Replicating Portfolios and Risk Management

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1. Needs and Applications
2. What is a Replicating Portfolio
3. Methodologies
4. Case Studies
5. Implementation Challenges
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Risk Management

Best Practice

Dynamic Hedging

Static Hedging

Matching

Traditional ALM

Asset Liability Segregation

Real World Replication

Replicating Portfolios

Market Consistent

Risk Neutral

Real World Prudent

Historic / Old World
Capital Management
APRA Pillar 2: ICAAP Requirements

“To maintain at all times a level and quality of capital commensurate with the level and extent of risks to which the insurer is exposed from its activities”

“To have adequate systems and procedure for identifying, measuring, monitoring and managing the risks arising from its activities on a continuous basis, and for assessing the capital needed in relation to those risks”

Frequent A&L valuations
Computational Limitations
Fast valuations needed → RPs
Four Replicating Approaches

- Asset Backing: Unit linked, allocated pensions, target date funds, annuities, deep ITM guarantees, debt
- Option Identification: Capital guarantees, some simple variable annuities (GMAB, GMDB, GMIB)
- Optimisation: Participating business, some variable annuities
- Stochastic (light or full): Most variable annuities

Increasing complexity
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Definition

A replicating portfolio is an asset portfolio consisting of:

- Real assets with liquid market prices
- Real illiquid assets
- Theoretical assets

that replicates the scenario-dependent cash flows of an asset or liability.
### Theoretical Framework

**Principle of No Arbitrage**

- If two instruments or portfolios have identical cash flows under all economic conditions...

- then the prices of the two instruments must be equal

**Implementation of Replicating Portfolios**

- If the cash flows of a portfolio of assets and the liabilities are identical under the base and stressed economic scenarios...

- then the market value of the liabilities must equal that of the asset portfolio
Illustrative Example 1

1.00 unit

1.5-years fixed coupon bond
6% semi-annual coupon

0.03 units
6-months ZCB

0.03 units
12-months ZCB

1.03 units
18-months ZCB
Illustrative Example 2

1.00 unit
Equity index forward
Forward price at $K$
Maturity $T$

1.00 unit
European equity call option
Strike $K$
Maturity $T$

-1.00 unit
European equity put option
Strike $K$
Maturity $T$
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Different Methodologies

The OLS (ordinary least square) model
- Asset selection, optimisation, diagnostics

The GAM (generalised additive model) model
- GAM fitting, segmentation, asset selection, diagnostics

The equity replication model
- Parametric fitting, piecewise replication, diagnostics
The OLS Model

Asset Selection
- Applicable to each individual time step
- Select candidate assets based on the liability profiles

Optimisation
- Applicable to liability cash flows or the net present values
- Choose asset portfolio to minimise overall error, for both base and stressed scenarios, subject to constraints

Diagnostics
- Goodness of fit measured with standard statistics, e.g. $R^2$
- Out-of-sample tests performed with additional economic scenarios
1. Economic Scenarios

2. Liability Cash Flow Projections

3. Candidate Asset Selection

<table>
<thead>
<tr>
<th>Liability Cash Flows</th>
<th>Candidate Assets</th>
</tr>
</thead>
<tbody>
<tr>
<td>Guaranteed interest rates</td>
<td>Zero-coupon bonds</td>
</tr>
<tr>
<td>Maturity payments</td>
<td>Stock derivatives</td>
</tr>
<tr>
<td>Annuities</td>
<td>Swaptions</td>
</tr>
</tbody>
</table>

4. Optimisation

\[
\min \sum_{t=1}^{T} \sum_{i=1}^{S} \left( cf_{t}^{\text{liabilities}}(t) - \sum_{k=1}^{N} x_k cf_{t+k}^{*}(t) \right)^2 \\
\text{Subject to: RP Total Value = Simulated FV Liabilities}
\]

5. Validation & Diagnostics

6. Solvency & Capital Requirements

Stress runs carried out on RP

Capital
The GAM Model

**GAM Fitting**
- Fit GAM curves to (base & stressed) liability cash flows at each time step
- Apply optimisation, which balances between both goodness of fit and smoothness (robustness)

**Segmentation**
- Segment / discretise the GAM curves
- Can be achieved via clustering or optimisation

**Asset Selection**
- Applicable to each individual time steps
- Choose appropriate replicating assets based on the segmented curves
- Determine final assets via optimisation on the GAM curves

**Diagnostics**
- Goodness of fit measured with standard statistics, e.g. generalised cross validation (GCV) and $R^2$
- Out-of-sample tests performed with additional economic scenarios
The GAM Model

Graphs showing the relationship between Liability Claim Amount and Index Level.
The Equity Replication Model

