a) varying weight series, (b) return series.
Figure 3: QQ-plots (dashed lines) between $\hat{K}, K_c$ for three different families of Archimedean copulas. Here $x$ and $y$ axes are the values of the theoretical $K_c(t)$ and the nonparametric estimate in (2.8) and (2.9), respectively, and the solid line is for the diagonal line.

c) Set $u = (1 + x / z)^{-1/\theta}, v = (1 + y / z)^{-1/\theta}$. Then $(u, v)$ is a vector generated from the Clayton copula function with the parameter $\theta$.

Step 3*-Step 5*: The same as Step 3–Step 5 in the algorithm in Section 2.4.

To simulate the returns on the portfolio, we need the predicted volatility and shape parameter in marginal distributions through the skewed-EWMA iteration in equations (2.3) and (2.4). The time-varying
volatility and shape parameters in the marginal distribution over the
testing sample period are plotted in Figure 4. It is seen that the shape
parameters are changing away from 0.5, the constant-shape parameter in
Laplace distribution.

In practice, the values of VaR at two significance levels of
$\alpha = 1\%$ and $\alpha = 5\%$ are particularly important in risk management.
Applying the Monte Carlo simulation algorithm, we can calculate the
$\text{VaR}_{t \alpha} (\alpha)$ series over the testing time period; see Figure 5 for the
constant-weight (50\%) portfolio and Figure 6 for the varying-weight
portfolio with the weighting scheme and portfolio return series depicted
in Figure 2. Here in Figures 5 and 6, ‘copula’ stands for the copula-based
skewed-EWMA forecasting proposed in this paper and ‘skewed-
EWMA’ for the VaR forecasting of Lu and Huang (2007) applied to the
portfolio return. Obviously, both methods produce similar outcomes for
the constant-weight portfolio (Figure 5), but may make quite big
differences for the varying-weight portfolio (Figure 6). From these
figures, we can roughly see that both methods work well for the
constant-weight portfolio, but may perform differently in the case of
varying-weight portfolio. We evaluate which method performs better in
the next subsection.

3.3 Backtesting

In order to evaluate the performance of our proposed portfolio VaR
model of using the skewed-EWMA forecasting via ‘copula’ method, we
made the backtesting of the VaR forecasts by our ‘copula’ method and
by the ‘skewed-EWMA’ of Lu and Huang (2007), respectively, by
applying the tests described in Subsection 2.5.
Figure 4: One-step-ahead predicted volatility and shape parameter in asymmetric Laplace distribution over the testing time period.

(a) Predicted volatility for Shanghai index

(b) Predicted volatility for Shenzhen index

(c) Predicted shape parameter for Shanghai index

(d) Predicted shape parameter for Shenzhen index
The constant-weight portfolio return series versus the one-step-ahead forecasted VaR series at different significance levels over the testing sample period. Note that we cannot distinguish the ‘Skew-EWMA 5%’ VaR series from the ‘Copula’ 5% VaR series (the middle line) and the ‘Skew-EWMA’ 1% VaR series from the ‘Copula’ 1% VaR series (the bottom line).

Table 4 lists the likelihood ratio (LR) testing statistics for unconditional coverage test $LR_u$, independence test $LR_{in}$ and conditional test $LR_c$ at different significance levels, in the case of constant-weight portfolio. We also provide the critical values of $\chi^2$ distribution at different significance levels in the last two columns in the table. From Table 4, we can see that all testing statistic values for the VaR calculated by both methods at both significance levels of $\alpha = 1\%$ and $\alpha = 5\%$ are less than the critical values even at the testing significance level of 1%. The results show that both models of VaR for portfolio are reasonable and acceptable in this case of constant-weight portfolio.

However, in the case of varying-weight portfolio, Table 5 provides the testing outcomes as done in Table 4. From Table 5, we can see that all testing statistic values for the VaR calculated by the ‘copula’ method
of this paper at both significance levels of $\alpha = 1\%$ and $\alpha = 5\%$ are less than the critical values even at the testing significance level of 1%, indicating that the model of the VaR for portfolio proposed in this paper is reasonable and acceptable. But obviously the ‘skewed-EWMA’ method that treats the portfolio like an individual asset is rejected by both tests of $LR_u$ and $LR_c$ for the 5% portfolio VaR, even at the testing significance level 5%. This well indicates our point that we cannot take for granted treating the component-adjusted or varying-weight portfolio as an individual financial asset in the estimation of its VaR.

Overall, we may conclude that our method proposed in this paper can work well for the extreme portfolio risks.

**Figure 6:** The varying-weight portfolio return series versus the one-step-ahead forecasted VaR series at different significance levels over the testing sample period. Note from the upper to the bottom are the portfolio return series, the ‘Skew-EWMA’ 5% VaR series, the ‘Copula’ 5% VaR series, the ‘Skew-EWMA’ 1% VaR series and the ‘Copula’ 1% VaR series, respectively.
Table 4: Backtesting for the constant-weight portfolio VaR forecasting at different significance levels.

<table>
<thead>
<tr>
<th>Significance Level</th>
<th>Approach</th>
<th>Frequency</th>
<th>( LR_u ) [( \sim \chi^2(1) )]</th>
<th>( LR_{i,n} ) [( \sim \chi^2(1) )]</th>
<th>( LR_c ) [( \sim \chi^2(2) )]</th>
<th>( \chi^2(1) ) Critical Value</th>
<th>( \chi^2(2) ) Critical Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1%</td>
<td>Skewed–EWMA</td>
<td>0.01213</td>
<td>0.6694</td>
<td>0.4667</td>
<td>1.1605</td>
<td>2.7055</td>
<td>4.6052</td>
</tr>
<tr>
<td></td>
<td>Copula</td>
<td>0.01148</td>
<td>0.33395</td>
<td>0.4186</td>
<td>0.77568</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5%</td>
<td>Skewed–EWMA</td>
<td>0.04722</td>
<td>0.2588</td>
<td>0.82348</td>
<td>1.17908</td>
<td>3.8415</td>
<td>5.9915</td>
</tr>
<tr>
<td></td>
<td>Copula</td>
<td>0.04722</td>
<td>0.2588</td>
<td>0.82348</td>
<td>1.17908</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 5: Backtesting for the one-step-ahead forecasting of VaR with a varying-weight portfolio at different significance levels.

