Stochastic Solvency Testing in Life Insurance

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Deterministic Solvency Testing

- Assets > Liabilities
- In the insurance context, the values of the insurer’s assets and liabilities are uncertain.
- This uncertainty should be allowed for in any insurer solvency calculations, but historically it has been ignored.
Stochastic Solvency Testing

- Involves determining probability distributions for $A(t)$ and $L(t)$ (or just $C(t)$).
- Insurer must hold an amount of capital at the valuation date sufficient to satisfy a probability-based criterion.
Stochastic Risk Measures

• x% VaR: Determine $C(0)$ such that:
  \[ \Pr(C(t) > 0) = x\% \]
  or \[ \Pr(-\Delta C(t) < C(0)) = x\% \]
  where \[ \Delta C(t) = C(t) - C(0) \]

• x% TVaR = $E(-\Delta C(t) \mid -\Delta C(t) > x\% \text{ VaR})$
Value at Risk (VaR)
Tail Value at Risk (TVaR)

![Chart showing Tail Value at Risk](chart.png)
Cost of Capital Risk Margins

• Allow for “the hypothetical cost of regulatory capital necessary to run off all of the insurance liabilities, following financial distress of the company”.

• Included so as to provide adequate risk compensation for a hypothetical insurer who may take over the portfolio in the future.
Australian LI Solvency Legislation

• Three actuarial valuation standards: LPS1.04, LPS2.04 and LPS3.04.
• No requirement is made for the actuary to use stochastic assumptions under any of these three standards.
• The probabilities of adequacy of the solvency and capital requirements are unknown.
Australian LI Solvency Legislation

- According to Karp (2002): “the solvency risk criterion was set at a 5% probability of assets falling below liabilities within any of the next three annual balance dates”.
Australian GI Solvency Legislation

- If the internal model based approach is used, the insurer must hold sufficient capital such that the insurer’s probability of default over a one year time horizon is reduced to 0.5% or below.
International Association of Actuaries

• A reasonable period for the solvency assessment time horizon is one year.

• The amount of required capital must be sufficient with a high level of confidence (eg. 99%) to meet all future obligations.

• The most appropriate risk measure for solvency assessment is the TVaR.
Data – Policy Types

- Type 1: Whole of life/endowment insurance.
- Type 2: Unbundled policies (capital guaranteed and investment-linked).
- Type 3: Level term insurance.
- Type 4: Yearly-renewable term insurance.
Stochastic Solvency Testing Model

- Compatible with the existing Australian valuation philosophy.
- Non-policy liabilities are ignored.
- Investment earnings, inflation, tax, expenses, mortality and policy discontinuance are all considered.
- Dependency relationships are considered.
Dependency Relationships

• Some evidence to indicate the presence of selective lapsation, but the evidence is inconclusive.

• Evidence to suggest a significant relationship exists between fluctuations in the short-term interest rate and mortality.

• Evidence to suggest a significant relationship exists between fluctuations in economic variables and lapsation.
Dependency Relationships

- Economic Sub-Model
  - Mortality Sub-Model
  - Lapsation Sub-Model
Stochastic Sub-Models

• Economic:
  – modified CAS/SOA model.

• Mortality:
  – Negative binomial distribution for Type 1 policies.
  – Poisson distribution for all other policy types.

• Lapsation:
  – Normal-Poisson model for all policy types.
Deterministic Capital Requirements

• Solvency Capital Requirement = LPS2.04 Solvency requirement – BEL

• Capital Adequacy Capital Requirement = LPS3.04 Cap. Ad. requirement – BEL
Stochastic Capital Requirements

- 99.5% VaR of the change in capital distribution over a one year time horizon.
- 99.5% TVaR of the change in capital distribution over a one year time horizon.
- 95% VaR of the change in capital distribution over a three year time horizon.
- 95% TVaR of the change in capital distribution over a three year time horizon.
Stochastic Asset Requirements

