Prediction Uncertainty in the Chain-Ladder Reserving Method

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joint work with

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Outline

- Introduction to claims reserving
- Chain-ladder method
- Claims development result
- Examples

General insurance company's balance sheet

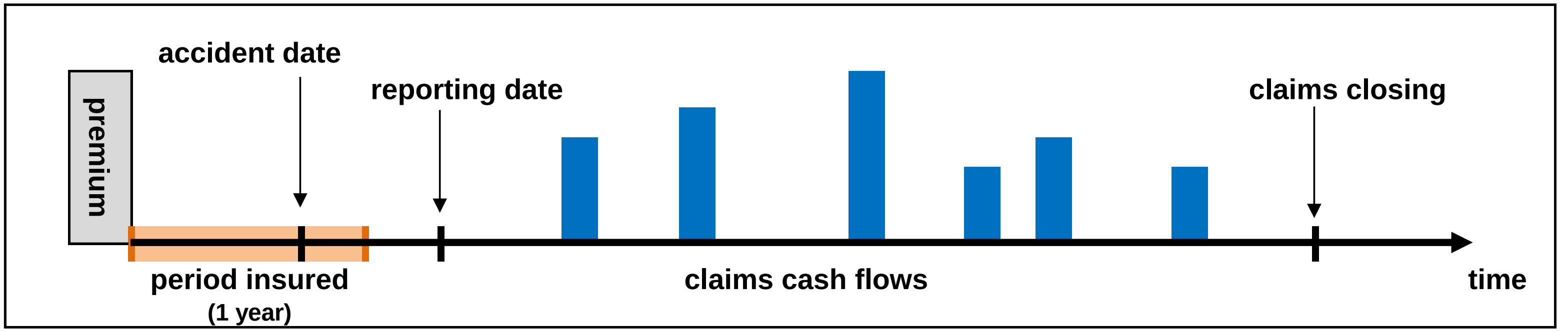
assets as of 31/12/2013	mio. CHF
debt securities	6'374
equity securities	1'280
loans & mortgages	1'882
real estate	908
participations	2'101
short term investments	693
other assets	696
total assets	13'934

liabilities as of 31/12/2013	mio. CHF
claims reserves	**7'189**
provisions for annuities	1'178
other liabilities and provisions	2'481
share capital	169
legal reserve	951
free reserve, forwarded gains	1'966
total liabilities & equity	13'934

Claims reserves are the biggest position on the balance sheet of a general insurance company!

Source: Annual Report 2013 of AXA-Winterthur Versicherungen AG

General insurance claims cash flows



▷ Typically, insurance claims cannot be settled immediately (at occurrence):

1. reporting delay (days, weeks, months or even years);
2. settlement delay (months or years).

▷ Predict and value all future claims cash flows (of past exposure claims).

▷ Build claims reserves to settle these future claims cash flows.

Claims development triangle

accident year i	development year j									
	0	1	2	3	4	5	6	7	8	9
2004	1'216	1'347	1'786	2'281	2'656	2'909	3'283	3'587	3'754	3'921
2005	798	1'051	1'215	1'349	1'655	1'926	2'132	2'287	2'567	
2006	1'115	1'387	1'930	2'177	2'513	2'931	3'047	3'182		
2007	1'052	1'321	1'700	1'971	2'298	2'645	3'003			
2008	808	1'029	1'229	1'590	1'842	2'150				
2009	1'016	1'251	1'698	2'105	2'385					
2010	948	1'108	1'315	1'487						
2011	917	1'082	1'484							
2012	1'001	1'376					$C_{i,j}$ **to be predicted**			
2013	841									

▷ $C_{i,j}$ = cumulative (nominal) claims cash flows for accident year $i \in \{1, \ldots, I\}$ and development year $j \in \{0, \ldots, J\}$.