<table>
<thead>
<tr>
<th>Significance Level</th>
<th>Approach</th>
<th>Frequency</th>
<th>( LR_u ) [( \sim \chi^2(1) )]</th>
<th>( LR_{i,n} ) [( \sim \chi^2(1) )]</th>
<th>( LR_c ) [( \sim \chi^2(2) )]</th>
<th>( \chi^2(1) ) Critical Value</th>
<th>( \chi^2(2) ) Critical Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1%</td>
<td>Skewed–EWMA</td>
<td>0.014039</td>
<td>2.2948</td>
<td>1.01133</td>
<td>3.334436</td>
<td>2.7055</td>
<td>4.6052</td>
</tr>
<tr>
<td></td>
<td>Copula</td>
<td>0.0114869</td>
<td>0.3339545</td>
<td>0.4186141</td>
<td>0.77568</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5%</td>
<td>Skewed–EWMA</td>
<td>0.06573</td>
<td>7.4603</td>
<td>0.10502</td>
<td>7.70135</td>
<td>3.8415</td>
<td>5.9915</td>
</tr>
<tr>
<td></td>
<td>Copula</td>
<td>0.047862</td>
<td>0.15285</td>
<td>0.9139569</td>
<td>1.164935</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Conclusion

In this paper, we propose a skewed-EWMA forecasting via copula method to estimate VaR for portfolios. We suggest using three families of Archimedean copulas as candidates for their convenience in application, and the optimal copula can be chosen by QQ-plots. We propose a Monte Carlo simulation algorithm to simulate the one-period-ahead forecast of the VaR for portfolios at a given significance level. To evaluate the validity of our method, we use ‘unconditional coverage test’, ‘independence test’ and ‘conditional coverage test’ to examine our model. Empirical application to the Chinese stock market with two equity indices (Shanghai index and Shenzhen index) illustrates that our method is valid and the outcome is acceptable by all the tests even at a testing significance level of 1%. This shows that the skewed-EWMA
forecasting via copula method appears to be a useful tool in estimating the VaR for portfolios.

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Salary Linked Home Finance: Reducing Interest Rate, Inflation and Idiosyncratic Salary Risks

A Asher*

Abstract

It is possible to develop an alternative housing finance instrument that matches the cash flow, and reduces the risks faced, by homeowners and pension funds. The instrument would also reduce the liquidity constraints faced by new and existing homeowners, and eliminate the cash flow tilt imposed by high inflation. Moral hazard and anti-selection risks are likely to restrict the market to employees of large institutions, but such an instrument would encourage a greater flow of funds from superannuation into housing. Other obstacles to its introduction can be overcome.

Keywords: mortgages, human capital contracts, income contingent loans, macro contracts

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1 Introduction

Salary-linked home-finance (SLHF) describes a home financing instrument repaid by a predetermined proportion of the home-owner’s income over a predetermined term. The cost, and the return to the investor, are therefore dependent on the homeowner’s income growth over the term – and is independent of interest rates.

As a financial instrument, the concept is closely related to that of income contingent loans (ICLs), such as Australia’s HECS system, which are increasingly used, internationally, to fund higher education. SLHF is however an equity rather than a debt instrument and is intended also to provide stable inflation-protected returns on the assets that back pensions in payment.

This paper first describes the motivation for SLHF. It then goes on to describe one practical embodiment in more detail and examines the advantages and disadvantages for homeowners and investors. It then considers the returns that could be earned based on historical salary growth in Australia, and how the expected rate of return might be adapted to satisfy investors and homeowners. A final section considers risks – particularly moral hazards – in more detail and the obstacles to SLHF’s introduction in the marketplace, suggesting how they might be managed.

The instrument could offer advantages for investors and homeowners.

2 Motivation

SLHF provides a way that asset-rich pensioners can provide finance to asset poor young homeowners in a manner that reduces the risks faced by both.

The idea for SLHF came from attempts to address three different challenges. This section discusses these challenges and then briefly compares SLHF with ICLs, which, while similar, have a significantly different focus.
2.1 Liquidity constraints and the inflation tilt

The first challenge is the liquidity constraints faced by young homeowners: if they were to smooth their consumption over their lifetimes, they would borrow more than they can typically do because of lenders’ (understandable) antipathy to the capitalisation of interest. This is aggravated by high interest rates in inflationary times, as the real value of repayments is tilted towards earlier years. Nominally level repayments on even small home loans are burdensome in the earlier years, but soon ease with salary inflation. The mathematics of inflation annuities could however be applied to the problem faced by younger borrowers to start repayments low and increase them with inflation.

This relatively obvious solution has been tried frequently. Mortgage repayments can be increased at a fixed rate or in line with a wage-related index, but the cost of borrowing is still related to market interest rates. Roldan and Spoor (1992) tell how these dual indexed mortgages were introduced to Mexico when they were in favour with the World Bank. They have been tried in other countries, including Australia. They have a significant drawback however in that the incomes of some borrowers did not keep pace with the required increases in the repayments. The loans of these borrowers became too large to be serviced and also exceeded the current market value of their homes.

SLHF addresses the tilt problem because nominal instalments automatically rise with income, and those with lower salary increases repay less.

