Weather Derivative Pricing and Risk Management Applications

Jon Tindall
Overview

1. Introduction
2. Pricing Principles
3. Temperature Derivatives
4. Rainfall Derivatives
1. Introduction
Introduction

First formal recorded transaction in 1996 – Enron and Energy-Koch.

- HDD swap – Milwaukee, winter 1997
- De-regulation of energy industries – mainly in US and Europe.
- Initially used as a hedge against variability in electricity supply.

US Department of Commerce estimates that weather adversely affects:

- 70% of all US companies;
- 22% of total GDP.
Weather Markets

➢ Mature Over-the-Counter (OTC) market:
  • Existed since early 1990’s.
  • Specifically ‘tailored’ products.
  • Large European banks and insurance brokers.

➢ Chicago Mercantile Exchange (CME):
  • operates electronic exchange for weather derivatives.
  • futures and option contracts over US and Canadian cities.
  • 55% of total global turnover in 2005.

➢ L.I.F.F.E – Closed in 2004
  • series of contracts based on daily average temperatures in London, Paris and Berlin.
Market Size

Market size 2001 - 2005

- Stagnant after Enron collapse.
- 2005 shows strong growth may be returning.

* Source: PwC 2005 Market Survey
Contracts

Contract Types:

- Futures - CME, OTC.
- Options – Majority of transactions to date.
- Swaps - increasing in popularity.

Underlying Variables:

- Temperature
- Rainfall
- Wind Speed
- Snow Fall
- Barometric Pressure
Contract Types

* Source: PwC 2005 Market Survey

- Large increases in rainfall contracts.
- CDD now equal with HDD contracts.
Temperature Derivatives

- Average daily temperature
  \[ T_i = \frac{T_{\text{max}} + T_{\text{min}}}{2} \]

- The most popular derivative contracts are over Heating Degree Days (HDD) and Cooling Degree Days (CDD).

  \[
  HDD = \sum_{\text{month}} \max\{ 0, (\bar{T} - T_i) \}
  \]

  \[
  CDD = \sum_{\text{month}} \max\{ 0, (T_i - \bar{T}) \}
  \]

- Where the reference level, T, is usually 18°.
- Heating is generally required below the reference temperature and cooling above it.
- Cumulative number of degrees the average temperature was below the reference level
Rainfall Derivatives

Much less common than temperature-based derivatives.

- Market was born out of temperature exposure.
- ‘Discreetness’ of Rainfall
  - Basis risk - greatest barrier to expansion.
  - Modelling difficulties.
- Lack of counter-parties – only water supply companies as a possible partner.
Risk Management Applications

Energy / Utility Companies

- Sales are highly correlated with temperatures.
- Definition of HDD and CDD contracts is based on an energy companies exposure.
  - Heating/cooling reference level \((18^\circ)\)
  - Cumulative based underlying variable.

- Enron
  - Oil and gas pipeline manager.
  - Used weather derivatives to reduce exposure to weather.
  - Soon became a ‘market-maker’ on CME and others.
Hedging Temperature

![Diagram showing the relationship between monthly gas delivery and temperature for Illinois residential users from January 1989 to November 2002. The graph includes a fitted line with an R² value of 0.9416.](image)
Risk Management Applications

Construction

- Temperature:
  - Concrete curing (setting) is temperature dependent.
  - Productivity reduces at unusually high and low temperatures.
  - ‘Stop work’ laws.

- Rainfall:
  - Precipitation delays can often represent 10% of contract.
  - Subsidence.

- Other exposures:
  - Snow fall.
  - Wind speed – cranes and other heavy equipment.
Weather Derivatives vs. Insurance

Some key differences:

- **Identifiable Loss**: There is no need to prove that a loss has occurred. Reduces costs – claims assessors, lawyers etc.

- **Moral Risk**: Nearly entirely removed – referenced to a transparent index

- **Minimal Underwriting**: Only counter-party risk requires investigation.

- **Immediate Payout**: Known magnitude.

- **Basis risk**
2. Pricing Principles
Pricing

Traditional Black-Scholes assumptions:

- A traded underlying asset that can be used to create a hedge, i.e. sold short.
- Log-normal distribution.