▷ Observations at time t are given by

$$\mathcal{D}_t = \{C_{i,j};\ i + j \leq t\}.$$

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Chain-ladder method

accident year i	development year j 0	1	2	3	4	5	6	7	8	9
2004	1'216	1'347	1'786	2'281	2'656	2'909	3'283	3'587	3'754	3'921
2005	798	1'051	1'215	1'349	1'655	1'926	2'132	2'287	2'567	
2006	1'115	1'387	1'930	2'177	2'513	2'931	3'047	3'182		
2007	1'052	1'321	1'700	1'971	2'298	2'645	3'003			
2008	808	1'029	1'229	1'590	1'842	2'150				
2009	1'016	1'251	1'698	2'105	2'385					
2010	948	1'108	1'315	1'487						
2011	917	1'082	1'484							
2012	1'001	1'376					$C_{i,j}$ **to be predicted**			
2013	841									

▷ Chain-ladder (CL) algorithm is based on the observation that data often

$$C_{i,j+1} \approx f_j \, C_{i,j},$$

for CL factors f_j not depending on accident year i.

▷ CL algorithm has been used for many decades in the insurance industry.

Mack's stochastic CL model

Model assumptions (Mack 1993).
Assume there are fixed positive constants $f_0, \ldots, f_{J-1}$ and $\sigma_0^2, \ldots, \sigma_{J-1}^2$ given.

- $(C_{i,j})_{j=0,\ldots,J}$ are strictly positive, independent (in i) Markov processes (in j);
- for $i = 1, \ldots, I$ and $j = 0, \ldots, J-1$

$$\begin{aligned} \mathbb{E}\left[C_{i,j+1} \middle| C_{i,j} \right] &= f_j \, C_{i,j}, \\ \mathrm{Var}\left(C_{i,j+1} \middle| C_{i,j} \right) &= \sigma_j^2 \, C_{i,j}. \end{aligned}$$

▷ Mack introduced this stochastic model for the study of the CL algorithm.

▷ Mack's CL model has the CL property: $C_{i,j+1} \approx f_j \, C_{i,j}$.

CL predictor

▷ For *known* CL factors f_j and given observations $\mathcal{D}_I$

$$\mathbb{E}\left[C_{i,J}\middle|\,\mathcal{D}_I\right] = C_{i,I-i} \prod_{j=I-i}^{J-1} f_j.$$

▷ We define the CL predictor for *unknown* CL factors f_j at time I by

$$\widehat{C}_{i,J}^{(I)} \;=\; \widehat{\mathbb{E}}\left[C_{i,J}\middle|\,\mathcal{D}_I\right] \;=\; C_{i,I-i} \prod_{j=I-i}^{J-1} \widehat{f}_j^{(I)},$$

with CL factor estimators at time $t \geq J$

$$\widehat{f}_j^{(t)} = \frac{\sum_{i=1}^{(t-j-1)\wedge I} C_{i,j+1}}{\sum_{i=1}^{(t-j-1)\wedge I} C_{i,j}}.$$

▷ Note that $\widehat{C}_{i,J}^{(I)}$ and $\widehat{f}_j^{(I)}$ share many good properties (unbiased, uncorrelated).

CL claims prediction

accident year i	development year j: 0	1	2	3	4	5	6	7	8	9
2004	1'216	1'347	1'786	2'281	2'656	2'909	3'283	3'587	3'754	3'921
2005	798	1'051	1'215	1'349	1'655	1'926	2'132	2'287	2'567	*2'681*
2006	1'115	1'387	1'930	2'177	2'513	2'931	3'047	3'182	*3'424*	*3'577*
2007	1'052	1'321	1'700	1'971	2'298	2'645	3'003	*3'214*	*3'458*	*3'612*
2008	808	1'029	1'229	1'590	1'842	2'150	*2'368*	*2'534*	*2'727*	*2'848*
2009	1'016	1'251	1'698	2'105	2'385	*2'733*	*3'010*	*3'221*	*3'465*	*3'619*
2010	948	1'108	1'315	1'487	*1'731*	*1'983*	*2'184*	*2'337*	*2'514*	*2'626*
2011	917	1'082	1'484	*1'769*	*2'058*	*2'358*	*2'597*	*2'779*	*2'990*	*3'123*
2012	1'001	1'376	*1'776*	*2'116*	*2'462*	*2'821*	*3'106*	*3'324*	*3'577*	*3'736*
2013	841	*1'039*	*1'341*	*1'598*	*1'859*	*2'130*	*2'346*	*2'510*	*2'701*	*2'821*
$\widehat{f}_j^{(I)}$	1.2343	1.2904	1.1918	1.1635	1.1457	1.1013	1.0702	1.0760	1.0444	