2.2 Usury and idiosyncratic salary risks

The second challenge can be posed as an ethical one. Usury (the charging of interest) was prohibited in much of the ancient world and is still prohibited by some versions of Muslim law. Mills (1989) looked at the religious and historical arguments for and against charging interest, and concludes that various partnership arrangements – some analogous to share cropping – and the joint stock company are preferable to charging interest, because they place less of the burden on the ‘borrower’ if things go badly.
The problem can alternatively be expressed as a market failure in the provision of insurance against idiosyncratic salary risks. The actuarial challenge is therefore to develop appropriate insurance contracts to protect borrowers against salary increases falling short of expectations.

SLHF is based on the recipient’s own income and is therefore an equity rather than debt instrument. As with insurance contracts, the SLHF contracts are fair *ex ante* in that the future value of expected repayments is equal to the initial value. They are also reasonable and fair *ex post* because those with salary increases higher than expected will ‘subsidise’ those with increases lower than expected.

### 2.3 Smoothed inflation-protected investment returns

The third challenge is to provide an inflation-protected low volatility asset to back pensions. For these, SLHF is expected to provide a low volatility cash flow directly related to the cash flow required by a pension, and growing with salary inflation.

### 2.4 Human capital contracts (HCCs) and ICLs

ICLs have a different origin and largely different purpose to SLHF.

It would seem that Friedman and Kuznets (1945) first suggested that students could pay for their higher education and share the risks of the accompanying increase to their future incomes using HCCs with a form similar to SLHF (ie. the return dependent on increases to incomes). Palacios (2002) describes them in more detail. They now appear to be available commercially in a number of countries.

Other versions (where the repayments but not the returns are linked to the borrower’s income and are therefore more debt than equity) have been developed over time and are now common internationally. While they address the liquidity constraints and create some insurance against idiosyncratic salary risk, they are aimed at a different demographic and are not intended to yield a smoothed real investment return. In fact, most of the schemes are significantly subsidised by government. Chapman (2005) provides a thorough summary.
They do, however, provide insights into the potential – particularly operational risks of a SLHF product.

3 The Financial Instrument

The new product is simply explained by the formula linking the finance amount with the repayments.

It should be explained that the word ‘loan’ is not used in order to make it clear that the pure form of the instrument is not a loan, and there is no interest in the sense of a predetermined cost of borrowing. Instead of the traditional interest rate, the investor has an equity interest in a share of the homeowner’s future earnings. Salary is used for income or wage as the product is likely to be more attractive to those earning a relatively fixed salary rather than a wage with overtime allowances.

\[ SLHF_t = \sum_{i=t}^{n-1} K_i S_i \]

Where

- \( SLHF_t \) is the amount of finance outstanding at the end of month \( t \),
- \( K_i \) is the predetermined proportion of salary to be repaid in month \( i \),
- \( S_i \) is the home-owner’s salary in month \( t \), and
- \( n \) is the term of the loan in months.

In a simplified example, if \( n = 240 \), \( K_i \) is fixed at 20%, then:

- \( SLHF_0 = 48 S_o \) is the amount advanced initially
- \( SLHF_1 = 47.8 S_1 \), which together with a repayment of 20% \( S_i \) gives a return of \( S_i / S_0 - 1 \)
- \( SLHF_2 = 47.6 S_2 \) etc.

(eg. If $480,000 were to be advanced, it would be repaid over 20 years by 20% of a salary that began at $10,000 per month. Future increases are not anticipated, and if salary remained constant, there would be no investment return.)
One might see this as a linear repayment of capital. The homeowner is able to repay some or all of the amount outstanding at any time, although some charge might be necessary to manage anti-selection.

The basic formula described above may well require embellishment for operational, legal and marketing reasons, but the adaptations may prove commercially valuable and add nothing to the description of the basic product. They are therefore not described in this paper.

3.1 The return and cost of finance

The return (or cost of the SLHF), in the simple example above, is therefore equal to the homeowner’s growth in salary.

This return may not, however, match investors and users of the SLHF. If there is insufficient investment, then the repayments will have to be loaded to attract more investors and discourage users of SLHF – and vice versa. Most obviously, the instalments could be increased by a fixed percentage loading. Such a loading is easily calculated as the term of the SLHF divided by the annuity factor determined at an interest rate equal to the required increase to the yield. This loading therefore can be regarded as an interest charge (increasing the return over and above salary growth).

The instrument therefore creates a novel link between investors and users of funds, which can be regarded financially as analogous to a new currency (unique to each homeowner). Investors and borrowers in this currency will be matched at a particular loading or interest rate that will depend on supply and demand and expectations of the rate of increase of each homeowner’s salary.

The loadings could be fixed over the lifetime of the loan – and therefore lead to a potential charge on early repayment as with conventional fixed interest loans, or vary as supply and demand for funds changes.
3.2 Advantages to homeowners

Ignoring, for the moment, the question of expected returns and charges, this instrument provides a number of advantages to the homeowner relative to the standard variable rate mortgages.

3.2.1 No exposure to interest rate movements

The instalments on variable rate mortgages can be volatile. Australian mortgage rates have varied from 5% to 17% over the past 50 years.¹ This translates into monthly instalments on a 20-year $100,000 loan that have varied from $660 to $1,467. On three occasions, instalments on these mortgages would have increased by more than 20% over the course of one year.

Direct interest rate risk is eliminated completely with this instrument – unless a variable loading is applied, in which case changes to the loading would represent a new interest-type risk.

3.2.2 Greater advances and lower initial instalments

The instalments under standard mortgages are not adapted to allow for the likelihood that the borrower’s income will grow. The expected returns on the SLHF instrument can however anticipate some salary growth, and therefore can allow for a greater initial advance relative to initial instalments.

