Institute of Actuaries of Australia

Managing Systematic Mortality Risk with Group Self Pooling and Annuitisation Schemes

Prepared by Chao Qiao and Michael Sherris

Presented to the Institute of Actuaries of Australia
Biennial Convention
10 - 13 April 2011
Sydney

The Institute of Actuaries of Australia
Level 7 Challis House 4 Martin Place
Sydney NSW Australia 2000
Telephone: +61 (0) 2 9233 3466 Facsimile: +61 (0) 2 9233 3446
Email: actuaries@actuaries.asn.au Website: www.actuaries.asn.au

Managing Systematic Mortality Risk with Group Self Pooling and Annuitisation Schemes

Chao Qiao
Australian School of Business
University of New South Wales
chaoqiao@hotmail.com

Michael Sherris
Australian School of Business
University of New South Wales
m.sherris@unsw.edu.au

23rd February 2011

Abstract

Group Self-annuitisation Schemes (GSAs), or Pooled Annuity Schemes, are designed to share uncertain future mortality experience including systematic improvements. They have been proposed because of the significant uncertainty and the financial impact of future mortality improvement on pension and annuity costs. The challenges for designing group pooled schemes include the decreasing average payments when mortality improves significantly, the decreasing numbers in the pool at the older ages and the dependence of systematic mortality improvements across different ages of members in the pool. This paper assesses the impact of systematic dependence and reducing numbers in the pool at extreme ages on the efficacy of longevity pooling. Current proposals for pooling schemes are designed to insure against idiosyncratic risk while leaving systematic risk to be borne by individuals. The paper uses a multiple-factor stochastic Gompertz-Makeham model of mortality, calibrated to Australian data, to demonstrate the significance of these issues. The model produces analytical results from extreme value theory for survival distributions and approximate annuity computations. Simulations are used to show how the pooling can be made more effective and to quantify the limitations of these pooling schemes. The results quantify the impact of pool size on risk sharing, especially at the older ages, demonstrate the significance of systematic mortality risks and dependence on pooling effectiveness, and highlight the potential need for reinsurance and solidarity between the young and old ages in the pool to improve the effectiveness of GSAs as a longevity insurance product solution.

Keywords: group self-annuitisation, pooled annuity, longevity risk, extreme value distribution, Gompertz-Makeham mortality

JEL Classifications: D02, G22, G23, J26

1 Introduction

Internationally there has been a significant shift to defined contribution schemes to fund retirement and a reduction in pension funds providing longevity protection post retirement. Increasingly attention has been turned to the post-retirement phase and financial arrangements to convert accumulated lump sums into retirement incomes. These policy issues are well covered in recent World Bank discussion papers (World Bank (2010a) [20] and World Bank (2010b) [14]). The ideal post retirement income provides consumption income with both longevity insurance and inflation indexation. Yaari (1965) [21] demonstrates the welfare benefits of ordinary life annuities because of the longevity insurance protection provided under a standard life-cycle consumption model with perfect markets, actuarially fair annuities with no loadings and rational individuals with no bequest motives. Davidoff et al. (2005) [4] showed that there were benefits from some level of annuitisation under more general assumptions. Although annuity prices are not actuarially fair, risk-averse individuals will still value annuities (Mitchell 2001 [10]).

There are many reasons advanced as to why there is a lack of well developed annuities markets despite the potential longevity risk benefits. From a demand perspective, these include lack of liquidity, bequest motives, poor value for money, and availability of public pensions (Friedman and Warshawsky (1990) [6], Hurd (1989) [8], Mitchell et al. (1999) [11]). From the supply perspective, insurers incur significant capital costs in guaranteeing life time incomes because of the significant uncertainty of individual longevity prospects. Adverse selection and lack of underwriting in the life annuity market also leads to significant costs for suppliers of life annuities. Evans and Sherris (2010) [5] review demand and supply factors in developing a life annuity market in Australia.

An approach to the management of uncertain future longevity presented in Piggott, Valdez and Detzel (2005) [13] is referred to as group self-annuitisation (GSA), which is designed to pool idiosyncratic risk with individuals bearing systematic risk. Individuals invest capital into the pool and are paid an annuity income that varies with the mortality experience in the pool. As individuals exit the pool from death, their remaining capital is shared amongst the survivors in the pool in the form of mortality credits. The motivation behind GSAs is that there are no payment guarantees so that systematic longevity risk and idiosyncratic mortality risk is borne by the individual and the pool but not the insurance provider. An advantage of a GSA scheme is that there is no need for guarantees and costly capital as compared with an ordinary annuity, especially if there is a lack of hedging markets for longevity risk.

Valdez, Piggott and Wang (2006) [19] addressed adverse selection and demand issues for GSAs. Based on research by Abel (1986) [2] and Sinha (1989) [17] on adverse selection for ordinary annuity funds with known heterogeneous mortality probabilities, Valdez et al. (2006) applied a similar utility framework to GSAs to show that annuitants will naturally adversely select against both conventional annuities and GSAs. The extent to which adverse selection is exercised against GSAs is lower than that of conventional annuities given certain utility function conditions.

Stamos (2008) [18] extended Valdez, Piggott and Wang' work by investigating the optimal consumption problem for fund participants in a portfolio choice framework.

Stamos showed that the expected mortality credit achieved by members who survived increased with the number of members in the pool. The expected optimal consumption as a proportion of initial wealth increased with the number of fund members. A similar pooling scheme was presented by Sabin (2010) [15] known as Fair Tontine Annuity or FTA, in which "fairness" is achieved by ensuring that pool members' expected benefits did not depend on the mortality experience of others in the pool.

Group pooling will be most effective and sustainable if the benefit payments received by individuals do not exhibit a significant downward trend over time and future benefit payments do not have too great a level of volatility. Dependence between individual lives created by common drivers of mortality improvements has the potential to undermine pooling effectiveness not only because of declining payments but also because of systematic longevity risk that cannot be reduced with increases in pool size.

Under existing sharing arrangements, as in Piggott, Valdez and Detzel (2005) [13], benefit payments will significantly decline over time, driven primarily by the improving trend of underlying human mortality. In addition, the volatility of benefit payments will increase over time, which is driven by uncertainty in mortality forecasts as well as the random deviation of experienced deaths away from population mortality especially at the older ages where numbers of survivors are small.

This paper assesses the effectiveness of group self annuitisation schemes allowing for systematic but uncertain longevity trends. A multi-factor stochastic version of the Gompertz-Makeham (GoMa) model is used to capture both dependence across ages and systematic mortality improvements over time. The model is fitted to historical data for Australian males and implemented in a manner such that positive mortality rates are generated. Simulation is used to quantify the extent to which benefit payments will decline over time, due to the inability of GSA benefit payment calculation rules to capture systematic mortality improvements. The extent to which the volatility of benefit payments widens over time is also quantified using the model. GSA's with a single cohort only allow for non-systematic risk. Introducing other cohorts of the same initial age makes it practical to pool systematic risk without undermining the overall fairness of the pooling arrangements.

The extent to which these pooling arrangements can be made effective is a main aim of this paper. Current pooling arrangements adopt new annuity factors based on the current experience. GSA benefit payments still decline at the older ages with this approach. The effectiveness and feasibility of GSAs is quantified using the distribution of benefit payments for realistic pooling scenarios. A method is proposed to share systematic mortality improvements in a GSA arrangement so that they are pooled and borne by the collective rather than the individual, extending and generalizing pooling arrangements in Piggott, Valdez and Detzel (2005) [13]. It is also shown that by including the systematic trend of future mortality improvements in the calculation of annuity factors for benefit payments, the expected decline over time is significantly reduced.

The paper is organised as follows. Section 2 discusses the technical specifications for GSA pooling. Section 3 discusses the mortality model and Section 4 outlines the GSA fund value simulation procedures. Results of the analysis are presented and discussed in Section 5 including the improvements to pooling methodology covered in this paper. Finally, the main findings of this paper are summarised in Section 6.

2 Analysis of GSA Schemes

The analysis of GSA schemes follows that in Piggott, Valdez and Detzel (2005). A generalization of their procedure is presented here and explained.

2.1 Benefit payments

In Piggott, Valdez and Detzel (2005), the benefit payments at time t for a fund of members in a pool is recursively determined by modifying the previous benefit with experience factors. For a surviving individual i at time time t in the pool the benefit payment is determined as

$$\begin{aligned} {}_x^kB^*_{i,t} &= {}_x^kB^*_{i,t-1} \cdot MEA_t \cdot IRA_t \cdot CEA_t \\ &= {}_x^kB^*_{i,t-1} \cdot \frac{F^*_t}{\sum_{k\geq 1}\sum_x \frac{1}{p_{x-1}}\sum_{A_t} {}_x^kF^*_{i,t}} \cdot \frac{1+R_t}{1+R} \cdot \frac{\ddot{a}^{old}_{x-1}}{\ddot{a}^{new}_{x-1}} \end{aligned} \tag{1}$$

where there are a total of A_t individuals alive, aged x at entry, currently having been in the fund for k years. F^*_t denotes the total fund value of all surviving members at time t and ${}_x^kF^*_{i,t}$ denotes the fund value of individual i conditional on survival. The $*$ superscript denotes survival to time t.