▷ What about prediction uncertainty?

▷ Consider the conditional mean square error of prediction (MSEP)

$$\mathrm{msep}_{C_{i,J}|\mathcal{D}_I}\left(\widehat{C}_{i,J}^{(I)}\right) = \mathbb{E}\left[\left(C_{i,J} - \widehat{C}_{i,J}^{(I)}\right)^2 \middle| \mathcal{D}_I\right].$$

Mack's conditional MSEP formula

Mack (1993) gives the following MSEP estimate for single accident years i:

$$\mathrm{msep}^{\mathrm{Mack}}_{C_{i,J}|\mathcal{D}_I}\left(\widehat{C}^{(I)}_{i,J}\right) = \left(\widehat{C}^{(I)}_{i,J}\right)^2 \sum_{j=I-i}^{J-1} \left[\frac{\sigma_j^2 / \left(\widehat{f}_j^{(I)}\right)^2}{\widehat{C}^{(I)}_{i,j}} + \frac{\sigma_j^2 / \left(\widehat{f}_j^{(I)}\right)^2}{\sum_{\ell=1}^{I-j-1} C_{\ell,j}} \right].$$

▷ Note that this is an estimate because of

- ⋆ *unknown* parameters f_j (and σ_j) need to be estimated, and
- ⋆ *estimation error* cannot be calculated explicitly.

▷ Aggregation over accident years i is slightly more involved, but can be done.

Example, revisited

acc.year i	$\widehat{R}_i^{(I)}$	$\sqrt{\mathrm{msep}^{\mathrm{Mack}}_{C_{i,J}\mid\mathcal{D}_I}}$	in % $\widehat{R}_i^{(I)}$
2004	0		
2005	114	89	78%
2006	395	235	60%
2007	609	256	42%
2008	698	261	37%
2009	1'234	324	26%
2010	1'139	275	24%
2011	1'639	374	23%
2012	2'360	493	21%
2013	1'980	468	24%
total	10'166	1'518	15%

▷ Consider the CL reserves at time I defined by

$$\widehat{R}_i^{(I)} = \widehat{C}_{i,J}^{(I)} - C_{i,I-i},$$

and the corresponding prediction uncertainty.

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Claims development result (1/2)

- Mack's formula considers the total prediction uncertainty over the entire run-off.

- Solvency considerations require a dynamic view: possible changes in predictions over the next accounting year (short term view).

- Define the claims development result of accounting year $t+1 > I$ by

$$\mathrm{CDR}_i(t+1) \;=\; \widehat{C}_{i,J}^{(t+1)} - \widehat{C}_{i,J}^{(t)}.$$

- Martingale property of $(\widehat{C}_{i,J}^{(t)})_{t \geq I}$ implies

$$\mathbb{E}\left[\mathrm{CDR}_i(t+1) \middle| \mathcal{D}_t\right] = \mathbb{E}\left[\widehat{C}_{i,J}^{(t+1)} - \widehat{C}_{i,J}^{(t)} \middle| \mathcal{D}_t\right] = 0.$$