• Stochastic Minimum Asset Requirement (SMAR) =
  Best estimate liability
  + Cost of capital risk margin
  + Solvency capital requirement

• Similar to the minimum asset requirement under the Swiss Solvency Test.
Model Asset Portfolios

- Composition of each of the Model Asset Portfolios:

<table>
<thead>
<tr>
<th></th>
<th>Portfolio 1</th>
<th>Portfolio 2</th>
<th>Portfolio 3</th>
<th>Portfolio 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equity</td>
<td>0%</td>
<td>25%</td>
<td>55%</td>
<td>75%</td>
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<tr>
<td>Property</td>
<td>0%</td>
<td>5%</td>
<td>5%</td>
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<tr>
<td>Fixed Interest</td>
<td>70%</td>
<td>45%</td>
<td>30%</td>
<td>15%</td>
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<tr>
<td>Cash</td>
<td>30%</td>
<td>25%</td>
<td>10%</td>
<td>5%</td>
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<td>Total</td>
<td>100%</td>
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</table>
Base Case Simulation Results

- Capital Requirements per Policy for the Base Case Scenarios ($)

<table>
<thead>
<tr>
<th>Portfolio</th>
<th>99.5% VaR</th>
<th>99.5% TVaR</th>
<th>95% VaR</th>
<th>95% TVaR</th>
<th>LPS 2.04 Cap. Req.</th>
<th>LPS 3.04 Cap. Req.</th>
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<tbody>
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Base Case Simulation Results

- Levels of Sufficiency (on a VaR Basis) of the LPS2.04 and LPS3.04 Capital Requirements for the Base Case Scenarios

<table>
<thead>
<tr>
<th>Liability Portfolio</th>
<th>Asset Portfolio</th>
<th>$-\Delta C(1)$ LPS2.04</th>
<th>$-\Delta C(1)$ LPS3.04</th>
<th>$-\Delta C_{\min}(0,3)$ LPS2.04</th>
<th>$-\Delta C_{\min}(0,3)$ LPS3.04</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1</td>
<td>&gt;99.99%</td>
<td>&gt;99.99%</td>
<td>&gt;99.99%</td>
<td>&gt;99.99%</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>1.36%</td>
<td>72.45%</td>
<td>0.00%</td>
<td>1.19%</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>&gt;99.99%</td>
<td>&gt;99.99%</td>
<td>0.00%</td>
<td>0.00%</td>
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Sensitivity Analysis

1. Using a different economic sub-model.
2. Ignoring mortality over-dispersion.
3. Ignoring lapsation over-dispersion.
4. Ignoring mortality and lapsation over-dispersion.
5. Ignoring mortality and lapsation over-dispersion and dependency relationships.
6. Simplifying the formulae used to determine the mean mortality and lapsation rates.
Sensitivity Analysis Results

• SMAR are not significantly affected by ignoring mortality or lapsation over-dispersion or the dependency relationships between the sub-models.

• SMAR tend to be higher for Type 3 and 4 policies if calculated using an alternative economic sub-model.

• SMAR tend to be lower for Type 3 and 4 policies if calculated using simplified mean formulae.
Sensitivity Analysis Results

• In all cases, it is still true that:
  – for Type 1 and 2 policies, deterministic capital requirements are much greater than the stochastic capital requirements.
  – for Type 3 policies the LPS2.04 solvency requirements are less than the stochastic capital requirements.
  – for Type 4 policies, the deterministic capital requirements are greater than the 99.5% VaR and TVaR, but less than the 95% VaR and TVaR.
Implications

• For Type 1 and 2 policies, the LPS2.04 and LPS3.04 requirements are unnecessarily high.

• For Type 3 and 4 policies, the LPS2.04 and LPS3.04 requirements are too low.
Suggested Actions

• Increase the deterministic solvency requirements for portfolios containing Type 3 or 4 policies.
• Move from a deterministic solvency capital calculation regime to a stochastic regime.