Other methods must be found for the pricing of these contracts:

- Alternative BS framework.
- Martingale approach.
- Numerical simulation.
Mean Reversion

- Weather variables do not rise or fall without bound
- Mean reversion strength depends on several factors – most significantly latitude.

Mean-reversion component:
\[
\frac{dX_t}{dt} = -\gamma (X_t - \bar{X})
\]

Ornstein-Uhlenbeck (OU) process:
\[
dX_t = \gamma (\bar{X} - X_t) dt + \sigma dW_t
\]

Modified OU process:
\[
dX_t = \left[ \gamma (\bar{X} - X_t) + \frac{dX_t}{dt} \right] dt + \sigma dW_t
\]
Alternative Black-Scholes

- Futures Price:
  \[ Y_t = X_t e^{r(T-t)} \]

- Process s.d.e:
  \[ dY_t = y[(\mu - r)dt + \sigma dW_t] \]

- Modified Black-Scholes p.d.e:
  \[ \frac{dV_t}{dt} = rV - \frac{1}{2} \sigma^2 y^2 \frac{d^2V}{dy^2} \]

- Solution:
  \[ V(y,t) = BS(ye^{-r\tau}, t, r, \sigma) \]
  \[ = e^{-r\tau} \cdot BS(y, t, 0, \sigma) \]
Numerical Methods

‘Burn’ Analysis:
- No assumptions needed re: the process dynamics;
- No parameters to be estimated;
- Agreement on price.

Monte Carlo Simulations:

\[
\mathbf{E} \left[ f \left( X_i \right) \right] = \frac{1}{N} \sum_{i=1}^{N} f \left( \bar{X} \left( t, \psi_i \right) \right)
\]

- Model dependant;
- Data intensive.
3. Temperature Modelling and Derivative Pricing
Data

Australian Bureau of Meteorology (BOM)

- Sydney Airport. (Jan 1940 – Dec 2005)
- Observatory Hill. (Jan 1940 – Dec 2005)
- Prospect Dam. (Jan 1965 – Dec 2005)

Missing data:

- Temperature – Backup stations.
- Rainfall – much more difficult. No accurate measure due to discreetness of rainfall.
Temperature Distributions

- Bi-modal Distribution
Modelling Temperature

Steps:

• De-trend data;
• Choose functional form for seasonal fluctuations;
• Estimate the parameters, including mean-reversion;
• Simulate the process;
• Analyse residuals.
Long-term Trends

All temperature data sets revealed a significant positive slope

\[ T_{\text{Long}} = a + b.t \]

\[ T_{\text{long}} = a + b.t + c.t^2 \]

- Time series over 70 years should de-trend with a quadratic function.
- Natural geological based heating + human induced global warming
Seasonal Trends

Fourier series to model seasonal component:

\[ T_{Seasonal} = \varepsilon_0 + \sum_i \alpha_i \cdot \sin(\gamma + \phi) + \sum_i \beta_i \cdot \cos(\lambda t + \theta) \]

- A first order series is sufficient to capture seasonal pattern.

Combining this with the linear trend we obtain:

\[ \bar{T} = a + b.t + \alpha \cdot \sin(\gamma + \phi) + \beta \cdot \cos(\lambda t + \theta) \]
Model Fit

Actual vs Expected – Sydney Airport (10 years)
Temperature Volatility

Daily Temperature Volatility – Sydney Airport (10 years)

- Degree-4 polynomial fitted to volatility distribution.
Parameter estimation

- Parameter estimation – least squares

<table>
<thead>
<tr>
<th></th>
<th>Syd. Airport</th>
<th>Observatory</th>
<th>Prospect</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>16.925</td>
<td>17.434</td>
<td>17.26</td>
</tr>
<tr>
<td>b</td>
<td>6.30*10^{-5}</td>
<td>5.16*10^{-5}</td>
<td>4.91*10^{-5}</td>
</tr>
<tr>
<td>α</td>
<td>5.14</td>
<td>4.91</td>
<td>5.194</td>
</tr>
<tr>
<td>β</td>
<td>0.69</td>
<td>-0.20</td>
<td>0.986</td>
</tr>
<tr>
<td>φ</td>
<td>1.097</td>
<td>1.25</td>
<td>1.100</td>
</tr>
<tr>
<td>θ</td>
<td>0.97</td>
<td>1.10</td>
<td>0.675</td>
</tr>
</tbody>
</table>

- Residuals – Sydney Airport.

