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A Comparison and Economic Analysis of International Solvency Regimes for Life Annuity Markets

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Abstract

This paper considers international solvency regimes for annuity products. A comparison of current international solvency regimes using an established qualitative framework is used to motivate a more detailed quantitative assessment of the impact of solvency on product pricing and shareholder value. A multi-period firm value model for a life annuity provider allowing for stochastic mortality and asset returns, imperfectly elastic product demand, as well as frictional costs, is developed. The model is used to derive optimal capital and pricing strategies for a range of solvency levels reflecting differences in regulatory regimes. The results show that value-maximizing insurers should target solvency levels that are higher than the regulatory 99.5% reflecting policyholder's aversion to insolvency risk. A discussion of the implications for regulatory policy to support the development of an annuity market concludes the paper.

Keywords: insurance regulation, solvency, longevity risk, life annuity markets
JEL Classifications: G22, G23, G28, G32

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1 Introduction

International demographic change has resulted in the prominence of longevity risk - uncertainty surrounding the risk of people living longer. Many developed nations are faced with the task of providing retirement solutions for an ageing population, whilst mortality continues to improve. Effective longevity risk management solutions that transform retirement savings into reliable retirement income sources are needed.

Life annuities provide an ideal hedge against longevity risk (Brown and Orzag, 2008 [20]), and risk averse individuals should value these annuities even more than the amount paid, since the probability of outliving individual retirement savings is significant (Mitchell, 2001 [48]). However, in reality, consumer demand for annuities is limited. This *annuity puzzle* is attributed to a variety of reasons, including bequest motives (Piggott and Purcal, 2008 [55]), the poor value for money of annuities (Brown et al., 1999 [19]), and the loss of liquidity and control over their finances in the case of unexpected and uninsured events (Piggott and Purcal, 2008 [55]).

Effective regulatory controls can help to develop and enhance market participation in longevity insurance products by ensuring that providers of these products will deliver on consumer contracts to a high degree of certainty. Higher capital requirements will lower insolvency risks, reducing insolvency costs to policyholders and shareholders, while at the same time increasing capital costs leading to higher frictional costs and premiums. Welfare losses arise from higher premiums since fewer individuals purchase longevity insurance, eschewing the longevity risk benefits associated with these products.[1]

Effective regulation needs to efficiently balance the trade-off between prudential security and consumer affordability. Many existing solvency regimes for annuities make an implicit choice of this trade-off, without finding an effective and optimal balance of the two sides. Even the most advanced annuity markets, such as that of the United Kingdom, still have many issues associated with the prudential regulation of these products (Davis, 2003 [26]).

The paper uses both qualitative and quantitative methods. The results from a comparison of current international solvency regimes using an established qualitative framework are initially discussed. Five regimes are chosen to represent a sample of the world with developed annuity

[1] Rees, Gravelle and Wambach (1999) [56] examine the arguments for solvency regulation and find that solvency regulation is unnecessary if consumers are fully informed about the risks of the insurer's insolvency. In reality, consumers cannot adequately inform themselves of insurer insolvency and its implications, hence prudential regulation is justified.

markets and/or developed regulatory systems: the United States, the European Union (Solvency II), the United Kingdom, Switzerland, and Australia. The comparison provides important insights into differences between these regimes and motivates a more detailed quantitative assessment of the impact of optimal solvency on product pricing and shareholder value.

The main contribution is the analysis of a multi-period value maximization model for an insurer offering lifetime guaranteed annuities, calibrated using realistic assumptions and market data. This is used to quantify the optimal solvency level that balances the trade-off between prudential security and consumer willingness to buy annuities. The model incorporates a stochastic mortality model, stochastic investment returns, and an imperfectly elastic demand function capturing consumer preferences for financial quality.

The structure of this paper is as follows. Section 2 summarises and reviews current solvency structures in the five regimes along with a discussion of the results of a comparative analysis of these regimes using an established qualitative framework. Section 3 presents an insurer value maximization model, along with calibration details. Section 4 presents the results of a quantitative study of an annuity provider's capitalization and pricing strategies under different regulatory requirements. Section 5 concludes.

2 Summary and Review of Solvency Regimes

The introduction of Basel II in 2004 brought with it the convergence of international capital measurement and standards for the banking industry. Basel II, in conjunction with the International Accounting Standards Board (IASB) and the International Financial Reporting Standards (IFRS), gave rise to a refocusing of the solvency requirements for the insurance industry with the development of the Individual Capital Adequacy Standards (ICAS) in the UK as well as the Swiss Solvency Test (SST) in Switzerland. These systems and frameworks are the foundation of the EU's Solvency II regulation (Arocha and Tan, 2008 [14]; Hill, 2008 [40]).