The actual amounts will depend on the credit rules applied by the investors in these instruments. Comparisons with standard mortgages also depend on the rules of particular lenders, but it is suggested that the numbers in Table 1 would be realistic for some borrowers and SLHF recipients at time of writing. The calculations assume that the instalments would have to be loaded in order to provide a return that would attract sufficient investors.

This additional advance increases the homeowner’s gearing to the price of housing – and the amount outstanding may well increase initially before reducing. This potentially increases the size of the loss if the home is sold. Figure 1 below illustrates using a simplified version to show how the amount outstanding can increase initially in nominal terms before declining to zero. Salaries have been increased at a higher 6% pa in order to illustrate the increase.

Table 1: Cash flow comparisons

<table>
<thead>
<tr>
<th>Earning $100,000 pa</th>
<th>Maximum amount advanced</th>
<th>Initial instalments</th>
</tr>
</thead>
<tbody>
<tr>
<td>20 year variable rate mortgage</td>
<td>$376,199</td>
<td>$35,000</td>
</tr>
<tr>
<td>20 year salary linked home finance</td>
<td>$400,000</td>
<td>$25,200</td>
</tr>
<tr>
<td>Difference</td>
<td>6.3%</td>
<td>-28.0%</td>
</tr>
</tbody>
</table>

Note: These calculations are based on interest of 7% (payable in arrears and compounded monthly) and include loadings (on the repayments) of a flat percentage of 20% in order to achieve this yield on the SLHF, assuming salary inflation of 5% including promotional increases. (See section 4 for discussion.)
3.2.3 Protection against higher inflation and nominal interest rates

While high inflation has not been an issue for more than a decade, it places significant pressures on homeowners in increasing nominal interest rates and therefore their cash-flow problems. Policy makers can also respond to inflation by increasing real interest rates, so exacerbating the problem.

History suggests that inflation always remains a risk. Massive government stimulus being offered to the world economy this year, combined with increasing demand from developing populations in China and India particularly may create demand-pull inflation. The likelihood of periods of higher inflation is further increased by the cost-push inflation that may be created by the peaking of world oil production in the next decade or two.

Inflation also harms retirees. Figure 2 below gives a history of inflation in Australia and the USA. Even the USA, the world’s largest economy, has suffered three bouts of significant inflation over this period. Someone retiring in 1940 on a fixed pension would have lost half the value by the mid-1950s and another half by the mid-1970s.

Figure 2: Inflation rates of the last century

3.2.4 Salary growth hedge

This instrument also provides insurance (or a hedge) for homeowners against their own salary growth being below expectations – either because of macroeconomic factors beyond their control, or considerations more closely related to their personal circumstances, such as difficulties experienced by their employers or poor health.

The SLHF would therefore provide one natural way of implementing ‘macro-markets’ as described in Shiller (1999). He suggested the development of a very wide range of income indices (based on different professions and economies) that would allow people to swap a portion of their own future income for someone else’s. SLHF has an advantage over Shiller’s swaps in that there is no basis risk between the index and the individual’s salary growth, but – on the other hand – SLHF faces the risks of anti-selection and moral hazards.

Moral hazards arise because people might put less effort into increasing their incomes if they have to share a significant proportion with investors (in addition to their tax obligations). This risk is clearly uninsurable; its impacts and management are considered in section 4.3.2 below.

If we ignore the impact of moral hazards, the expected return on SLHF at issue will depend on expected income growth. Expectations may however differ between investors and recipients. This creates the anti-selection risk, which is discussed further in section 4.3.1 below.

If we further assume that potential investors and recipients have the same expectations of future income, random deviations from the expectation are ‘insurable’ in the sense that the risks can be pooled and those with below average increases are supported by those with above average increases. As discussed in section 2.2 above, SLHF contracts can be seen as incorporating an insurance element.

I am not aware of any attempt to measure the deviation of these random deviations only, although personal observation would suggest that it is considerable. Deaton and Paxson (1998) do measure the considerable dispersion of incomes over the life cycle and show that it
is increasing and that there is a positive correlation between health and income. This suggests that at least part of the differences is involuntary and insurable.

3.3 Advantages to funds paying income streams

Ignoring again the question of expected returns, a portfolio of SLHF investments provides some significant advantages to superannuation funds attempting to match retirement income streams. It matches the liabilities of the employer in the case of defined benefit (DB) funds and of the objectives of the members in the case of defined contribution (DC) funds.

3.3.1 Returns linked to salary inflation

A portfolio of SLHF would yield a return equal to a weighted average of the growth in the salaries of the homeowners who had been financed.

It is suggested that salary linkages provide the ideal match for superannuation fund liabilities – DB and DC. In the absence of any investment that will yield a return precisely equal to the particular needs of individual retirees, it is generally argued the best linkage is either to a pensioner price index or to salary inflation.

Salary inflation is generally expected to be higher than price inflation, but this just creates a tilt in the payment, if the present values of cash flows are expected to be the same. A price index would be more appropriate to match a particular standard of living, while a salary link would allow the pensioner to participate in the prosperity – or otherwise – of the current generation of workers. For those on a comfortable retirement income, the salary linkage would seem more appropriate, arguably an ideal match.

3.3.2 Regular cash flows to match income stream payments

While not exact, the regular cash flows from a portfolio of SLHF investments would be strongly related to the outflows from a portfolio of pensions.
An exact match would not be possible given that the SLHF investments:

- are likely to have a maximum term shorter than the maximum likely term of a life pension; and
- must include an option to repay, which will create a varying reinvestment risk.