**Parametric Fitting**
- Models net assets (assets less liabilities) at next time step as a multivariate polynomial of pre-selected risk factors
- Determine coefficients via optimisation, by utilising both normal and extreme economic scenarios

**Piecewise Replication**
- Construct a portfolio of instruments to replicate each term of the multivariate polynomial via optimisation
- Combine with polynomial coefficients to establish replicating portfolio for the entire liability profile

**Diagnostics**
- Goodness of fit measured with standard statistics, e.g. $R^2$
- Out-of-sample tests performed with additional economic scenarios
The Equity Replication Model

Sub portfolio consisting of cash and stocks (equity derivatives for convexity adjustment)

Sub portfolio consisting of equity put and call options

Sub portfolio consisting of cash and zero-coupon bonds

Sub portfolio consisting of zero-coupon bonds, interest rate caps / floors
GAM Preferred Method

• The **GAM model** is our preferred method, based upon years of implementation and experience.
  
  – Not impacted by **outliers in data** and **more robust** (compared to the standard model)
  
  – Allow **replication of individual liability items** (compared to the equity replication model)
  
  – GAM framework well understood and is **supported by most standard statistical softwares**
  
  – **Cross terms** may be **difficult to replicate** under the equity replication approach
  
  – **Optimisation** problem more **plausible** (compared to the standard model)
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## Case Studies

| Simple Guaranteed Minimum Accumulation Benefit (Simple GMAB) | • To illustrate the validity of the concept  
• Product guarantees a minimum investment return over a fixed investment horizon **without** the real-world features |
|-------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Complex Guaranteed Minimum Accumulation Benefit (Complex GMAB) | • To assess the effectiveness of the replication of a real-world investment guarantee  
• Product guarantees a minimum investment return over a fixed investment horizon |
| Range Forward (a.k.a. Cylinder, Tunnel or Collar) | • To illustrate some shortfalls of the GAM approach  
• Consists of a short European put option and a long European call option |
Case Study – Simple GMAB

- A portfolio of 10 male policies each with $100,000 of account value
- Age profiles range from 35 to 85
- Death benefit (GMDB) available
- Term of GMAB = 20 years
- Range of guaranteed balances from $70,000 (out-the-money) to $130,000 (in-the-money)
- Deterministic lapse rates
- No roll-ups nor ratcheting of the guaranteed benefits
Case Study – Simple GMAB

**Graph 1:**
- **Equity Index Level (w.r.t. Initial Value)**
- **Liability Values**
  - MG-Hedge Liabilities
  - GAM
  - RP Portfolio

**Graph 2:**
- **MG-Hedge Liability Values**
- **RP Liability Values**
- **R² = 99.95%**
Case Study – Simple GMAB

- Replicating portfolio consists of only European options and zero-coupon bonds
- Excellent replication of base liability values
- Minimal errors in the sensitivities to key risk variables
- Very accurate capital strain from equity stress

<table>
<thead>
<tr>
<th>Liabilities</th>
<th>MG-Hedge Values</th>
<th>RP Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bond</td>
<td>6</td>
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<tr>
<td>European Options</td>
<td>56,118</td>
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<tr>
<td>Total Base Values</td>
<td>56,933</td>
<td>56,124</td>
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<td>-20% Equity Stress</td>
<td>75,561</td>
<td>74,424</td>
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<td>Capital Strain</td>
<td>18,629</td>
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<td>Delta</td>
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<th>Rho</th>
<th>MG-Hedge Values</th>
<th>RP Values</th>
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<tr>
<td>0-1 Year</td>
<td>3</td>
<td>4</td>
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<tr>
<td>2-5 Year</td>
<td>13</td>
<td>13</td>
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<tr>
<td>6-10 Year</td>
<td>21</td>
<td>21</td>
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<td>11-20 Year</td>
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<td>21-30 Year</td>
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<td>0</td>
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<tr>
<td>Parallel</td>
<td>128</td>
<td>127</td>
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<thead>
<tr>
<th>Vega</th>
<th>MG-Hedge Values</th>
<th>RP Values</th>
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<tr>
<td>1-2 Year</td>
<td>284</td>
<td>174</td>
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<tr>
<td>3-4 Year</td>
<td>176</td>
<td>289</td>
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<td>5-6 Year</td>
<td>228</td>
<td>303</td>
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<td>7-8 Year</td>
<td>417</td>
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<td>9-10 Year</td>
<td>1352</td>
<td>1293</td>
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<tr>
<td>Parallel</td>
<td>2,457</td>
<td>2,359</td>
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</table>
Case Study – Complex GMAB