![Residual Distribution](image)
Pricing Example

- CDD option - January

<table>
<thead>
<tr>
<th>Period</th>
<th>January</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measure</td>
<td>Cumulative CDD</td>
</tr>
<tr>
<td>Exercise Prices</td>
<td>170 / 180 / 190 / 200 CDD's</td>
</tr>
<tr>
<td>Tick</td>
<td>$100,000 /CDD</td>
</tr>
<tr>
<td>Location</td>
<td>Sydney Airport (Kingsford Smith)</td>
</tr>
</tbody>
</table>

- Pricing via:
  1. Normal approximation.
  2. ‘Burn’ analysis – 66 years of data.
  3. Monte Carlo simulations
Monte Carlo

- Stochastic form:

\[ T_t = T_0 + (T_0 - T_o) . e^{-\gamma \Delta t} + \int_{s}^{t} e^{-\gamma \Delta \tau} \cdot \sigma_{\tau} dW_{\tau} \]

- Euler approximation - discrete

\[ T_{t+1} - T_t = \gamma (T - T_y) . + \frac{dT_t}{dt} + \sigma Z \]
Pricing Example

<table>
<thead>
<tr>
<th>Sydney Airport</th>
<th>Exercise (CDD)</th>
</tr>
</thead>
<tbody>
<tr>
<td>December</td>
<td>170 180 190 200</td>
</tr>
<tr>
<td>Method</td>
<td>'Burn' Analysis</td>
</tr>
<tr>
<td></td>
<td>$473,306</td>
</tr>
<tr>
<td></td>
<td>$268,763</td>
</tr>
<tr>
<td></td>
<td>$137,865</td>
</tr>
<tr>
<td></td>
<td>$59,582</td>
</tr>
</tbody>
</table>

Pricing Comparison - January

- 'Burn' Analysis
- Monte Carlo Simulation
- Normal Approximation
4. Rainfall Modelling
Rainfall Correlations

- Annual

- Monthly
Modelling Rainfall

Compound Model – size & frequency.

- Frequency: Markov Chain.
- Size: Gamma distribution (4 segments).

Transition probabilities:

<table>
<thead>
<tr>
<th></th>
<th>Rain</th>
<th>No rain</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rain</td>
<td>0.55</td>
<td>0.45</td>
</tr>
<tr>
<td>No rain</td>
<td>0.28</td>
<td>0.72</td>
</tr>
</tbody>
</table>
Errors are greatest in winter - i.e. errors not proportional.

Clearly defined seasonal patterns.
Magnitude

- 4 segments conditional on t-1 and t+1.

<table>
<thead>
<tr>
<th>Rain t-1</th>
<th>Rain t-1</th>
</tr>
</thead>
<tbody>
<tr>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td><strong>Avg</strong></td>
<td><strong>StdDev</strong></td>
</tr>
<tr>
<td>No</td>
<td>3.74</td>
</tr>
<tr>
<td></td>
<td>6.21</td>
</tr>
<tr>
<td>Yes</td>
<td>6.42</td>
</tr>
<tr>
<td></td>
<td>11.22</td>
</tr>
</tbody>
</table>

- Fit 4 Gamma distributions

<table>
<thead>
<tr>
<th>Gamma distribution parameter values</th>
</tr>
</thead>
<tbody>
<tr>
<td>NRN</td>
</tr>
<tr>
<td>-----</td>
</tr>
<tr>
<td>alpha</td>
</tr>
<tr>
<td>beta</td>
</tr>
</tbody>
</table>
Magnitude

Actual vs Expected Magnitude – Sydney Airport

- Close Gaussian fit.
- Clearly defined seasonal patterns.
Simulation

Actual vs Expected Magnitude – Sydney Airport

Feb.
Where to from here?

- New Markets:
  - Australian market practically non-existent – agricultural based economy.
  - Must promote to seek out suitable counter-parties.
  - Improve product design – reduce basis risk.
  - Centralised data recording methodologies – Europe in particular.

- New Interest:
  - Hedge funds – attracted to immature market.
  - Diversification tool – minimal correlation to debt and equity markets.
  - Weather-based indexed insurance contracts.