Payments are adjusted over the time period t with an interest rate factor, where R is the expected interest rate and R_t the realised interest rate, a mortality adjustment factor (MEA) and a separate changed expectation factor (CEA) to allow for changes in experienced mortality and in future expected mortality.

Individual i who is aged aged x at entry, subscribes ${}_x^0\hat{F}^*_{i,0}$ and an initial payment is determined based on

$${}_x^0B^*_{i,0} = \frac{{}_x^0\hat{F}^*_{i,0}}{\ddot{a}_x}$$

The main concern in this paper are the adjustments for mortality and will not include interest rate variability, so that the IRA_t adjustment factor will not be included. This is not only to simplify the analysis but also to ensure results are not confounded by other factors. Adjustments for random interest rates is readily incorporated since these are normally assumed independent of mortality adjustments. The annual effective nominal interest rate, expected and realised, used throughout this paper is 5%, equivalent to a force of interest of $\delta = 0.048790164$.

At any given point in time

$${}_x^kB^*_{i,t} = \frac{{}_x^k\hat{F}^*_{i,t}}{\ddot{a}_{x+k}}$$

so that the benefit payment is an average amount based on the expected present value factor of an ordinary whole life annuity. The hat, $\hat{\cdot}$, denotes the inherited amount of fund balance given survival after the remaining wealth of deaths of pool members is distributed to those who survive, at time t.

2.2 Modification of Pooling Benefit Calculation

In Piggott, Valdez and Detzel (2005) the mortality adjustment factor (MEA) is separate to the changed expectation factor (CEA). The mortality adjustment factor allows for deviation from the expected number of deaths experienced by a GSA fund, while the changed expectation adjustment allows for a permanent shock to the underlying mortality. However these two effects are in practice difficult to differentiate between. Any pattern of mortality changes can be arbitrarily split between temporary and permanent changes in mortality. For example, if additional deaths were experienced in a GSA they could either be the random deviation away from the expectation, or it could be the result of a permanent increase of the underlying mortality rates. It is only future unknown experience that will allow us to differentiate in any meaningful way. These two adjustment factors are incorporated into a single factor in this paper.

The mortality model is used to determine annuity factors. As the underlying rate of mortality evolves and the survival probabilities, p_x, change, these will be represented by $p_{x,t}$ where t is the time that the survival probabilities apply. The actuarial annuity factor $\ddot{a}$ changes every year, as the new mortality rates are revealed and an updated current life table is generated. These annuity factors will be denoted as $\ddot{a}_{x,t}$, representing the actuarial present value of a \$1 life annuity for a life aged x, calculated at time t, using all available mortality information at that point in time. These factors do not include expected future mortality rates. They are based on the updated dynamic life table generated by the mortality model at the current time.

Mortality is modeled with a stochastic model, at any time future mortality rates are denoted by $\mu_{x,t}$ which is the mortality rate assumed to be experienced by lives aged x at time t. Consequently, the probability of a life aged x at time t surviving s years is given by

$$_{s}p_{x,t} = \exp\left[-\int_{x}^{x+s} \mu_{z,t} dz\right]$$

The actuarial present value of an ordinary lifetime annuity paid in annually in advance for a life aged x at time t is given by

$$\ddot{a}_{x,t} = \sum_{s=0}^{\infty} e^{-\delta s} {}_{s}p_{x,t}$$

The rest of the analysis in this paper is formally based on applying an updated life table at each future time point, and hence all variables will be subscripted by t to denote calculation at time t, given mortality information at time t.

The benefit payment received by an individual aged x at entry at time t, having been a member of the pool for k years, is given by

$$ {}_{x}^{k}B_{i,t}^{*} = \frac{{}_{x}^{k}\hat{F}_{i,t}^{*}}{\ddot{a}_{x+k,t}} \tag{2} $$

The calculations are performed at time t based on mortality information at time t.

For the total GSA fund, assuming multiple cohorts, with individuals of various ages at entry, having been with the fund for various periods of time, the benefit payment and fund balance are, respectively,

$$ \begin{aligned} B_t^* &= \sum_{k\geq 1}\sum_{x}\sum_{A_t} {}_{x}^{k}B_{i,t}^{*} \\ F_t^* &= \sum_{k\geq 1}\sum_{x}\sum_{A_t} {}_{x}^{k}\hat{F}_{i,t}^{*} \end{aligned} $$

where A_t is the number of fund members alive at time t for the cohort (x, k).

The individual fund and benefit amounts are derived using the following recursive formula:

$$ {}_{x}^{k}F_{i,t}^{*} = \left({}_{x}^{k-1}\hat{F}_{i,t-1}^{*} - {}_{x}^{k-1}B_{i,t-1}^{*}\right)e^{\delta} \tag{3} $$

where $\delta = \ln\left[1 + R\right]$ is the rate of investment earned over the previous period.

This relationship defines the fund balance after investment accumulation but before any inheritance from mortality credits from deaths. For the fund as a whole, $F_t^* = \hat{F}_t^*$ since the fund balance is always preserved.

The relationship between ${}_{x}^{k}F_{i,t}^{*}$ (the survived fund balanced before inheritance) and ${}_{x}^{k}\hat{F}_{i,t}^{*}$ (after inheritance) is important and reflects the mortality experience sharing.

The relationship between benefit payments in consecutive periods where there is only a single mortality experience adjustment factor is derived as follows:

$$
\begin{aligned}
F_t^* &= \sum_{k\geq 1}\sum_x\sum_{A_t}\left({}^k_x B^*_{i,t}\ddot{a}_{x+k,t}\right) \\
&= \sum_{k\geq 1}\sum_x\sum_{A_t}\left[\left({}^{k-1}_{x} B^*_{i,t-1}\cdot TEA_t\right)\cdot \ddot{a}_{x+k,t}\right] \\
&= \sum_{k\geq 1}\sum_x\sum_{A_t}\left[\left({}^{k-1}_{x} B^*_{i,t-1}\cdot TEA_t\right)\cdot \ddot{a}_{x+k,t}\cdot\frac{(\ddot{a}_{x+k-1,t-1}-1)}{p_{x+k-1,t-1}}e^{\delta}\cdot\frac{1}{\ddot{a}_{x+k,t-1}}\right] \\
&= \sum_{k\geq 1}\sum_x\sum_{A_t}\left[\left({}^{k-1}_{x} B^*_{i,t-1}\cdot TEA_t\right)\cdot \frac{\ddot{a}_{x+k,t}}{\ddot{a}_{x+k,t-1}}\cdot\frac{(\ddot{a}_{x+k-1,t-1}-1)}{p_{x+k-1,t-1}}e^{\delta}\right] \\
&= TEA_t\sum_{k\geq 1}\sum_x\sum_{A_t}\left[\frac{\ddot{a}_{x+k,t}}{\ddot{a}_{x+k,t-1}}\cdot\frac{\left({}^{k-1}_{x} B^*_{i,t-1}\ddot{a}_{x+k-1,t-1}-{}^{k-1}_{x} B^*_{i,t-1}\right)}{p_{x+k-1,t-1}}e^{\delta}\right] \\
&= TEA_t\sum_{k\geq 1}\sum_x\sum_{A_t}\left[\frac{\ddot{a}_{x+k,t}}{\ddot{a}_{x+k,t-1}}\cdot\frac{{}^k_x F^*_{i,t}}{p_{x+k-1,t-1}}\right]
\end{aligned}
$$

This then gives an adjustment factor of

$$
TEA_t = \frac{F_t^*}{\sum_{k\geq 1}\sum_x \frac{1}{p_{x+k-1,t-1}}\frac{\ddot{a}_{x+k,t}}{\ddot{a}_{x+k,t-1}}\sum_{A_t} {}^k_x F^*_{i,t}}
$$

where TEA_t is the total adjustment factor. It is important to note that TEA_t is not $MEA_t \times CEA_t$, as in the Piggott, Valdez and Detzel (2005) sharing arrangement. They have

$$
\begin{aligned}
MEA_t &= \frac{F_t^*}{\sum_{k\geq 1}\sum_x \frac{1}{p_{x+k-1,t-1}}\sum_{A_t} {}^k_x F^*_{i,t}} \\
CEA_t &= \frac{\ddot{a}_{x+k,t-1}}{\ddot{a}_{x+k,t}}
\end{aligned}
$$

which implies that CEA_t is dependent on the cohort to which it is being applied. This requires a subjective judgement as to how to translate the realised survival experience into a cohort specific permanent improvement factor CEA dependent on cohort, and a universal volatility factor MEA independent of cohort. The exact way in which this can be done is subjective. A single factor TEA_t universal across cohorts avoids this ambiguity in the sharing rules.