There are many possible approaches to assessing and comparing solvency regimes. Criteria need to be carefully developed and then applied taking into account the main objectives of solvency regulation. Cummins, Harrington and Niehaus (1994) [25] describe the main objectives of solvency regulation as protecting policyholder interests by: (1) providing appropriate incentives which encourage insurers to minimize insolvency risk, (2) facilitating the rehabilitation of weak insurers, if possible, and (3) ensuring the orderly exit of unsuccessful companies as closely as possible to the point in time of economic insolvency. They identify seven criteria that focus mainly on the risk-based capital formula. Holzmüller (2009) [42] extends the framework proposed by Cummins, Harrington and Niehaus (1994) [25] with an additional four criteria that also integrate recent changes in regulation and in the dynamic nature of insurance capital markets. This extended framework is applied in Holzmüller (2009) [42] to the US risk-based capital system, Solvency II and the Swiss Solvency Test (SST). She finds that the US system does not satisfy many of the criteria to the same extent as Solvency II and the SST. Her findings are in agreement with the results of a previous application of the original framework by Cummins, Harrington and Niehaus (1994) [25] to Solvency II provided in Doff (2008) [28].

Eling, Schmeiser and Schmit (2007) [32] compare existing solvency regimes with the aim of assessing Solvency II based on the objectives of a solvency system as outlined in the Sharma report (Sharma, 2002 [60]). They classify solvency systems into four model typologies: no model, static factor model, dynamic cash flow based models, and a combination of static factor models and dynamic cash flow models. The classification proposed by Eling, Schmeiser and Schmit (2007) [32] does not consider qualitative factors when assessing a solvency regime as a whole. Holzmüller and Eling (2008) [31] address these shortcomings by constructing a comparison framework based on the four main elements of a regulatory system: general information, definition of capital required, definition of available capital, and levels of intervention.

Sharara, Hardy and Saunders (2010) [59] compare the current US and Canadian statutory capital frameworks against the Solvency II standardized formula by quantitatively considering the minimum capital requirement for each of the three jurisdictions. The paper calculates the capital requirements for several hypothetical term life insurance portfolios in accordance with the requirements for each regime.

For our assessment the framework developed by Holzmüller (2009) [42] is used with a particular focus on lifetime guaranteed annuities. A detailed assessment of the solvency regimes using these criteria, which is the basis of this summary and review, is provided in Nirmalendran (2011) [52]. The current UK and Australian systems in addition to the US risk-based capital system, Solvency II and the SST are compared. The framework provides a balance between a detailed focus on quantitative calculation methods of capital and a qualitative assessments of the regimes, including assessment of management, anticipation of systemic risk, main objectives of risk management as well as flexibility over time. It offers a holistic view of the solvency regimes as opposed to focusing on particular components of the system only.

The original framework by Holzmüller (2009) [42] proposed the following criteria.

1. Giving the appropriate incentives: The risk-based capital formula should provide incentives for weak companies to hold more capital and/or reduce their exposure to risk without significantly distorting the decisions of financially sound insurers.

2. Formula should be risk sensitive: The risk based capital formula should reflect the major types of risk that affect insurers and be sensitive to how these risks differ across insurers.

3. Formula should be appropriately calibrated: The risk-based capital charges (or weights) for each major type of risk should be proportional to their impact on risk of insolvency.

4. Focus on the highest insolvency costs for the economy as a whole: The risk-based capital system should focus on identifying insurers that are likely to impose the highest costs of insolvency.

5. Focus on economic values: The formula and/or the measurement of actual capital should reflect economic values of assets and liabilities whenever practicable.

6. System should discourage misreporting: To the extent that is possible, the risk-based capital system should discourage under-reporting of loss reserves and other forms of manipulation by insurers.

7. Formula should be as simple as possible: The formula should avoid complexity that is of questionable value in increasing accuracy of risk-measurement.

8. Adequacy in economic crises and anticipation of systemic risk: Solvency regulation should anticipate systemic risk and prevent the insurance industry from being trapped in a vicious cycle when economic crises occur.

9. Assessment of management: A solvency system should take into consideration "soft" factors including, particularly, management capabilities.

10. Flexibility of framework over time: A model should be flexible with regard to its general concept and its parameters. Empirical insights and theoretical development, such as new models and concepts, should lead to continuous improvement.