It will be difficult therefore to manage a portfolio to match the term of a pension portfolio. Nevertheless, a portfolio of SLHF would produce an increasing cash flow that would leave relatively little liquidity risk and, because the pension payments could be linked to the value of the SLHF, no market risk.

3.3.3 Minimal credit risk

The actual credit experience of these instruments will depend on the credit restrictions adopted and the enforceability of the contracts, but properly managed portfolios should enjoy the minimal losses that are normally associated with home loans. The primary security will be the income of the homeowners, with the home itself providing additional protection through a secured mortgage should this fail through unemployment.

Credit risk, and the possible need for government intervention, is discussed in more detail below.

4 Likely returns

The return on a portfolio of SLHF would be equal to the weighted average salary increase of recipients. This section suggests that the average increase in the salaries of recipients is likely to be at least 3% above inflation, and so justifies the 5% increase used in Table 1, as a conservative estimate.

Increases would differ from a national wage index, because:

- the salaries would represent only a sample of the total; and
the salary increases would include a ‘promotional’ element arising from the increases experienced by the homeowners. This increase arises partly from promotions to more senior positions and partly from increases arising from the benefits of experience. In considering the factors that affect these increases, it is important to distinguish between annual changes in cross-sectional data and the increases of a particular cohort. A notional index of a stationary population would not include any promotions. The average SLHF recipient will however be younger than average and promotions should make an important contribution to the return.

The returns can therefore be seen as depending firstly on the average increase to salaries arising from inflation plus an additional element of promotion. They will also depend on the individual salary expectations of SLHF recipients and how the behaviour of SLHF recipients is modified as a consequence of their receipt of the loan. This section addresses these questions in turn. There is no attempt to be particularly scientific in measuring the past, as any precision would be spurious. The past is at best a guide to the future, but it is likely that potential returns will best be estimated by measuring a potential SLHF recipient’s salary history and that of his or her employer.

4.1 Salary growth

Average salary growth in Australia has exceeded inflation by a little over 1% pa over the past 40 years, as shown in Figure 3. This is more or less consistent with other developed and urbanised countries, and is unlikely to change significantly in future.

The fact that the working population is not entirely stationary means that the average can be distorted by the inclusion or exclusion of a greater or lesser promotional element. Some rough calculations based on a fairly steep promotional curves and the current age structure suggests that such distortions are likely to be small.
4.2 Increases over the working life

The precise effect of promotional increases on SLHF return will depend on the characteristics of the recipients that influence their salary growth. Polachek and Siebert (1993) summarise the factors affecting salaries, which depend significantly on education levels and age.

- For educated men, income normally rises with age until the late forties (later in organisations with promotions based on seniority), and then appears to be relatively flat until retirement.
- Non-skilled men’s incomes peak at around age 30.
- Single women’s incomes largely follow the pattern for single men.
- Married women’s incomes drop at the birth of the first child, and they seldom catch up.

Not surprisingly however actual salary increases vary significantly over time and the type and location of the employer.

There are three main sources of information available on the progress of income over the working life.
4.2.1 Actuaries of defined benefit funds

Actuaries have produced salary scales including promotions for DB schemes for many years. They can be found in most valuation reports, and generally show fairly steep increases in the twenties, dropping to zero before retirement. These may not be applicable to SLHF portfolios:

- Older ages are normally more important financially to defined benefit schemes and so increases at the younger end of the scale may be given less attention and may be less reliable, unless the age and benefit structure of the fund dictate otherwise.

- There may be an anti-selection impact in that people taking out SLHF may not be typical, and those leaving the employer may subsequently have an experience of increases in income that differ from those that stay.

With these caveats, it is interesting to speculate what annual promotional type increases are likely to add to the return on SLHF portfolios. The scales of a couple of large Australian government funds suggest that, for ages where the finance is received under 30, the return would be enhanced by between 1% and 3% pa.

4.2.2 National statistics

Most of these appear to be based on cross-sectional studies. An OECD (1998) report provides a number of graphs of the progression of salaries over the working lifetime scale. It shows steep increases in the early twenties reducing in later decades.

The curves appear to be similar for fairly different countries. In Figure 4, France and the Scandinavian countries have rather different educational and tax arrangements to Australia and Canada, yet still have similarly shaped curves. The curve for the Czech Republic is very much flatter, but its recent emergence from a centrally controlled market may explain the difference.

While tracing a similar shape, the precise curvature differs significantly by country and over time. The OECD report uses the ratio of the income of those aged 45 to 54 to the income of those aged 25 to 29
to measure steepness. In 1995, the ratio for graduates varied from 1.42 in the Czech Republic to 2.61 in Spain. Also reported was the change over the last 20 years in the USA, where the ratio had increased from 1.23 in 1975 to 1.67 in 1995. These ratios would suggest promotional increases between 1% and 4.5% pa, which would be consistent with the increases reported in the section above and, with salary increases of at least 5%, when inflation is of the order of 2%.

These cross-sectional studies invariably report a decline in average incomes at later ages – a pattern not repeated in the longitudinal studies of the next section. It is generally agreed that the difference is caused by higher income people retiring earlier.

Figure 4 Cross-sectional wage levels in 1995

Source: OECD (1998)

4.2.3 Panel studies

Studies that have examined salary progression over a significant period of the lives of the same individuals show significantly steeper curves that do not seem to differ much in different economic and cultural circumstances.
Figure 5 shows the earnings of a sample of 70,000 Italian workers, who entered the labour market at the age of 25 or 26, and are therefore assumed to have had some higher education. The thick line represents the initial real wages of those entering the labour market in the years shown, while the thin line shows the subsequent real increases of each cohort. It confirms that actual increases are likely to be steeper at younger ages, but to vary over time.

Figure 5: Entry wages and career development of young workers

Source: Rosolia and Torrini (2007)

Similar results are found by an analysis by Beach and Finnie (2004) of a 10% sample of Canadian taxpayers.