The approach allows mortality experience to be shared across cohorts. Sharing arrangements are always conflicted in balancing solidarity for the whole pool and actuarial fairness for individuals in the pool. By sharing systematic risk across cohorts, provided each cohort is made better off in terms of benefit payments or benefit volatility, sharing rules will be beneficial to all cohorts. Because the age of the cohorts is still used in the sharing rules at an individual level, allowance for cohort experience is still captured.

2.3 Fund value inheritance

The method for sharing mortality credits in the pool developed in Piggott, Valdez and Detzel (2005) relies on the computation and application of the two components MEA and CEA. The recursive relationship $\ddot{a}_{x+t,t} = \frac{(\ddot{a}_{x+t-1,t}-1)e^{\delta}}{p_{x+t-1,t}}$ is used. This constrains the assumptions used to compute $\ddot{a}_{x+t,t}$.

The method for the computation of an individual's share of the fund value and the subsequent benefit payment can be improved by noting that

$$\begin{aligned}
\sum_{k\geq 1}\sum_{x}\sum_{A_t} {}^{k}_{x}\hat{F}^{*}_{i,t} &= \sum_{k\geq 1}\sum_{x}\sum_{A_t} \frac{{}^{k}_{x}\hat{F}^{*}_{i,t-1}}{\ddot{a}_{x+k-1,t-1}} \cdot TEA_t \cdot \ddot{a}_{x+k,t} \\
&= \sum_{k\geq 1}\sum_{x}\sum_{A_t} \left[\frac{{}^{k}_{x}\hat{F}^{*}_{i,t-1}}{\ddot{a}_{x+k-1,t-1}} \cdot TEA_t \cdot \ddot{a}_{x+k,t} \cdot \frac{(\ddot{a}_{x+k-1,t-1}-1)}{p_{x+k-1,t-1}} e^{\delta} \cdot \frac{1}{\ddot{a}_{x+k,t-1}} \right] \\
&= \sum_{k\geq 1}\sum_{x}\sum_{A_t} \left[{}^{k}_{x}\hat{F}^{*}_{i,t-1} \left(1 - \frac{1}{\ddot{a}_{x+k-1,t-1}}\right) \cdot TEA_t \cdot \frac{\ddot{a}_{x+k,t}}{\ddot{a}_{x+k,t-1}} \cdot \frac{e^{\delta}}{p_{x+k-1,t-1}} \right] \\
&= \sum_{k\geq 1}\sum_{x}\sum_{A_t} \left[{}^{k}_{x}F^{*}_{i,t} \cdot TEA_t \cdot \frac{\ddot{a}_{x+k,t}}{\ddot{a}_{x+k,t-1}} \cdot \frac{1}{p_{x+k-1,t-1}} \right] \\
&= \sum_{k\geq 1}\sum_{x}\sum_{A_t} \frac{\frac{{}^{k}_{x}F^{*}_{i,t}}{p_{x+k-1,t-1}}}{\sum_{k\geq 1}\sum_{x}\sum_{A_t} \frac{{}^{k}_{x}F^{*}_{i,t}}{p_{x+k-1,t-1}}} \cdot F^{*}_{t}
\end{aligned}$$

This shows that

$$ {}^{k}_{x}\hat{F}^{*}_{i,t} = \frac{\frac{{}^{k}_{x}F^{*}_{i,t}}{p_{x+k-1,t-1}}}{\sum_{k\geq 1}\sum_{x}\sum_{A_t} \frac{{}^{k}_{x}F^{*}_{i,t}}{p_{x+k-1,t-1}}} \cdot F^{*}_{t} \tag{4}$$

where ${}^{k}_{x}F^{*}_{i,t}$ is derived using equation (3), and the benefit payment amount is then subsequently calculated using equation (2). This is an improved approach to computing the recursive equations that bypasses the necessity of calculating TEA_t. This method is used to derive benefit payment amounts.

This expression has a simple interpretation. The fund value inherited by an individual is simply a weighted portion of the total available fund value. The weight is determined by the combination of

- ${}^{k}_{x}F^{*}_{i,t}$, the fund value of the individual prior to inheritance; and
- $p_{x+k-1,t-1}$, the expected survival probability of the individual for the previous calendar year.

3 Mortality model

The mortality model adopted to demonstrate the impact of systematic risk on GSA benefit payments is a stochastic dynamic Gompertz-Makeham (GoMa) model similar to the Gaussian Makeham model in Schrager (2006) [16].

The model is specified as

$$\begin{aligned} \mu_{x,t} &= Y_{t1} + Y_{t2}c^x & (5)\\ dY_{t1} &= a_1 dt + \sigma_1 dW_{t1} & (6)\\ dY_{t2} &= a_2 dt + \sigma_2 dW_{t2} & (7)\\ dW_{t1}dW_{t2} &= \rho dt & (8) \end{aligned}$$

with the conditions

$$\begin{aligned} Y_{t1} &> 0\\ Y_{t2} &> 0\\ c &> 1\\ \sigma_1 &> 0\\ \sigma_2 &> 0\\ -1 \leq \rho &\leq 1 \end{aligned}$$

where W_{t1} and W_{t2} are Brownian motions, and the initial condition is

$$\mu_{x,0} = y_1 + y_2 c^x \qquad (9)$$

The model formulation allows the derivation of closed form survival probabilities and annuity factors (Schrager (2006) [16] and Milevsky (2006) [9]). The model allows for systematic improvements in mortality and captures dependence in mortality changes across ages. Dependence in the factors across ages is captured by the standard correlation coefficient.

From the model descriptions given above, the following observations can be made

$$\begin{aligned} Y_{t1} &\sim N\left(a_1 t + y_1, \sigma_1^2 t\right)\\ Y_{t2} &\sim N\left(a_2 t + y_2, \sigma_2^2 t\right)\\ cov\left[Y_{t1}, Y_{t2}\right] &= \rho\sigma_1\sigma_2 t \end{aligned}$$

So that

$$\mu_{x,t} \sim N\left(u_{x,t}, s_{x,t}^2\right)$$

where

$$\begin{aligned} E\left[\mu_{x,t}\right] = u_{x,t} &= \left(a_1 t + y_1\right) + \left(a_2 t + y_2\right)c^x\\ var\left[\mu_{x,t}\right] = s_{x,t}^2 &= \sigma_1^2 t + \sigma_2^2 t c^{2x} + 2\rho\sigma_1\sigma_2 c^x t \end{aligned}$$

and $X \sim N\left(\mu, \sigma^2\right)$ denotes a normally distributed random variable X with mean μ

and variance σ^2. That is, $E[X] = \mu$ and $var[X] = \sigma^2$.

Long-term mortality improvements arise from negative parameter values for a_1 and a_2, whereas a deterioration would be represented by positive estimates for a_1 and a_2.

The model is fitted to data from the Human Mortality Database [1] for Australian deaths and population data for ages 0 to 110 from the years 1965 to 2007. Male data for ages 60 to 99 is selected since GSAs are designed for retired individuals. Older ages are not included in fitting the model because of smaller number of deaths at these ages. Data before 1965 was omitted, due to a data collection limitations by the HMD prior to that year. This results in the data being reduced to incorporate 40 different ages across 43 years for a total of 1720 observations.

The observed mortality rate is

$$\hat{m}_{x,t} = \frac{\text{number of deaths in year } t \text{for those aged } x \text{last birthday}}{\text{average size of the population aged } x \text{in year } t} = \frac{\hat{D}_{x,t}}{\hat{E}^c_{x,t}} \tag{10}$$

where a hat,$\hat{}$, denotes an observed value and a tilde,$\tilde{}$, denotes an estimated value.

Under standard Poisson deaths assumptions, with a central population exposure of $E^c_{x,t}$ an individual aged x at time t has a death rate of $\mu_{x,t}E^c_{x,t}$. The conditional death rate $\left.\frac{D_{x,t}}{E^c_{x,t}}\right| \mu_{x,t}$ has a mean of $\mu_{x,t}$ and variance $\frac{\mu_{x,t}}{E^c_{x,t}}$. This is approximately normally distributed from the Central Limit Theorem (CLT), so that $\left.\frac{D_{x,t}}{E^c_{x,t}}\right| \mu_{x,t} \sim N\left(\mu_{x,t}, \frac{\mu_{x,t}}{E^c_{x,t}}\right)$.