11. Strengthening of risk management and market transparency: Solvency regulation should require insurers to handle the predominantly quantitative risks with sound risk management. Increased market transparency will, in the long run, reduce the need for regulation.

Table 1 summarises our subjective ratings scored for these criteria for each of the five solvency regimes with respect to life annuity business. We provide a summary of the main features of these regimes based on the more detailed assessment in Nirmalendran (2011) [52].

The United States

The current risk-based capital (RBC) standards were implemented by the National Association of Insurance Commissioners (NAIC) in 1994. There are two main components in this system; an RBC formula and an RBC model law. The RBC formula establishes the minimum capital level required for an insurer; this amount is compared with the actual level of capital, termed as the Authorized Capital Level (NAIC, 2009a [50]). There are three separate formulas for each line of business, incorporating the principle that capital must be assigned to the corresponding variety of risks, to which the insurer is exposed. The life insurance formula takes into account a covariance adjustment for potential correlations between asset, mortality, market, business and insurance risks.

The actual level of capital is defined as the total adjusted capital; that is the insurer's statutory capital plus a regulatory surplus. As insurance is regulated at a state level in the US, each state also has its own minimum capital floors ranging from $0.5 million to $6 million. In California, the minimum floor for life insurance products is $2.25 million plus a surplus which is defined as 2.7 times the authorized control level for life insurance (according to s10510 to s10511 of the California Insurance Code). In New York, the floor is $2 million plus a $4 million initial surplus (according to s4202 of the New York Insurance Law) (NAIC, 2009b [51]).

The RBC model law grants automatic authority for the state insurance regulators to intervene based on insurers' actual capital levels. The five levels of intervention, range from a No Action Level, when the insurer's total adjusted capital is double or more the risk based capital level, known as the Authorized Control Level, up to a Mandatory Control Level, when the total adjusted capital is below 70% of the Authorized Control Level and the regulator takes control of the insurer and places it in liquidation, since the insurer is likely to be technically insolvent (NAIC, 2009a [50]).

Criteria	Australia	US	UK	Switzerland	Germany
1. Giving the appropriate incentives	◕	○	◑	◕	◕
2. Formula should be risk sensitive	◕	○	◕	◕	◕
3. Formula should be appropriately calibrated	◑	◔	◕	◕	◑
4. Focus on the highest insolvency costs for the economy as a whole	◕	○	◑	◕	◑
5. Focus on economic values	●	○	●	●	●
6. System should discourage misreporting	◕	○	◕	◔	◑
7. Formula should be as simple as possible	◕	◑	◕	◕	◕
8. Adequacy in economic crises and anticipation of systemic risk	◔	○	◔	◔	◔
9. Assessment of management	◕	○	◕	◔	◑
10. Flexibility of framework over time	◕	○	◕	◕	◑
11. Strengthening of risk management and market transparency	◕	○	◕	◕	●

Table 1: Summary of ratings for each criterion for life annuity business. A dark fully shaded moon indicates the highest level of meeting the criteria and an unshaded full moon indicates the lowest level.

Based on the criteria and analysis summarized in Table 1, the US regulatory system performs poorly in comparison with the other regimes. Only two criteria are considered to be partially satisfied. This is in general agreement with the results in Holzmüller (2009) [42] who notes that the US system was conceived and implemented in the early 1990's when risk based capital systems were an innovative introduction. In comparison, the other systems have been developed and/or implemented in the last decade or so. A regulatory reform movement towards a more principles based approach in the US is currently under development. Under the US system a diversity exists in levels of solvency as shown in insurer credit ratings.

European Union - Solvency II

Solvency II is scheduled to come into effect at the beginning of 2014. Once in effect, all insurers operating in the European Union will be compliant with Solvency II. Solvency II has a three pillar structure adapted from Basel II to the insurance sector (Basel Committee on Banking Supervision, 2004 [17]). The following description of the three pillars is mainly based on Hill (2008) [40].

The first pillar regulates the quantitative aspects of risk measurement for insurers, in particular focusing on the use of standard or internal models to monitor market, credit, operational and life insurance risks. Whilst the internal models are company specific and require regulatory approval, the standard model takes a more prescriptive approach with fundamental risk parameters (such as volatility, yield, credit defaults and correlations) specified by the regulator. Under both approaches, capital requirements are calculated by valuing assets and liabilities on a market consistent basis using best estimate assumptions. A risk margin is then estimated using cost of capital methods and added to the best estimate. From this the minimum capital requirement (MCR), the minimum amount of capital necessary to protect policyholder interests, and the solvency capital requirement (SCR), the target capital in addition to the MCR, are determined.