Claims development result (2/2)

accident year i	development year j 0	1	2	3	4	5	6	7	8	9
2004	1'216	1'347	1'786	2'281	2'656	2'909	3'283	3'587	3'754	3'921
2005	798	1'051	1'215	1'349	1'655	1'926	2'132	2'287	2'567	*
2006	1'115	1'387	1'930	2'177	2'513	2'931	3'047	3'182	*	
2007	1'052	1'321	1'700	1'971	2'298	2'645	3'003	*		
2008	808	1'029	1'229	1'590	1'842	2'150	*			
2009	1'016	1'251	1'698	2'105	2'385	*				
2010	948	1'108	1'315	1'487	*					
2011	917	1'082	1'484	*						
2012	1'001	1'376	*							
2013	841	*								

▷ Martingale property of $(\widehat{C}_{i,J}^{(t)})_{t \geq I}$ implies

$$\mathbb{E}\left[\mathrm{CDR}_i(I+1) \middle| \mathcal{D}_I\right] = \mathbb{E}\left[\widehat{C}_{i,J}^{(I+1)} - \widehat{C}_{i,J}^{(I)} \middle| \mathcal{D}_I\right] = 0.$$

▷ Solvency: study the one-year uncertainty of next accounting year $t = I + 1$

$$\mathrm{msep}_{\mathrm{CDR}_i(I+1)|\mathcal{D}_I}(0) = \mathbb{E}\left[\left(\mathrm{CDR}_i(I+1) - 0\right)^2 \middle| \mathcal{D}_I\right].$$

One-year uncertainty formula

Merz-W. (2008) give the following MSEP estimate for single accident years i:

$$\mathrm{msep}^{\mathrm{MW}}_{\mathrm{CDR}_i(I+1)|\mathcal{D}_I}(0) = \left(\widehat{C}^{(I)}_{i,J}\right)^2$$
$$\times \left[\frac{\sigma^2_{I-i}/\left(\widehat{f}^{(I)}_{I-i}\right)^2}{C_{i,I-i}} + \frac{\sigma^2_{I-i}/\left(\widehat{f}^{(I)}_{I-i}\right)^2}{\sum_{\ell=1}^{i-1} C_{\ell,I-i}} + \sum_{j=I-i+1}^{J-1} \alpha^{(I)}_j \frac{\sigma^2_j/\left(\widehat{f}^{(I)}_j\right)^2}{\sum_{\ell=1}^{I-j-1} C_{\ell,j}} \right],$$

with (credibility) weight

$$\alpha^{(I)}_j = \frac{C_{I-j,j}}{\sum_{\ell=1}^{I-j} C_{\ell,j}} \in (0,1).$$

▷ Eric Dal Moro (SCOR) and Joseph Lo (Aspen UK) basically write:
the problem is solved for the CL method,
see industrial discussion paper, ASTIN Bulletin 44/3, September 2014.

Total uncertainty vs. one-year uncertainty

Mack's formula for **total uncertainty**:

$$\mathrm{msep}^{\mathrm{Mack}}_{C_{i,J}|\mathcal{D}_I}\left(\widehat{C}^{(I)}_{i,J}\right) = \left(\widehat{C}^{(I)}_{i,J}\right)^2 \sum_{j=I-i}^{J-1} \left[\frac{\sigma_j^2 / \left(\widehat{f}^{(I)}_j\right)^2}{\widehat{C}^{(I)}_{i,j}} + \frac{\sigma_j^2 / \left(\widehat{f}^{(I)}_j\right)^2}{\sum_{\ell=1}^{I-j-1} C_{\ell,j}} \right].$$

MW formula for **one-year uncertainty**:

$$\mathrm{msep}^{\mathrm{MW}}_{\mathrm{CDR}_i(I+1)|\mathcal{D}_I}(0) = \left(\widehat{C}^{(I)}_{i,J}\right)^2$$
$$\times \left[\frac{\sigma_{I-i}^2 / \left(\widehat{f}^{(I)}_{I-i}\right)^2}{\widehat{C}^{(I)}_{i,I-i}} + \frac{\sigma_{I-i}^2 / \left(\widehat{f}^{(I)}_{I-i}\right)^2}{\sum_{\ell=1}^{i-1} C_{\ell,I-i}} + \sum_{j=I-i+1}^{J-1} \alpha_j^{(I)} \frac{\sigma_j^2 / \left(\widehat{f}^{(I)}_j\right)^2}{\sum_{\ell=1}^{I-j-1} C_{\ell,j}} \right].$$

Process uncertainty, parameter estimation uncertainty and its reduction in time.