Of particular interest in both samples is the interaction between the change in the real starting average wage and the impact on subsequent increases. It would appear – particularly for the women in the Canadian sample – that the rate of increase is similar for different cohorts although the starting points may vary.

Perhaps the best data on which to base estimates of Australian increases is that provided by HILDA (the Household, Income and Labour Dynamics in Australia Survey), which has been surveying over 12,000 people since 2001. Wilkins et al. (2011, ch.12) reports that average wage increases in the seven years to 2008 were 5.5% p.a. above inflation. The results confirm that there is considerable variation year by year.
and between individuals, with increases much higher for younger, lower income and better educated individuals particularly. They also show that increases were much higher for those who changed jobs, which would explain the lower salary increases reported by DB schemes, and which would reduce the return on SLHF – as people would often repay when changing jobs.

While each of the data sources above shows significant variability over time and between individuals, there are also overall patterns that appear universal and justify the expectation that average salary increases for those remaining with the same employer will outpace inflation by perhaps 2% to 4% pa under normal circumstances.

4.3 The impact of anti-selection and moral hazard

The link between income and SLHF instalment does introduce additional risks arising from anti-selection and moral hazard.

4.3.1 Managing anti-selection

‘Prior endowments’ (the capabilities inherent in a person’s genes, education and experience) are not insurable to the extent that their impact on salary growth is already known at inception. Asymmetric knowledge of the impact of these prior endowments and other factors that may impact salary increases creates an anti-selection risk.

The risk can be mitigated by developing a classification model for potential borrowers. Individuals’ lifetime wage patterns will depend on age, income, education and other variables. The models could then be applied to determine loadings so that the expected return on every advance would be the same at inception. The models do not have to be perfect. The anti-selection risks will be acceptable to the extent that the relationship between the recipients’ characteristics and their salary increases remains more or less stable and therefore a reliable basis for projecting returns.

Developing the models initially presents one of the significant challenges to SLHF, but experience with HCCs and ICLs is promising. O’Neill and Antcliff (2009), in their abstract, reports that Australian
HECS data has enabled them to build ‘feasible ... microsimulation models of incomes to project future repayments’ for those entering the workforce. The anti-selection and moral hazards attached to SLHF may be greater than those of the HECS scheme as the amounts being repaid will be significantly larger. Investors will however have access to some years of salary growth history, which gives significantly more data for individuals than that available before they begin repaying their HECS. It can be noted that Nerlove (1975, p.160) felt that developing a model to evaluate the prospects of students applying for human capital contracts would not be practical: ‘Risk rating and independent appraisals of income prospects for individuals, or even for broadly defined groups of common socioeconomic background or race, would have far-reaching, indeed politically and socially intolerable, consequences, quite apart from the high informational costs involved.’ Investors in HCCs have obviously overcome the problem, part of which arose from the politics of the time. Information costs have also reduced dramatically with computer technology.

Anti-selection risks also include the risk that SLHF recipients will repay the finance just before they get significant promotions – that would make also it easier for them to afford conventional finance. This risk can be managed by a combination of break fees and ceilings on the maximum rate of increase.

It is not entirely clear who will have the best knowledge of future salary increases. The individual has a better insight into their relative skills and future plans but limited knowledge of average salary progressions. The investor will however have detailed knowledge of how salary increases are developing in the market. The actual ‘deal’ struck between the two is likely to be fair in the sense that both will believe that the risk adjusted costs/yields are reasonable relative to alternatives.

4.3.2 Managing moral hazard

The moral hazard risks are directly analogous to those that arise from proportional income tax. Income may be underreported, shifted to other non-counting sources, and reduced by working less. Managing this risk will probably be easier if SLHF recipients are selected from the employees or large employers who have minimal ability to manipulate
their income. Information on income can also be obtained from the employers, who are also likely to be associated with the superannuation fund advancing the finance.

The possible work disincentives cannot be gauged with any accuracy, but there are some studies of the problem. Tuomala (1990, p. 43) reports that ‘most labour supply studies of men seem to indicate backward-sloping supply curves.’ Higher income leads to men taking more leisure (described as an income effect), but the leisure is more expensive relative to other goods (which creates a substitution effect and reduces the leisure taken.) He lists 11 studies undertaken in the 1970s, of which seven showed the backward slope. Studies of women have, however, normally shown a normal slope. Brown (1983) gives more detail of some of these studies. More recent studies similarly find different and barely significant results, except perhaps for very low incomes. Kalb (2002), for instance, finds negative slopes for both men and women in Australia.

This suggests that moral hazard will not be a significant issue. Even if it is, as with anti-selection, it is not so much the existence but its unpredictability that might make SLHF unattractive. At this point, there appears to be no strong reasons to believe it will be an insurmountable problem. It will probably be necessary, anyway, to introduce floors and ceilings to the rate of increase to reduce the impact of anti-selection and be fair to those who obtain very high increases.

The experience of defined benefit schemes can give considerable comfort to potential investors in SLHF portfolios. They are, in a sense, mirror images, with SLHF providing a lump sum benefit in advance in return for a predetermined proportion of income, while a DB scheme provides the benefit in arrears. The benefits are determined in very much the same way, with relatively significant cross subsidies (in the case of DB schemes to those whose salaries have risen faster, but vice versa in the case of SLHF arrangements). While DB schemes are fast disappearing, their demise would appear to relate mainly to the investment-risks involved rather than the instability or unpredictability of incomes, or of cross subsidies. Given that SLHF will reduce investment related risks significantly, there may be good reason to believe that SLHF schemes could ultimately be more durable than fast-disappearing DB schemes.
4.3.3 The need for government involvement

An argument can be made for government involvement in ICLs and HCCs. The fact that there is no insurance against income risks – in spite of the need – suggests market failure. Market failure in insurance markets arises, in theory, because of asymmetric information and the presence of moral hazards.