To fit the model parameters to the data, the unconditional distribution of deaths, $M_{x,t} = \frac{D_{x,t}}{E^c_{x,t}}$, is required. This is determined by integrating the distribution function of $\left.\frac{D_{x,t}}{E^c_{x,t}}\right| \mu_{x,t}$ with the distribution of $\mu_{x,t}$. Although the unconditional distribution should be a normal distribution, closed form analytical results do not exist if both the mean and variance in the distribution of $\left.\frac{D_{x,t}}{E^c_{x,t}}\right| \mu_{x,t}$ are functions of $\mu_{x,t}$ and hence random.

An approximation is used to overcome this problem. Using the law of iterated expectations

$$\begin{aligned} E\left[\frac{D_{x,t}}{E^c_{x,t}}\right] &= E\left[E\left[\left.\frac{D_{x,t}}{E^c_{x,t}}\right| \mu_{x,t}\right]\right] \\ &= E[\mu_{x,t}] \\ &= u_{x,t} \end{aligned}$$

Using the law of iterated variances

$$\begin{aligned} var\left[\frac{D_{x,t}}{E^c_{x,t}}\right] &= E\left[var\left[\left.\frac{D_{x,t}}{E^c_{x,t}}\right| \mu_{x,t}\right]\right] + var\left[E\left[\left.\frac{D_{x,t}}{E^c_{x,t}}\right| \mu_{x,t}\right]\right] \\ &= E\left[\frac{\mu_{x,t}}{E^c_{x,t}}\right] + var[\mu_{x,t}] \approx E\left[\frac{\mu_{x,t}}{\hat{E}^c_{x,t}}\right] + var[\mu_{x,t}] = \frac{u_{x,t}}{\hat{E}^c_{x,t}} + s^2_{x,t} \\ &\approx \frac{\hat{m}_{x,t}}{\hat{E}^c_{x,t}} + s^2_{x,t} \end{aligned}$$

The approximation replaces the population parameters with sample estimates.

The volatility of the observed number of deaths varies with age. Even if the underlying variability of the true rate of mortality for a particular age was zero, the observed deaths data exhibits variability from the random date of death.

The expectation and variance of observed deaths is used to estimate the model parameters. Since $M_{x,t}$ is normal, with

$$\begin{aligned} E\left[M_{x,t}\right] &= u_{x,t} = (a_1 t + y_1) + (a_2 t + y_2)\, c^x \\ var\left[M_{x,t}\right] &= v_{x,t} = \frac{\hat{m}_{x,t}}{\hat{E}^c_{x,t}} + \sigma_1^2 t + \sigma_2^2 t c^{2x} + 2\rho\sigma_1\sigma_2 c^x t \end{aligned}$$

the likelihood is given by

$$\begin{aligned} L\left(\hat{m}_{x,t}\right) &= \prod_x \prod_t \frac{1}{\sqrt{2\pi\left(\frac{\hat{m}_{x,t}}{\hat{E}^c_{x,t}} + \sigma_1^2 t + \sigma_2^2 t c^{2x} + 2\rho\sigma_1\sigma_2 c^x t\right)}} \times \\ &\quad \exp\left[-\frac{1}{2}\left(\frac{\hat{m}_{x,t} - \left[(a_1 t + y_1) + (a_2 t + y_2)\, c^x\right]}{\sqrt{\frac{\hat{m}_{x,t}}{\hat{E}^c_{x,t}} + \sigma_1^2 t + \sigma_2^2 t c^{2x} + 2\rho\sigma_1\sigma_2 c^x t}}\right)^2\right] \end{aligned}$$

The log likelihood function to be minimised in estimating the parameters, using the observed 1720 data points, is

$$\begin{aligned} l\left(\hat{m}_{x,t}\right) &= -\sum_{x=60}^{99}\sum_{t=0}^{43} \ln\left[\sqrt{2\pi\left(\frac{\hat{m}_{x,t}}{\hat{E}^c_{x,t}} + \sigma_1^2 t + \sigma_2^2 t c^{2x} + 2\rho\sigma_1\sigma_2 c^x t\right)}\right] \\ &\quad -\frac{1}{2}\sum_{x=60}^{99}\sum_{t=0}^{43}\left[\left(\frac{\hat{m}_{x,t} - (a_1 t + y_1) - (a_2 t + y_2)\, c^x}{\sqrt{\frac{\hat{m}_{x,t}}{\hat{E}^c_{x,t}} + \sigma_1^2 t + \sigma_2^2 t c^{2x} + 2\rho\sigma_1\sigma_2 c^x t}}\right)^2\right] \\ &= -\sum_{x=60}^{99}\sum_{t=0}^{43} \ln\left[\sqrt{2\pi v_{x,t}}\right] - \frac{1}{2}\sum_{x=60}^{99}\sum_{t=0}^{43}\left[\left(\frac{\hat{m}_{x,t} - u_{x,t}}{\sqrt{v_{x,t}}}\right)^2\right] \end{aligned} \tag{11}$$

The fitted parameters from the data for Australian males aged 65 to 99 from 1965 to

2007 are:

$$\begin{aligned}
\tilde{y}_1 &= 0.000322448 \\
\tilde{y}_2 &= 0.000058809 \\
\tilde{c} &= 1.096559466 \\
\tilde{a}_1 &= -1.144811496 \times 10^{-10} \\
\tilde{a}_2 &= -3.832494756 \times 10^{-7} \\
\tilde{\sigma}_1^2 &= 3.639275565 \times 10^{-19} \\
\tilde{\sigma}_2^2 &= 1.145473323 \times 10^{-11} \\
\tilde{\rho} &= 0.929491793
\end{aligned}$$

where time zero is the year of 1965.

The parameters estimates show that the two factors driving mortality include improvement over time and that these improvements are strongly correlated so that systematic risk is significant.

Pearson's chi-squared test was applied to the fitted results using the test statistic

$$\chi^2 = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i}$$

where O_i is the observed data point and E_i is the theoretical data point according the assumed underlying distribution. The null hypothesis that the data comes from the assumed distribution will be rejected if $X^2 > \chi^2_{k-1-p;@\alpha\%}$ where p is the number of parameters estimated. The Pearson's chi-squared test statistic is

$$\chi^2 = \sum_{x=60}^{99} \sum_{t=0}^{43} \frac{(\widehat{m}_{x.t} - \widetilde{m}_{x,t})^2}{\widetilde{m}_{x,t}} = 6.321235$$

with $k = 1720$ and $p = 6$. Given that $6.321235 < \chi^2_{1713;@95\%} = 1810.400192$, the null hypothesis is not rejected.

3.1 Mortality model assumptions

The Gaussian assumption in the model has the potential to produce negative mortality rates which is an issue with any Gaussian model. Schrager (2006) suggests that parameter values can be estimated to reduce the probability of negative rates. For a mean-reverting model the possibility of negative mortality rates will be small. However for a non-mean-reverting model as adopted here, a modification is required to eliminate the possibility of negative mortality rates. Negative mortality rates are generated by the model when there are too many larger sized improvements occurring one after the other.

The GoMa model, generally fits older age mortality period rates well, and explicitly produces survival probabilities that are double exponential. Survival distributions are extreme value distributions allowing for analytical approximations and survival curves consistent with empirical data. Closed form analytical expressions for survival probabilities and annuity factors is an advantage of the GoMa model. The GoMa mortality model has benefits in estimation and computation and reflects the uncertainty in mortality of the data period used. The choice of the GoMa model also allows for systematic mortality improvements. Alternative mortality models with systematic mortality improvements would be expected to produce qualitatively similar results.

Using the parameters fitted to the historical death rates of Australian males from 1965 to 2007, the probability of a negative mortality rate in at least one of the future 40 years is 10.66%, based on 83910 simulations. The chance of such an event increases for a longer time horizon.

In order to modify the model for negative mortality rates these simulations are excluded and replaced by new simulations that do not have negative mortality rates to produce a more realistic model.

The GoMa model has a fixed factor c applying to all ages at any given time period. Consequently, the dependence between ages is a significant feature of the model. Mortality models that have lower age dependence will produce benefit payments that show greater advantages from pooling across cohorts, due to reduced levels of dependence. In this sense, the GoMa model will produce conservative results.

The GoMa model does not capture cohort improvements explicitly. Alternative models such as the Renshaw and Haberman (2006) model may produce differing benefits from pooling across cohorts. The GoMa model is still able to capture a portion of cohort improvements through the relative changes in the two factors Y_1 and Y_2, which are positively, but not perfectly, correlated.