Residual uncertainty for $t \geq I+2$

This suggests for accounting year $t = I+2$:

$$\mathbb{E}\left[\operatorname{msep}^{\mathrm{MW}}_{\mathrm{CDR}_i(I+2)|\mathcal{D}_{I+1}}(0) \middle| \mathcal{D}_I \right]$$

$$\approx \left(\widehat{C}^{(I)}_{i,J}\right)^2 \left[\frac{\sigma^2_{I-i+1} / \left(\widehat{f}^{(I)}_{I-i+1}\right)^2}{\widehat{C}^{CL(I)}_{i,I-i+1}} + \left(1-\alpha^{(I)}_{I-i+1}\right) \frac{\sigma^2_{I-i+1} / \left(\widehat{f}^{(I)}_{I-i+1}\right)^2}{\sum_{\ell=1}^{i-2} C_{\ell,I-i+1}} \right]$$

$$+ \left(\widehat{C}^{(I)}_{i,J}\right)^2 \sum_{j=I-i+2}^{J-1} \left[\alpha^{(I)}_{j-1} \left(1-\alpha^{(I)}_{j}\right) \frac{\sigma^2_j / \left(\widehat{f}^{(I)}_j\right)^2}{\sum_{\ell=1}^{I-j-1} C_{\ell,j}} \right].$$

- ▷ This can be derived analytically and iterated for $t > I+2$!
- ▷ This allocates Mack's MSEP formula across different accounting periods, i.e., this provides a run-off pattern of the total prediction uncertainty.