This is the justification made for government intervention that is made by:

- Chapman (2005) when discussing ICLs for higher education;
- Chapman and Higgins (2009) in the context of a suggestion to issue ICLs to fund parental leave; and

In each of these cases, however, the ICL is seen as a way in which existing government subsidies can be reduced or made more efficient. In each case, government is seen as the bearer of the extreme risks.

There are strong arguments against government intervention into areas where markets are already functioning. Government intervention, and particularly the possibility of subsidies, has a destructive impact on markets: why pay a full price when you can obtain a better deal? One of the major reasons why Friedman’s ideas for the financing of higher education took so long to be introduced was the significant level of subsidised finance available. The Yale scheme described by Nerlove (1975) began only as subsidised finance was phased out because it became too expensive.

Government intervention can address anti-selection by making insurance compulsory. It cannot avoid moral hazards in lending: the large losses of the US housing agencies in the 1980s and again in the past three years, demonstrate this. As discussed in section 4.3.1 above, however, it seems quite feasible given current data processing abilities, to develop models to address anti-selection. It is moreover not possible to make SLHF compulsory.
Higher education loans may also require government assistance in the collection of debt. Students are particularly mobile and without fixed assets, and the loans are made in the absence of collateral. SLHF however will be made to people who have had employment for some years, are settled, and provide significant collateral in the form of their houses. There should be no difficulty in collecting instalments in the majority of cases.

Share cropping, where a tenant pays a proportion of the produce to the landlords, is an ancient practice. The percentage is often 50%, and the practice is often called ‘farming in the halves’. Share cropping faces risks of anti-selection and moral hazard very similar to SLHF, but it is in extensive use in many countries with landless farmers and small land holdings. It requires fairly close monitoring by the landlord.

It is therefore suggested that it would be possible for markets to provide SLHF-type instruments without government action. The problems of anti-selection and moral hazard are well understood by insurers and the existence of a number of analogous products shows that they can be managed.

4.4 Comparisons to other investment returns

If SLHF is to be sufficiently attractive to potential homeowners and investors, it will have to offer a risk-adjusted investment return comparable to other financial instruments. From an actuarial perspective, compared with the liabilities of investors and the cash flows of recipients, the market risks are lower than all alternatives. The operational risks and lack of liquidity are however such that investors are likely to require an expected return perhaps of the same order as obtainable on ordinary mortgages – or about 3% above average wage growth. Finance at that expected rate would probably be attractive to homeowners. Supply and demand would no doubt lead to deviations from this level at times.

Figure 6 below shows how the returns on an SLHF portfolio earning 3% more than salary inflation would have fared relative to Australian shares and mortgages since 1961. Table 2 shows the average returns (geometric for equities) and the annual standard deviations.
It is believed 2% more than wage growth (3% more than inflation or 5% nominal) should be readily attainable from younger homeowners, and that (supplemented with minimal loadings to increase the return to about 7% nominal) this will not be so high as to reduce the attraction to homeowners, and yet be sufficient to attract investors.

Figure 6: Comparable historical returns in Australia

Table 2: Average returns and volatility

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<thead>
<tr>
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<th>15 years</th>
<th>30 years</th>
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<tbody>
<tr>
<td></td>
<td>Average return</td>
<td>Standard deviation</td>
</tr>
<tr>
<td>Equities</td>
<td>14.7%</td>
<td>15.1%</td>
</tr>
<tr>
<td>Housing loans</td>
<td>7.1%</td>
<td>1.1%</td>
</tr>
<tr>
<td>Wages +3%</td>
<td>7.3%</td>
<td>1.0%</td>
</tr>
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Obstacles to its introduction

While the author remains convinced of the advantages of the instrument, it must be asked why no-one has seen fit to attempt its introduction in the two decades since the idea was first suggested publicly.
5.1 Additional risks

It is clear that there are real credit and operational risks attached, although not obvious that any of these are unmanageable.

5.1.1 Credit and reputational risks

The US subprime experience proves it is not impossible to suffer considerable credit and reputational losses on home loans. The novelty of SLHF makes it particularly vulnerable to unexpected sources of risk.

One possible source of brand and reputational risk that will require careful management is that some of the recipients will end up paying more than they would under a conventional mortgage (much like subprime borrowers), and have the potential of embarrassing any institution that may have invested. Investors will need to be convinced that the homeowners have understood and acknowledged this possibility.

Some thought also needs to be given to possible legal challenges to the novel contract required for SLHF. Two that have been raised previously were:

- the ‘ultra duplum’ principle that prevented capitalised interest from exceeding the initial principal. Legal opinion was that it was possible to specifically contract out of this; and

- the possibility that the contract would be interpreted as one of forced labour or slavery. The investor however does not exercise any domination over the recipient of the funds, and has almost no power over the nature of work performed; the interest is purely pecuniary.

Appropriate disclosures, training and education will however be needed to manage this risk.

It was credit losses of another type that prevented the product launch in South Africa in the early 1990s. Asher (1991) calculated that SLHF would have enabled over a million Black South African families to afford to enter the formal housing market. It should be explained that Black South Africans had been unable to buy property until 1988,
and the high nominal interest rates effectively prevented all but 750,000 households from buying even the least expensive house. At the time the product should have been launched however, over 300,000 loans were subject to ‘bond boycotts’ for a combination of political and economic reasons.

Setting aside the anti-selection and moral hazard concerns noted above, a sensibly managed home finance instrument should be subject to minimal credit risks. Recent experience, does however suggest that loan originators and administrators should participate in credit losses in order to reduce moral hazards arising from their ability to compromise credit standards.