3.2 Impact of Mortality Improvements and Systematic Risk

High levels of systematic risk reduce the effectiveness of pooling. Larger pools and pooling across multiple cohorts should reduce the volatility of benefit payments to people at older ages and reduce the extent to which benefit payments decline over time. Pooling provides higher benefit payments than for the case of self insurance. Regardless of the pool size or how often younger cohorts are admitted to the pool, benefit payments will decline over time due to

- The existence of systematic mortality improvements and dependence between lives; and
- The long-tailed distribution for aggregate survival distributions, as reflected by the GoMa model, where there is a downside skewed tail.

While it is important for systematic risk to be borne by the fund members, hence alleviating the need for capital, excessive systematic risk will discourage effective pooling because of declining benefit payments.

In order to resolve this feature of current pooling methods, a mechanism through which a portion of systematic risk can be shared in the pool is proposed.

The mortality model provides the random time t evolution of the factors Y_1 and Y_2. This produces a survival curve based only on past mortality information. However, what is also known is that, conditional on $\mu_{x,t}$ known at time t, these rates will improve over time as captured by the model. That is

$$E\left[\mu_{x,t+s} \,|\, \mu_{x,t}\right] = (y_{1t} + a_1 s) + (y_{2t} + a_2 s)\, c^x \tag{12}$$

which projects the mortality rates for the future s years starting at time t.

The expected future trend is allowed for using survival probabilities that take into account the expected future mortality improvements. To do this an approximation is required to allow convenient calculation of annuity values. By noting that the variance of $(\mu_{z,t+s} \,|\, \mu_{x,t})$ is very small compared to its expected value, an approximation for expected future survival probabilities is

$$E\left[p_{x,t+s} \,|\, \mu_{x,t}\right] \approx \exp\left[-\int_x^{x+1} E\left[\mu_{z,t+s} \,|\, \mu_{x,t}\right] dz\right] \tag{13}$$

The annuity value allowing for future expected mortality is then given by

$$\ddot{a}^f_{x,t} = \sum_{k=0}^{\infty} e^{-\delta k} E\left[{}_k p_{x,t}\right]$$

where

$$E\left[{}_k p_{x,t}\right] = \prod_{s=0}^{k-1} E\left[p_{x+s,t+s} \,|\, \mu_{x,t}\right]$$

and $\ddot{a}^f_{x,t}$ denotes the *updated* actuarial present value annuity factor, taking into account expected systematic mortality improvements. In the case of long-term mortality improvements, $\ddot{a}^f_{x,t} > \ddot{a}_{x,t}$.

The benefits in the GSA risk pool are then based on forecasts of the expected future mortality rates at each point in time allowing for the sharing of mortality credits from the deaths in the pool, using the known rates at t. A new projected expected survival rates life table is generated for each t, resulting in a new $\ddot{a}^f_{x,t}$ for each x that takes into account future expected mortality improvements based on the mortality model.

This approach incorporates mortality expectations dynamically without the need to measure changes in mortality against an assumed future experience. The annuity factors $\ddot{a}^f_{x,t}$ and $E\left[p_{x+s,t+s} \,|\, \mu_{x,t}\right]$ can be computed numerically or analytically depending on the mortality model used.

The incorporation of mortality expectations allows systematic mortality risk, based on the expected future mortality, to be shared among cohorts in the pool. Pool members still experience the unexpected portion of systematic mortality risk driven by mortality uncertainty.

Benefit payments are determined by equation (2), using the annuity factors to account for improving mortality. By using $\ddot{a}^f_{x,t}$, and hence increasing $\ddot{a}_{x,t}$ at all ages, benefit payments at entry will be lower but will be closer to level at future times.

4 Simulation methodology

Monte Carlo simulations are used to assess pooling effectiveness. The methodology for the simulations is:

1. Future mortality rates are forecasted by simulation of the random dependent factors Y_{1t} and Y_{2t} driving mortality changes.
   - These are bivariate normal simulations.
   - A mortality matrix x age by t time is generated.
   - Simulations with negative rates are not included inducing autocorrelation.
2. Determine survival probabilities from the simulated mortality rates.
   - Probability of survival for an individual in a particular year is determined using equation (15) and setting $s = 1$, for each year. This is then repeated every year for all fund members, for their corresponding ages x for each time t in the fund.
   - Determine the actuarial present value annuity factor using equation (16) for every year, and every member, for their corresponding ages x at time t in the fund.
3. Simulate the death and survival of fund members every year with a $Bernoulli\ (p_{x,t})$ random variable, where 1 denotes survival and 0 denotes death. Death events are doubly random, where the deaths are random, and the survival probabilities $p_{x,t}$ are also random, driven by random underlying mortality rate simulations.
4. Determine the inherited fund balance from the deaths and benefit payment for each of the survived fund members at the beginning of every year, using the algorithm in equations (2), (3) and (4).

This procedure produces one complete simulation for all the fund members. This process is repeated so that many simulations are generated, and the resulting benefit payment amounts are summarised in a distribution for differing future times (and ages).

All simulations assume

- A starting contribution value of \$100 (benefit payments as a percentage of the starting contribution are unchanged by differing starting contributions);
- A time horizon of 40 years (so the oldest age included is 105);
- All pools start at time zero, which is the year of 2007 for the mortality model;
- The mortality rates for time zero are known, this provides approximately \$8.99 in benefit payments, computed by $\frac{100}{\ddot{a}_{65,0}}$ to 65 year old males at time zero (not including expected mortality improvements);
- Initial entry age of participants is 65 unless stated otherwise;
- All fund members are males since the mortality data used for calibration of the model was Australian males; and

- An annual effective rate of interest of 5%, which equates to a constant force of interest of $\delta = 0.04879016$. In all simulations the realised interest rate return is the same as the expected rate of interest.

4.1 Mortality projections

Future mortality rates are projected by simulating the two factors Y_1 and Y_2 as bivariate normally distributed random variables. In projecting future mortality rates, the initial parameters at $t = 0$ (2007) are

$$\begin{aligned} y_{1,t=0} &= 0.00032244347614 \\ y_{2,t=0} &= 0.00004271285405 \end{aligned}$$

With each simulation a current mortality curve for $\mu_{x,t}$ is generated from

$$\mu_{x,t} = y_{1t} + y_{2t}c^x \tag{14}$$

The probability of a single individual aged x at time t surviving for s years is then

$$\begin{aligned} {}_sp_{x,t} &= \exp\left[-\int_x^{x+s} \mu_{z,t}dz\right] \\ &= \exp\left[-\int_x^{x+s} \left(y_{1t} + y_{2t}c^z\right) dz\right] \\ &= \exp\left[-y_{1t}s - \frac{y_{2t}}{\ln c}\left(c^{x+s} - c^x\right)\right] \end{aligned} \tag{15}$$

This rate is evaluated at time t, where information about the future has not yet been revealed. These survival probabilities do not take into account future mortality evolution, since this is unknown at point t in time. This closed form solution is a computational advantage in the use of a GoMa mortality model.

Under the GoMa survival probabilities given by equation (15), the actuarial present value of an ordinary annuity for a single individual aged x at time t is

$$\begin{aligned} \ddot{a}_{x,t} &= \sum_{s=0}^{\infty} e^{-\delta s}{}_sp_{x,t} \\ &= \sum_{s=0}^{\infty} \exp\left[-\left(y_{1t} + \delta\right)s - \frac{y_{2t}}{\ln c}\left(c^{x+s} - c^x\right)\right] \end{aligned} \tag{16}$$

The annuity factor allowing for expected future mortality improvements is obtained from

$$
\begin{aligned}
E\left[\, p_{x,t+s} \middle|\, \mu_{x,t}\right] &\approx \exp\left[-\int_x^{x+1} E\left[\,\mu_{z,t+s} \middle|\, \mu_{x,t}\right] dz\right] \\
&= \exp\left[-\int_x^{x+1} \left[(y_{1t}+a_1 s)+(y_{2t}+a_2 s)\, c^z\right] dz\right] \\
&= \exp\left[-(y_{1t}+a_1 s)-\frac{(y_{2t}+a_2 s)}{\ln c}\left(c^{x+1}-c^x\right)\right] \qquad (17)
\end{aligned}
$$

These computations are made for every year for use in equation (2) to arrive at individual benefit payments for both the case where only current mortality information is used in the annuity factors and the case where expected future mortality is included.

5 Results and Discussion

The methodology is used to assess the impact of pool size on future benefits by deriving the distributions of these benefits at future times (and ages). The cases considered include the group self annuitisation scheme based on Piggott, Valdez and Detzel (2005) but also improvements proposed to enhance the effectiveness of pooling systematic risk. The impact of admitting new entrants to the pool is also assessed in terms of the pattern of future pool payments and the impact on volatility of payments at different ages.