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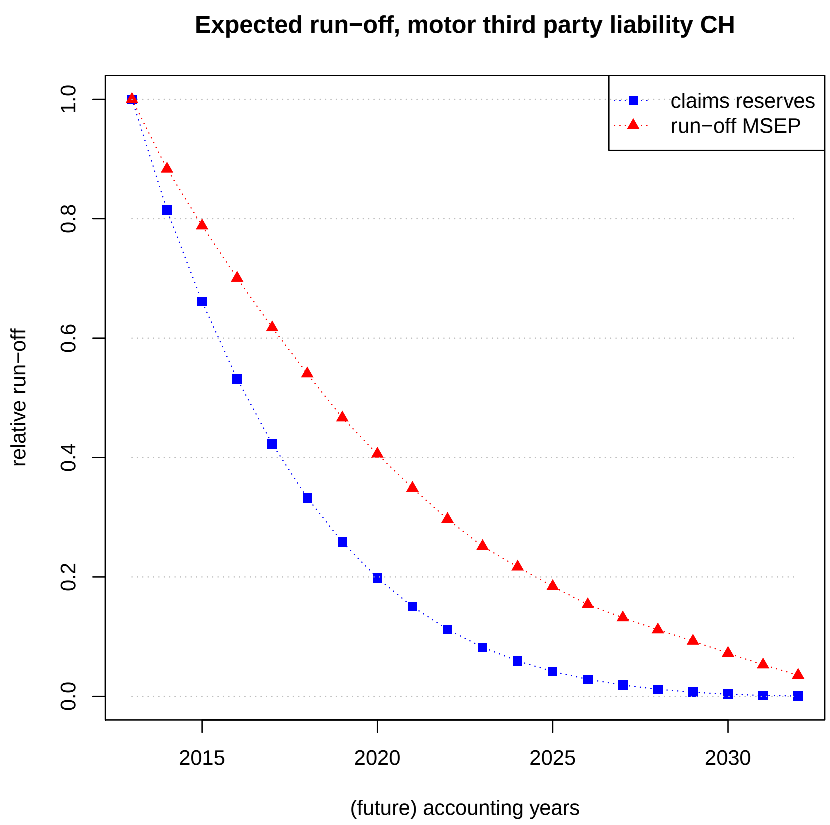
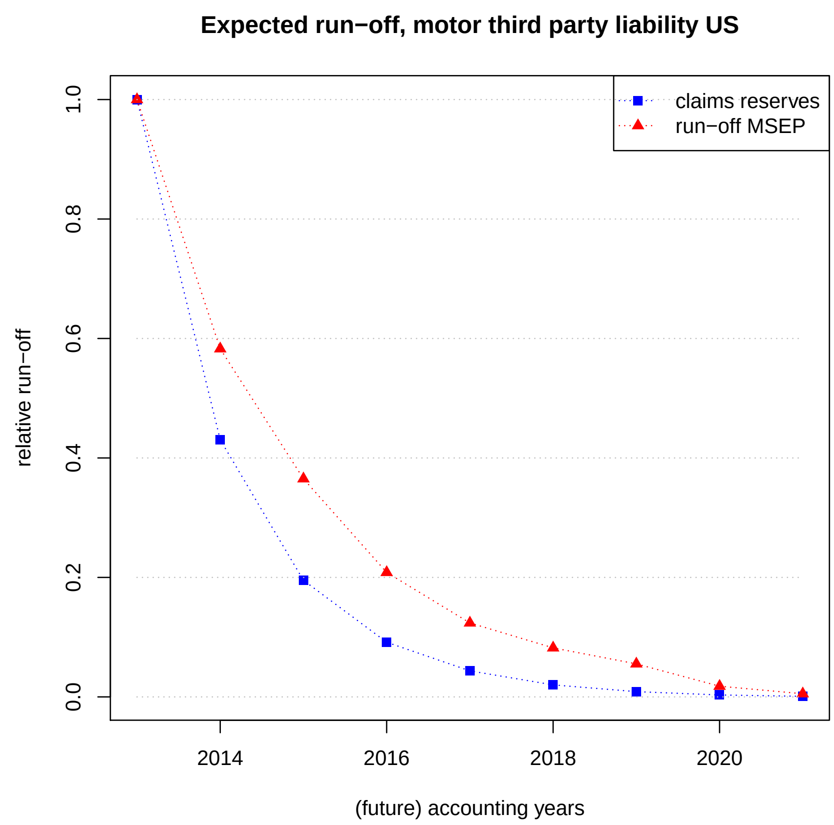
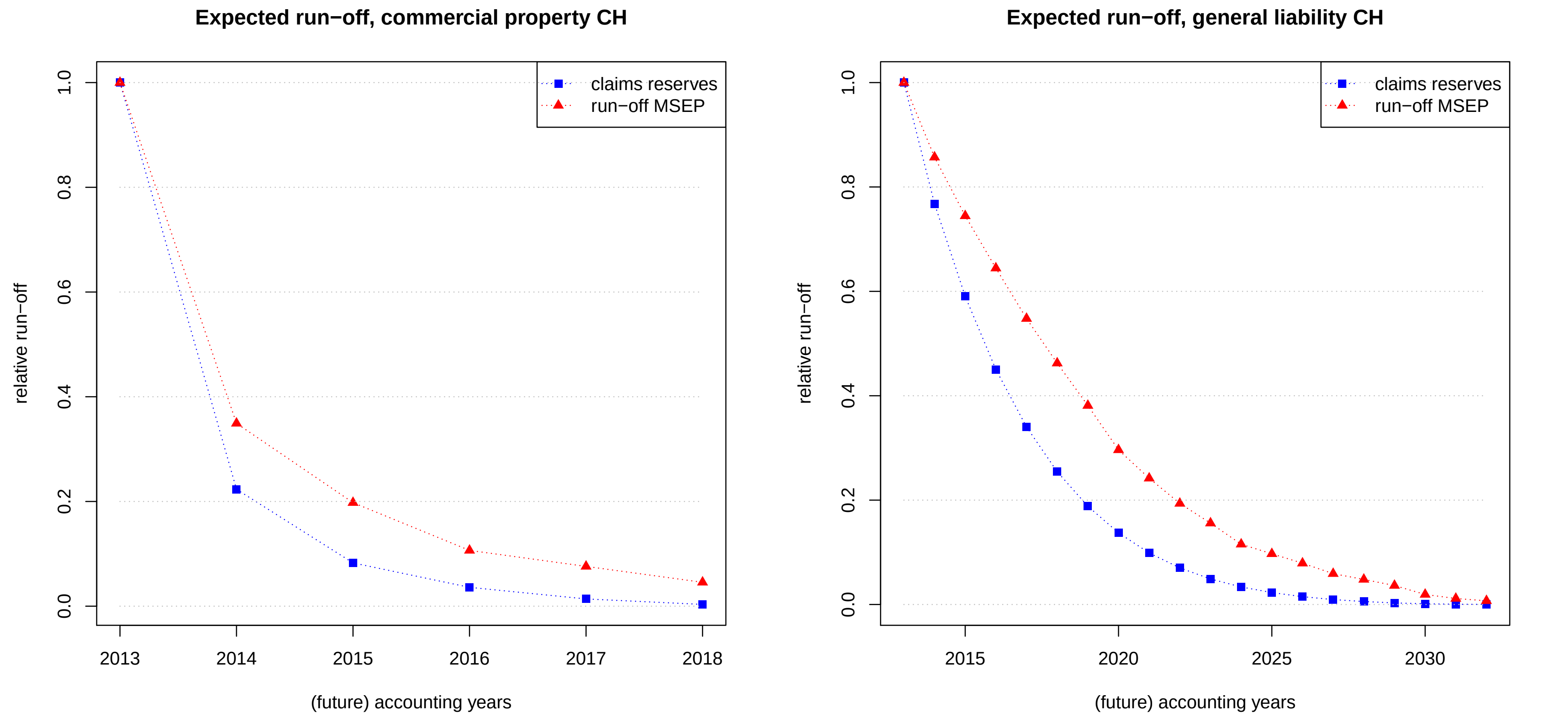
Motor third party liability: CH & US