5.1.2 Operational risks

The operational risks are more significant:

- Legal documentation for the instrument may require novel contracts with the recipients and investors, the tax consequences (what is income and what capital?) being entirely unclear.

- The collection and auditing of data are more demanding than most equivalent instruments. The definition of salary is an issue on its own as items subject to manipulation, such as overtime, need to be excluded in some way.

- There will be a need to rapidly build up the ability to analyse expectations of salary increases to prevent anti-selection and be fairer to different classes of borrowers.

- The supply and demand for funds is likely to take some time to find a balance that does not lead to large changes in price (loadings on the instalments) or the rationing of funds.

- Initial investors will have to commit to an untried instrument that may not be entirely repaid for 20 years, so are likely to be particularly wary of operational risks.

The operational risks are however potentially manageable; the fact that relatively predictable income tax is collected everywhere in the world demonstrates that salary contingent instalments can be collected.
5.2 Challenges faced by similar instruments

Thought must be given as to whether one of these risks, or other unthought of obstacles, are insurmountable. There are reasons for believing that instruments analogous to SLHF do face particular risks of being misunderstood. Such challenges to their introduction need to be addressed by more careful explanation of the risks that they address, and the manner in which these risks can be managed.

5.2.1 Inflation linked bonds

Stiglitz (1998, pp. 6-7) wonders why inflation indexed-bonds took so long to be introduced in the USA, given that they appear to be ‘Pareto improvements ... which make everyone better off ... provide a way for households and government to reduce their risks. At the same time they create a market that did not previously exist, and the government reaps some of the benefits of the new market in the form of lower interest charges on its debt.’ His reasons are worth quoting extensively as they come from first-hand experience from a man who had recently been chief economist of the World Bank and won the Nobel Prize three years later. My humbler experience is that the reasons continue to apply to indexed bonds, SLHF and other instruments that reduce inflation risk.

Despite these obvious attractions – and the fact that very few people would be hurt by the innovation – getting the Clinton administration to accept indexed bonds was a long and difficult process. There were three reasons for this. First, it was enormously difficult explaining the nature of the real risk faced by the government. Critics worried that if inflation increased, interest payments would increase. Try as we might, I think some never understood that the government’s tax receipts also went up with inflation and thus indexed bonds actually reduced the government’s real risk.

Second, some misguided inflation hawks thought that indexing would reduce the resolve of government to fight inflation. As is so often the case with such inflation hawks, they did not bother to look at the relevant empirical literature ... or at the counterargument that with indexed bonds, inflation has an immediate and direct budgetary impact, thus encouraging
governments to act against it.

The third reason was that Treasury turned to bond traders— their natural clientele – for advice. The experience in England from the perspective of bond traders was that these bonds were a failure; that is, people bought them for their retirement and did not trade them. Without trades, where were their commissions? Of course, from the perspective of someone trying to create an instrument to enhance retirement security, this was ideal: we did not want a gambling instrument. The bond traders raised anxiety levels: Would Treasury throw a party to which no one would come? (Stiglitz, 1998, p. 7)

5.2.2 Equity mortgages

Another housing innovation, which addresses both the cash flow problems of prospective homeowners and reduces the risks of house price volatility, is to tie the cost of the loan to changes in the value of the home.

As Joye et al. (2003) note, ‘shared equity’ products mitigate the indivisibility of the housing asset which otherwise binds together the homeowner’s consumption and investment decisions with the undesirable result of increasing their economic exposure to housing. Joye also notes that, in Australia, the volatility of a single family home is some 15–20% per annum which contrasts with a national property index’s volatility of around 3–4% per annum.

These ‘shared equity’ innovations offer not only benefits to homeowners, but also the opportunity for non-homeowning investors to hedge their future housing costs without the administrative complications of direct investment.

The idea dates back to Follain and Struyk (1977) if not earlier, but is now commercially available through Bendigo & Adelaide Bank (see www.efm.info).
The product can be used as a complimentary product with SLHF, subject to careful credit checking particularly for those equity shared mortgages that are geared to inflation.

The difficulties companies have had in making this innovation also appear to be greater than would be expected by the product itself. Stiglitz’s list (underestimation of the real risks; misunderstanding of the instrument; and the antipathy of institutional investors to illiquid assets) also appears to apply.

SLHF instruments will be more liquid than equity mortgages because instalments are paid; both however are likely to suffer from anti-selection and moral hazard risks.

6 Conclusion

Salary linked housing finance as described in this paper can largely eliminate the interest rate and idiosyncratic salary or inflation risks faced by homeowners, as recipients of funds, and pensioners as investors. The return at which investors and homeowners will exchange contracts is likely to be some 3% pa above the rate of average salary inflation.

It is suggested that the risks of such a product appear to be manageable as the operational risks are comparable with those of DB superannuation schemes, and the moral hazards are analogous to those arising from the collection of taxes. But anti-selection and moral hazard risks will nonetheless be nontrivial obstacles that any participants seeking to bring such innovations to market will have to comprehensively address.

There are at least two other obstacles to the introduction of salary linked housing finance as alternatives to conventional housing finance. One relates to misunderstandings as to the nature of the product and its advantages and disadvantages, which this paper hopes to highlight. Another relates to a lack of appreciation of market and inflation risks that apply to investors and borrowers alike. The current financial volatility is persuasive evidence that historical causes of economic instability do not go away. In this context, neither homeowners nor pensioners should ignore the significant inflationary threats posed over
the decades to come, and in the near-to-medium term as a function of fiscal and monetary stimulus combined with the finite supply of key commodities

As it also provides a way of assisting younger people to buy their own homes, the instrument appears to be worth further consideration.
References


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