5.1 Presentation of results

Benefit payments to individuals are illustrated based on the 5$^{\text{th}}$ percentile, median, and the 95$^{\text{th}}$ percentile of the simulated outcomes at each age in the future. These distribution measures highlight the variability of future payments and the risk of payments in a very clear way. These measures show the benefit payment level that pool payments will fall below (5$^{\text{th}}$ percentile) 5% of the time, and increase above (95$^{\text{th}}$ percentile) 5% of the time. The median is the payment value with an equal likelihood of being either above or below. These measures are similar to those recommended in the Review into the Governance, Efficiency, Structure and Operation of Australia's Superannuation System completed under Cooper (2010) [3].

Simulation percentiles are estimates and the confidence level for these quantiles is a function of the number of simulations run. The 95% confidence intervals (CIs) around the numerical percentile measures are computed using the Binomial Method developed by Hardy (2006) [7], where for N simulations, the 95% CIs for the α^{th} percentile are given by the

$$
\left(\alpha N \pm \left\lceil \Phi^{-1}(0.975)\sqrt{(1-\alpha)\,\alpha N}\right\rceil\right)^{\text{th}}
$$

ordered simulation.

This paper has mostly used 5000 simulations for scenarios, unless specified otherwise, since the confidence intervals demonstrate this is sufficient to produce reliable simulation estimates.

All graphical results are presented in a comparative format with identical axes. 95% CIs have been plotted in grey. All benefit payments are for the surviving pool individuals and conditional on a pool member surviving.

5.2 Self Insurance: Single cohort, single individual

For an individual who self insures longevity risk the benefit payment distribution under existing GSA rules is displayed in the top left of Figure 1 for a single individual aged 65 entering the pool at $t = 0$. For self insurance there are no mortality credits from other members in the pool and the mortality improvements result in declining payments throughout the individuals life. The original annuity factor used for the GSA assumes an average life time based on a large pool of independent individuals. As individuals survive to older ages the chart shows the clear impact of systematic longevity risk. Too much is consumed too early since survival can occur past the average life time reflected in the annuity factor at time $t = 0$. There is not much variability since there is only one member and experience is adjusted for in the annuity factors.

5.3 Increasing pool size and pooling benefits

Figure 1 also shows the distribution of payments as the pool size is increased to 10, 1000, and 10000. For smaller pool sizes the variation in the benefit payments at future ages is very wide and the upside for higher benefit payments increases early on. This is because for those scenarios where mortality deteriorates with a small group in the pool, the survivors benefit from larger mortality credits early on. However as the pool size diminishes through time the mortality credits reduce. This is referred to as a "lucky hump" or "tontine effect" for small pools and reflects the tontine benefit for survivors in small pools.

As the pool size increases this tontine effect is much reduced and occurs at later ages since it occurs only for small survivor pool sizes. Benefits are higher for longer than in the self insurance case since the effect of idiosyncratic risk is pooled. There are higher probabilities of paying benefits for the length of time assumed in the annuity factor used to determine payments. Eventually the effect of the systematic longevity risk dominates and benefits drop for the older aged survivors in the pool. Even with 1000 initial members in the pool, which would normally be sufficient to benefit from risk pooling with only idiosyncratic risk, the upside tontine effect remains significant at the older ages. With 10,000 initial members in the pool the effect is much reduced, but the wide range of benefit outcomes at the older ages remains along with the reduction in average benefits as the systematic risk effect dominates.

With a single cohort in the pool, even with significant sized pools, there are a number of factors that clearly undermine group self annuitisation. One is the decline in benefits at the older ages reflecting the impact of systematic improvement in mortality. The other is the widening range, and increasing volatility, of future benefits in the pool.

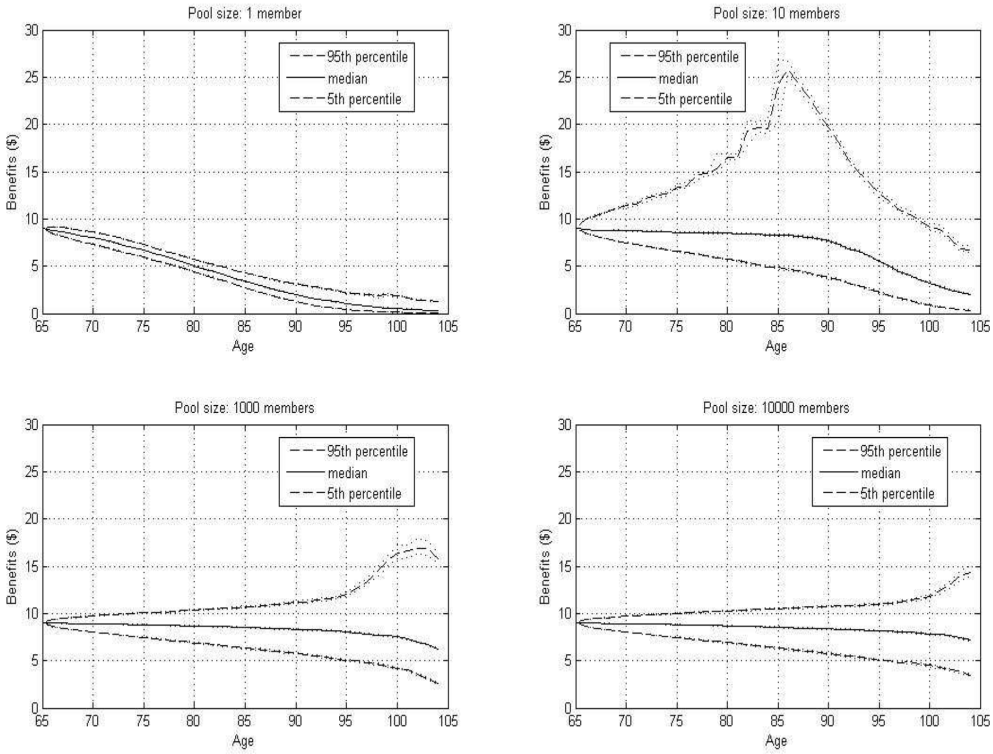

Figure 1: Comparison of benefit distributions for increasing pool sizes

Pool size	5th percentile	Median	95th percentile
1	(1.24,1.27)	(2.00,2.03)	(3.09,3.14)
10	(3.69,3.79)	(7.50,7.70)	(19.58,20.19)
1000	(5.49,5.77)	(8.18,8.33)	(10.81,11.15)
10000	(5.50,5.86)	(8.27,8.47)	(10.62,10.83)

Table 1: CIs for percentile comparisons with varying the pool sizes for age 90

The extent of the downside is of particular concern since the 5% worse cases at the older ages are a reduction of around 50% in benefit payments.

To provide a numerical comparison of these effects at the older ages, Table 1 shows the 95% CIs for the three percentile estimates for the amount of benefits received by a single individual at $t = 25$, when he is aged 90. This highlights the benefits of risk pooling and group self annuitisation in terms of improvements over self insurance but also the limitations.

To summarise the results where the annuity factor does not include expected future mortality improvements:

- Increasing the pool size will increase the median and the 5th percentile of benefit payments for GSA members. It will also decrease the absolute volatility of the payments.
- As the number of members in the pool increases, it becomes more difficult for *further* pooling benefits to be realised. Most of the significant pooling benefits are

realised when the pool size reaches 1000.

- The volatility of benefit payments at old ages is far greater than those at younger ages reflecting the impact of systematic longevity risk and reducing pool size.
- The median and 5^{th} percentile value of payments declines over time regardless of the number of people in the pool. This arises because of the expected mortality improvement in mortality and demonstrates that the current approaches to group self annuitisation benefit determination result in payments being too high early on. Smaller pool sizes at the older ages result in more significant reductions.
- The 95^{th} percentile increases first before declining, due to the tontine effect in small pools. There is always a small chance that a sizeable proportion of people will die early and this increases for small pool sizes. For the members who are lucky enough to survive, they benefit from those who died, leaving them with substantially larger benefit payments. Eventually, the benefit payments decline regardless of the tontine effect.
- As the pool size increases, the tontine effect is deferred to later, as the actual deaths are closer to the deaths expected.
- The absolute volatility of payments is large even allowing for increasing pool size.

5.4 Pooling with new cohorts

Piggott, Valdez and Detzel (2005) proposed pooling multiple cohorts in the same pool. This has to be carefully considered since it will be favourable for the 65 year old cohorts to pool with older cohorts entering the pool at the same time unless pooled benefit payments are determined separately for cohorts of different age at entry to the pool. The benefits of risk pooling would otherwise be diminished and older cohorts would be less willing to participate.