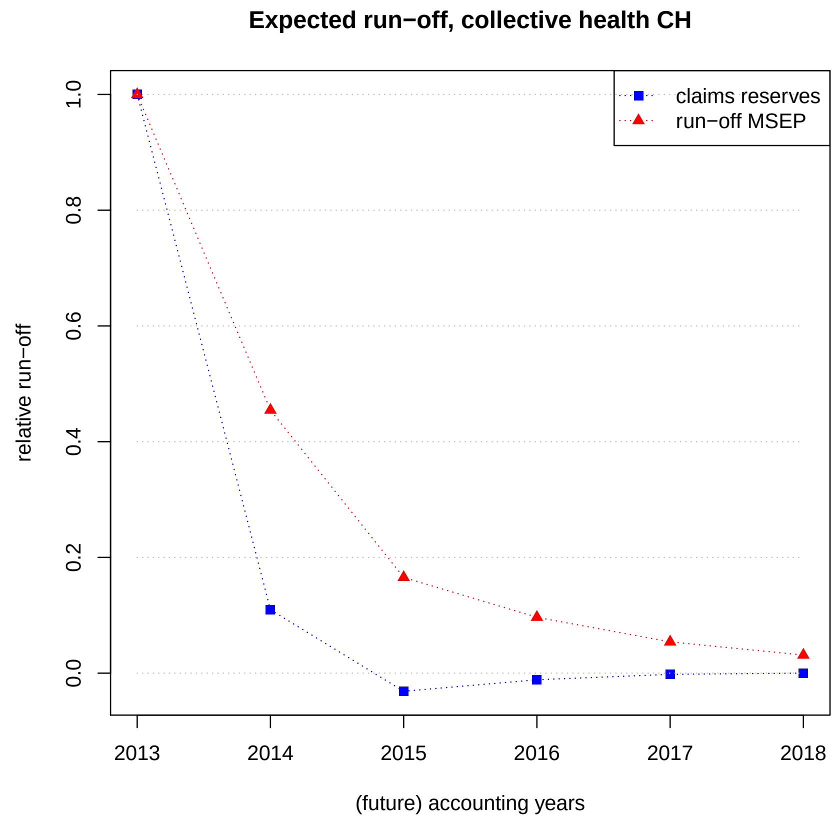
- ▷ Expected run-off of claims reserves is faster than the one of underlying risks.
- ▷ Legal environment is important for run-off.