• A portfolio of 10 male policies each with $100,000 of account value
• Age profiles range from 35 to 85
• Term of GMAB = 20 years
• Range of guaranteed balances from $70,000 (out-the-money) to $130,000 (in-the-money)
• Dynamic lapse rates assumed
• Roll-ups (0% - 9% p.a.) and ratcheting of the guaranteed death benefits (GMDB)
Case Study – Complex GMAB

- Liability Values
  - Equity Index Level (w.r.t. Initial Value)

- MG-Hedge Liabilities
- GAM
- RP Portfolio

- RP Liability Values

- MG-Hedge Liability Values

- $R^2 = 99.65\%$
Case Study – Complex GMAB
Case Study – Complex GMAB
Case Study – Complex GMAB

- Asian and look-back equity options required for replicating dynamic lapses and ratchets
- Excellent replication of base liability values
- Minimal errors in the sensitivities to key risk variables
- Very accurate capital strain from equity stress

<table>
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<tr>
<th>Liabilities</th>
<th>MG-Hedge Values</th>
<th>RP Values</th>
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<tbody>
<tr>
<td>Bond</td>
<td>116,948</td>
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<tr>
<td>European Options</td>
<td>366,834</td>
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<td>Asian Options</td>
<td>22,067</td>
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<tr>
<td>Lookback Options</td>
<td>-346,121</td>
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<tr>
<td>Total Base Values</td>
<td>164,800</td>
<td>161,729</td>
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<tr>
<td>-20% Equity Stress</td>
<td>192,287</td>
<td>188,423</td>
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<td>Capital Strain</td>
<td>27,487</td>
<td>26,694</td>
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<tr>
<th>Delta</th>
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<td>1,007</td>
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<table>
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<tr>
<th>Rho</th>
<th>MG-Hedge Values</th>
<th>RP Values</th>
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<tbody>
<tr>
<td>0-1 Year</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>2-5 Year</td>
<td>20</td>
<td>17</td>
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<td>6-10 Year</td>
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<td>89</td>
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<td>11-20 Year</td>
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<td>199</td>
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<tr>
<td>21-30 Year</td>
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<td>0</td>
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<tr>
<td>Parallel</td>
<td>306</td>
<td>306</td>
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</table>

<table>
<thead>
<tr>
<th>Vega</th>
<th>MG-Hedge Values</th>
<th>RP Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-2 Year</td>
<td>535</td>
<td>475</td>
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<tr>
<td>3-4 Year</td>
<td>370</td>
<td>721</td>
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<tr>
<td>5-6 Year</td>
<td>733</td>
<td>807</td>
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<tr>
<td>7-8 Year</td>
<td>889</td>
<td>820</td>
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<tr>
<td>9-10 Year</td>
<td>3403</td>
<td>3187</td>
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<tr>
<td>Parallel</td>
<td>5,931</td>
<td>6,010</td>
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</tbody>
</table>
Case Study – Range Forward

![Graph 1: Liability Values vs. Equity Index Level](image)

- **MG-Hedge Liabilities**
- **GAM**
- **RP Portfolio**

![Graph 2: MG-Hedge Liability Values](image)

- **R² = 99.85%**
Case Study – Range Forward

• GAM model smoothes input data
• More knots are required to produce unsmooth payoffs
• Replication still excellent – both fair value and risk sensitivities, i.e. replicating portfolio not unique!

<table>
<thead>
<tr>
<th>Range-Forward</th>
<th>MG-Hedge Values</th>
<th>RP Values</th>
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</thead>
<tbody>
<tr>
<td>Fair Value</td>
<td>-42,946</td>
<td>-41,580</td>
</tr>
<tr>
<td>Delta</td>
<td>-7,598</td>
<td>-7,699</td>
</tr>
<tr>
<td>Rho</td>
<td>-80</td>
<td>-81</td>
</tr>
<tr>
<td>Vega</td>
<td>-748</td>
<td>-737</td>
</tr>
</tbody>
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Implementation Requirements

- **Systems**: data collection, pre-processing, optimisation
- **Processes**: data review, asset selection, optimisation design, results review, signoff
Challenges

- Business requirements
  - **Scope**: single / multiple blocks, full balance sheet
  - **Characteristics**: accuracy, frequency, level of detail
- Choice of most appropriate method
- Timelines of data dependencies
- Robustness versus fit trade-off
- Goodness of fit thresholds of acceptability

Expertise and experience can make all the difference
Conclusion

• Replicating portfolios have numerous applications:
  – Risk management: assessment, hedging,
  – Capital management: APRA Pillar 2 requirements
  – Internal MI and education

• Suite of methods and tools available

• Implementation challenges exist but can be overcome with suitable experience and expertise
Thank You and Questions

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