Effective risk pooling arrangements with intergenerational solidarity are possible if members of older cohorts share experience with younger cohorts, where all the cohorts enter the pool at the same age. New cohorts of 65 year olds would continuously join existing pools in subsequent years. The benefits are that improvements, or adverse mortality changes, at the older ages would be shared across all ages resulting in a more level benefit payment across ages of the members in the pool. Also the volatility at the higher ages would be shared with the lower volatility at the younger ages resulting in more chance of adequate benefits at the older ages.

In order to quantify the benefits of such a pooling arrangement with new entrants at the same age, a scenario with new members entering every year at age 65 is considered, and this is compared with scenarios where new members aged 65 enter every five years. For each of the scenarios, a thousand members aged 65 enter at time 0 and are pooled with the survivors of the cohorts of the thousand 65 year olds who entered at earlier times.

The results of these dynamic pooling arrangements are shown in Figure 2. Table 2 shows the 95% CIs for the three percentile estimates for the amount of benefits received

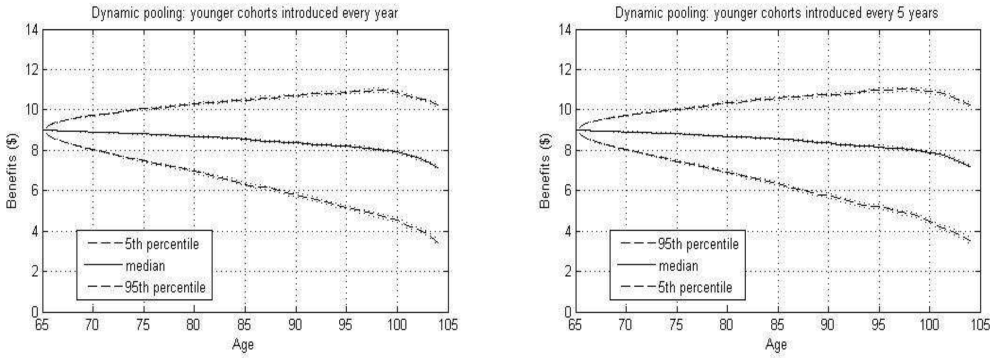

Figure 2: Benefit distributions dynamic pooling every year (one thousand new 65 year old entering every year and every five years)

Dynamic pooling frequency	5th percentile	Median	95th percentile
every year	(5.47,5.78)	(8.26,8.47)	(11.11,11.39)
every 5 years	(5.66,5.88)	(8.31,8.41)	(10.67,10.84)
never	(5.49,5.77)	(8.18,8.33)	(10.81,11.15)

Table 2: Confidence intervals for percentile comparisons for age 90 with scenarios where dynamic pooling occurs at differing frequencies

by a single individual at $t = 25$, when he is aged 90, under the dynamic pooling arrangements, with dynamic pooling occurring every year and every five years.

Dynamic pooling also reduces the volatility of benefit payments of older members significantly. This occurs because the idiosyncratic volatility of deaths experienced in the pool will diminish when younger cohorts are allowed to enter. Annuity factors for pooling and benefit payments allow for the age of members in the pool. The size of the pool is more stable over time and the higher volatility of older ages is shared with lower volatility at younger ages in the pool. Dynamic pooling every five years does not produce significantly different results to pooling new cohorts every year.

Dynamic pooling where individuals from other cohorts join existing funds on a frequent basis will reduce the high volatility of benefit payments experienced by people surviving to old ages. These benefits will be available to all those entering the pool if they survive to old ages and new entrants are admitted to the pool at the same entry age on a regular basis.

5.5 Dynamic pooling with future expected improvements

The pooling results so far use the annuity factor based on the simulated mortality experience as it evolves without explicit allowance for future expected improvement. By using an updated $\ddot{a}^f$ factor future benefit payments will be less susceptible to reductions from systematic mortality improvements and the equity of pooling arrangements between different cohorts will be improved.

To demonstrate the impact of this assumption, a group of a thousand 65 year olds are

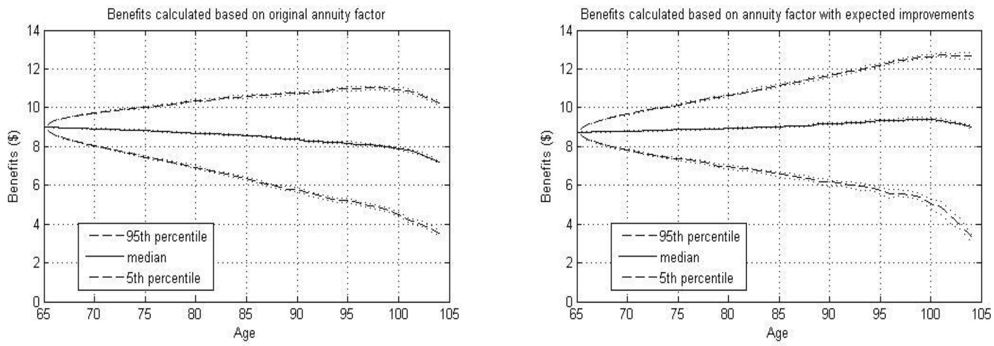

Figure 3: Benefit payments calculated using annuity factors with expected improvements

Age	5$^{\text{th}}$percentile	Median	95$^{\text{th}}$ percentile
70	(7.78134,7.84896)	(8.76987,8.81321)	(9.64436,9.70525)
80	(6.84956,7.06950)	(8.89730,8.97150)	(10.57074,10.67242)
90	(5.94495,6.28521)	(9.10051,9.24033)	(11.57233,11.73409)

Table 3: Benefit distributions at times $t = 5,\ 15,\ 25$, for a thousand 65 year olds at entry, with dynamic pooling where new members enter every 5 years, and with the updated $\ddot{a}^{\odot}$ factor with systematic allowance used to calculate benefit payments

assumed to enter the pool at $t = 0$, with subsequent groups of a thousand 65 year olds entering the same pool every 5 years. Benefit payments are determined based on the updated $\ddot{a}^{f}$ factor taking into account expected mortality improvements.

The benefit payment distribution is displayed in Figure 3 and Table 3. Table 3 presents the benefit payment received by a pool member when he is 70 years of age at $t = 5$, 80 years of age at $t = 15$ and 90 years of age at $t = 25$.

By allowing for systematic mortality improvements in the determination of the actuarial present value annuity factor, the amount of benefit payments received by individuals shows much reduced declines over time.

The updating of the annuity factor for determining benefit payments of surviving members in the pool to include expected improvements has been shown to largely offset the impact of systematic mortality improvements on median payments. The systematic nature of mortality changes means that absolute volatility will increase through time regardless of the risk sharing rules in the pool. Mortality uncertainty increases over time and impacts older ages more significantly than younger ages.

At the older ages there is always a declining trend in benefit payments affecting the median and the 5$^{\text{th}}$ percentile of outcomes. The survival distribution for the Go-Ma model is an extreme value distribution and with systematic mortality improvement the pooled distribution has extreme value properties so that the distribution has a significant skew to the downside at the older ages.

These issues call for other solutions. Since these solutions are not related to the GSA fund structure directly, they are not considered here. Reinsurance is a possible ap-

Age	5^{th}percentile	Median	95^{th} percentile
70	(59215.98,59730.61)	(66738.72,67068.57)	(73393.57,73856.99)
80	(52125.14,53798.93)	(67708.46,68273.09)	(80443.35,81217.13)
90	(45241.09,47830.43)	(69254.85,70318.9)	(88065.41,89296.42)

Table 4: Benefit distributions for a thousand 65 year olds at entry, with dynamic pooling where new members enter every 5 years, and with the updated $\ddot{a}^f$ factor with systematic allowance used to calculate benefit payments, and where the initial contribution is $761,000.

proach where younger members would reduce benefit payments earlier on to purchase a guaranteed narrower band for future payments from an external reinsurer.

5.6 Illustration of results

To illustrate the absolute level of payments consider as an example a male individual, earning on average $70,000 per year in salaries from age 25 with an average investment return rate over 40 years of 5%. This male individual will accumulate to $761,038 ($70000 \times 0.09 \times \ddot{s}_{\overline{40|}@5\%}$) in retirement savings by the age of 65 under a 9% contribution SG scheme.

In a pool with a thousand 65 year olds entering the fund every 5 years, and where the annuity factors take into account expected mortality improvements, the corresponding benefit payments based on Figure 3 and Table 3 for an initial contribution to the GSA of $761,038 are shown in Table 4. Tables with other initial contribution amounts can be readily constructed.