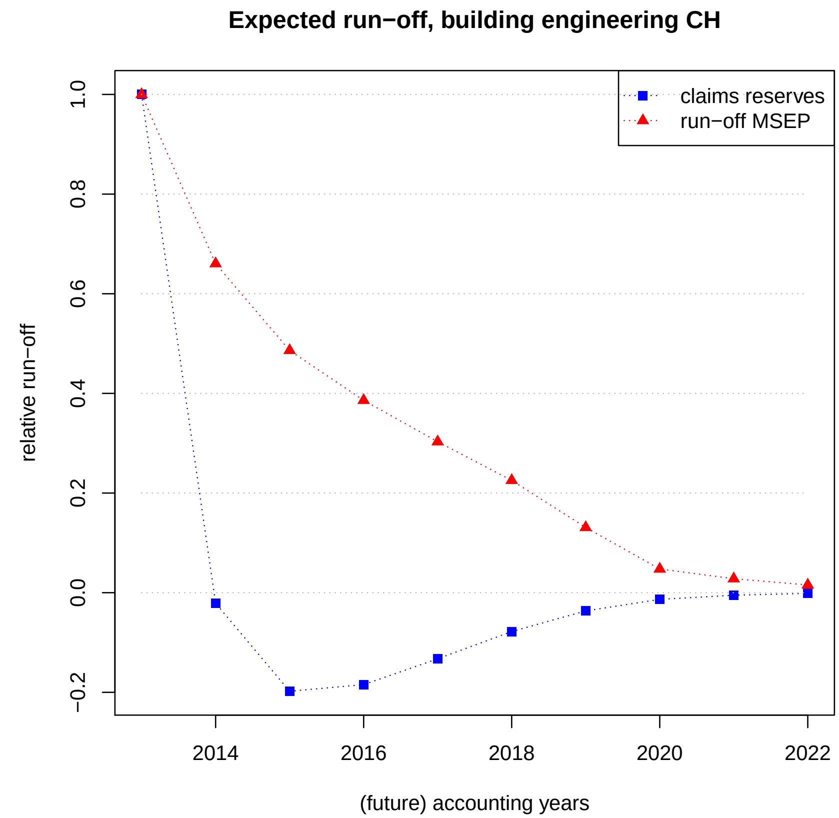
Commercial property & general liability (CH)



▷ Different lines of business behave differently (short- and long-tailed business).

Collective health & building engineering (CH)





▷ Subrogation and recoveries need special care.

Conclusions

- ▷ The Merz-W. formula was generalized to arbitrary accounting years.
- ▷ This allocates Mack's $total$ uncertainty formula across accounting years.
- ▷ This provides a run-off pattern for "risk".
- ▷ This improves and interprets risk margin calculations (Solvency II versus APRA).
- ▷ Standard approximation techniques typically under-estimate run-off risk.
- ▷ Portfolio characteristics and legal environment are important for risk margins.

References

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Workshop at UNSW