This male will receive around $68,000 annually from a GSA fund on the balance of probabilities (reflected by the median), should he choose to participate in such a fund with the entire amount of his retirement savings.

In such a GSA fund, there will be a 5% probability that he will receive less than around $45,000 annually in his retirement, an amount that would still be adequate as a retirement income. GSAs, implemented as discussed in this paper, will provide a sustainable and feasible retirement product for Australian retirees, even after allowing for adverse outcomes.

Substantial pooling benefits are gained when the pool size reaches around a thousand. The Australian Bureau of Statistics reported that there were 97,708 Australian males turning 65 in 2009. A hypothetical major retail GSA fund with a market share of 10% of Australian males at age 65, would have approximately 10,000 members. A thousand members would represent a relatively small fund. The major issue for such small funds is the expense of managing the fund. However the methodology for managing a GSA is efficient to implement and smaller funds may be viable, especially if the technology is available in a standard adminstration system.

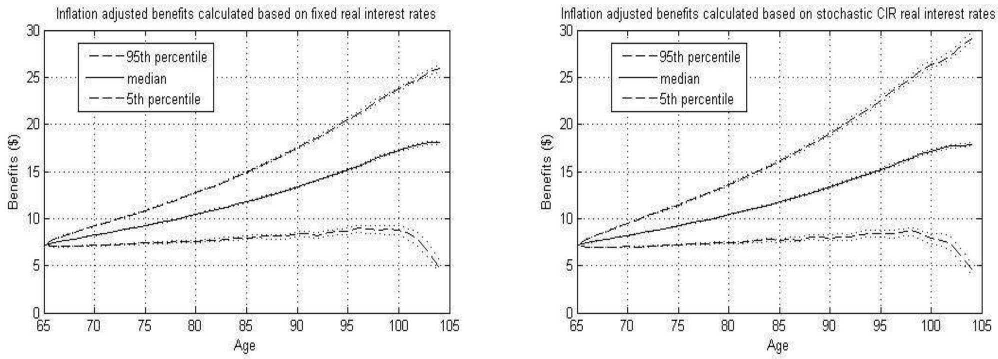

Figure 4: Inflation adjusted benefit payments based on historical real interest rates

5.7 Indexed benefit payments

Retirees need to ensure benefit payments are inflation indexed, such that their income does not diminish in purchasing power over time. The simulations with 5 year dynamic pooling and a pool size of a thousand is considered with real interest rates (inflation indexed benefit payments and nominal interest rates). The real rate assumed was the long run historical average real interest rate for Australia from 1991, or 2.72% per annum. Uncertainty in the real rate of interest is assumed based on a Cox-Ingersoll-Ross (CIR) stochastic real interest rate model, with a volatility factor of 0.0709, consistent with historical observations.

The results are shown in Figure 4.

In real terms the adverse payment scenarios are roughly constant when increased by inflation. By including real interest rate uncertainty, the range for the upper and lower benefit amounts widens. Systematic longevity risk remains the driving factor for volatility. Investments are effectively assumed to be made in real return instruments with low volatility. If an inflation indexed annuity bond market was available for investment then it would be possible to reduce the systematic mortality risk through the investment of the pool funds. The interaction between investments and mortality risk has not been included in the analysis in this paper partly because such a government bond is not available currently in Australia. An analysis of the hedging effect of such bonds on annuity payments is provided in Ngai and Sherris (2010) [12].

6 Conclusions

The paper extends and generalizes the pooling arrangements for GSAs as proposed by Piggott, Valdez and Detzel (2005). The extent to which benefit payments decline over time and the extent to which the volatility of benefit payments increases over time have been mitigated through innovations in the risk pooling methodology. Under currently proposed GSA pooling arrangements, benefit payments are expected to decrease, and the volatility of payments is expected to increase, over time.

This paper has provided enhancements to understanding the implementation of group self annuitisation schemes and shown how the impact of systematic longevity risk can undermine their effectiveness. At the same time, strategies for pooling in the schemes with the introduction of new cohorts at the same age through time are shown to improve the performance of the risk sharing in the pools.

Increasing the pool size was shown to increase the benefit payments for GSA members and to decrease the absolute volatility of the payments. Dynamic pooling, where individuals of the same age join existing funds on a frequent basis was shown to reduce the high volatility of benefit payments experienced by pool members surviving to old ages and to limit the decline in benefits for older ages. Using annuity factors that take into account future expected mortality trends also ensures the amount of benefit payments received by pool members will remain relatively stable over time.

A multi-factor stochastic version of the Gompertz-Makeham (GoMa) was used to project mortality rates. The model was calibrated to Australian male mortality. The model included systematic and idiosyncratic mortality risk and allows analytical computations to be used to improve efficiency of the determination of pool benefit payments. Simulation was used to demonstrate the performance of improved risk pooling arrangements. The results hold for more general mortality models.

There are many aspects of longevity risk pooling that this paper did not aim to address. A viable GSA pooling scheme must formally allow for heterogeneity for individuals of the same age. An example is pooling males with females. Sharing arrangements will have to recognise different longevity prospects for females and males. This is readily incorporated in the framework presented here.

7 Acknowledgement

The authors acknowledge the support of ARC Linkage Grant Project LP0883398 Managing Risk with Insurance and Superannuation as Individuals Age with industry partners PwC and APRA. Comments on an earlier version from Bruce Gregor, Katja Hanewald and Ralph Stevens are gratefully acknowledged.

References

[1] Human mortality database. http://www.mortality.org, May 2010.

[2] A. B. Abel. Capital accumulation and uncertain lifetimes with adverse selection. *Econometrica*, 54(5):1079–1097, 1986.

[3] J. Cooper. Review into the Governance, Efficiency, Structure and Operation of Australia's Superannuation System. *Australian Government*, 2010.

[4] T. Davidoff, J. R. Brown, and P. A. Diamond. Annuities and individual welfare. *The American Economic Review*, 95(5):1573–1590, 2005.

[5] J.R. Evans and M. Sherris. Longevity risk management and the development of a life annuity market in australia. *SSRN Working Paper. (See also Longevity*

Management Issues for Australia's Future Tax System, Commissioned Paper for Australia's Future Tax System Review Panel, 2009), 2010.

[6] B. M. Friedman and M. J. Warshawsky. The cost of annuities: Implications for saving behavior and bequests. *The Quarterly Journal of Economics*, 105(1):135–154, 1990.

[7] M. Hardy. An introduction to risk measures for actuarial applications. *SOA Syllabus Study Note*, 2006.

[8] M. D. Hurd. Mortality risk and bequests. *Econometrica*, 57(4):779–813, 1989.

[9] M. A. Milevsky. *The calculus of retirement income: financial models for pension annuities and life insurance*. Cambridge University Press, 2006.

[10] O. S. Mitchell. Developments in decumulation: The role of annuity products in financing retirement. *NBER Working paper No. 8567*, 2001.

[11] O. S. Mitchell, J. M. Poterba, M. J. Warshawsky, and J. R. Brown. New evidence on the money's worth of individual annuities. *The American Economic Review*, 89(5):1299–1318, 1999.

[12] A. Ngai and M. Sherris. Longevity risk management for life and variable annuities. effectiveness of static hedging using longevity bonds and derivatives. *SSRN Working Paper*, 2010.

[13] J. Piggott, E. A. Valdez, and B. Detzel. The simple analytics of a pooled annuity fund. *The Journal of Risk and Insurance*, 72(3):497–520, 2005.

[14] R. Rocha and D. Vittas. Designing the payout phase of pension systems : Policy issues, constraints and options. Technical report, World Bank Policy Research Working Paper 5289, 2010.

[15] M. J. Sabin. Fair tontine annuity. *SSRN Working Paper*, 2010.

[16] D. F. Schrager. Affine stochastic mortality. *Insurance: Mathematics and Economics*, 38(1):81–97, 2006.

[17] T. Sinha. A model of annuities with adverse selection. *Scandanavian Actuarial Journal*, pages 161–171, 1989.

[18] M. Z. Stamos. Optimal consumption and portfolio choice for pooled annuity funds. *Insurance: Mathematics and Economics*, 43(1):56–68, 2008.

[19] E. A. Valdez, J. Piggott, and L. Wang. Demand and adverse selection in a pooled annuity fund. *Insurance: Mathematics and Economics*, 39(2):251–266, 2006.

[20] D. Vittas, H. Rudolph, and J. Pollner. Designing the payout phase of funded pension pillars in central and eastern european countries. Technical report, World Bank Policy Research Working Paper 5276, 2010.

[21] M. E. Yaari. Uncertain lifetime, life insurance, and the theory of the consumer. *The Review of Economic Studies*, 32(2):137–150, 1965.