The second pillar focuses on the supervisory review process and the levels of intervention regulators should undertake when observing the ratios of available capital to SCR and MCR. It also outlines the qualitative factors that need to be addressed by an insurer including Own Risk Self Assessments (ORSA) (Holzmüller, 2009 [42]).

The third pillar mandates public disclosure rules, in order to enhance market transparency of the insurance industry, and sets out the supervisory process.

Table 1 shows how Solvency II and the SST generally are considered to satisfy the criteria for the most part. Compared with Holzmüller (2009) [42] and Doff (2008) [28] lower levels for Criterion 3 and 8 have been assessed for life annuity business. Criterion 3 considers the appropriate calibration of capital calculations. For life insurers who offer life annuities, the time horizon of the capital calculation should reflect the longer term nature of these policies, as opposed to the current one year period. Criteria 8 is anticipation of systemic risk. Both Solvency II and the SST encourage internal models. Insurers will in practice use similar risk models leading to similar capital and risk management decisions in times of economic crises. If a significant number of the larger insurers react in the same way then this can introduce systemic risk.

The United Kingdom

The UK implemented the Individual Capital Adequacy Standards (ICAS) in 2005 as a risk-based capital approach as opposed to the previous formula-based system. UK insurers will also need to comply with future Solvency II minimum standards when they are finalized and implemented into European Union member countries. The following description of the current ICAS system is mainly based on Hill (2008) [40].

The ICAS system has a three pillar structure that is similar to the setup of Solvency II. The first pillar refers to the quantitative capital requirements, which include the minimum capital requirement (MCR) and Internal Capital Assessment (ICA). MCR defines the minimum level of capital needed to protect policyholder interests and if breached would result in immediate regulatory action. It is set in accordance with Solvency II requirements for the European Union. The ICA is a company specific measure of risk-based capital and is often benchmarked to the previously used Enhanced Capital Requirement (ECR). The ICA can be calculated using one of two methods: a) firm assessment of capital needs based on prescribed rules and guidance included in the Integrated Prudential Sourcebook of the Financial Services Authority (FSA); or b) via an internal economic capital model, supplemented with capital add-ons when risks are not adequately covered.

The second pillar refers to the supervisory assessment of a firm's regulatory capital requirement. The regulator, FSA, will provide insurers whose internal capital assessment is inadequate with an Individual Capital Guidance, which is the regulator's perspective and judgement on the adequate level of capital necessary for the insurer's exposure to risk (FSA, 2002 [34]). The third pillar specifies market disclosure rules.

The UK regime also performs well against the qualitative criteria alongside Solvency II and the SST as summarized in Table 1. There are some differences but these are not major.

Switzerland

Switzerland employs the Swiss Solvency Test (SST) as its prudential regulation for the insurance industry and pension funds. It is monitored by its regulator, Swiss Financial Market Supervisory Authority (FINMA). Though not a member country of the European Union, Switzerland has many agreements with the EU and hence has attempted to align its regulation, the SST, with that of the forthcoming Solvency II (Avanzi, 2010 [15]).

The Swiss Solvency Test is a risk-based and principles-based system, based on a realistic valuation framework (Keller, 2007 [46]). The main outputs of the Swiss Solvency Test are the Target Capital (focusing on the quantitative requirements) and the SST Report (focusing on the qualitative requirements).

The target capital requires the output of two values; a higher level, which is a risk-based valuation of required capital using market consistent values of assets and liabilities, and a lower level, which is the minimum solvency requirement and is calculated using a volume-based indicator approach with statutory values (Holzmüller, 2009 [42]). In order to calculate capital requirements, standard or internal models can be used to assess the corresponding market risk, credit risk and other specific risks associated with life insurance. In addition, predefined scenarios are used to analyze external shocks to the capital distribution. The standard model compares the sensitivities of risk bearing capital with respect to risk factors (for example mortality, longevity, recovery rate, lapse rate) and then aggregates the sensitivities with respect to their volatilities and correlations (Swiss Federal Office of Private Insurance, 2004 [61]).

The SST report is a summary of a company's risk position, containing descriptions of models and assumptions used as well as the justification of their use in calculating target capital levels. Quantitative and qualitative results of the scenario analysis as well as qualitative assessments of other more complex risks, such as operational risk, are included. Finally, risk mitigation strategies that are currently being employed are disclosed to the regulator in this report. In addition to the SST Report, a Risk Management Report describes the insurer's risk management and risk governance strategies including the insurer's enterprise risk management processes (Keller, 2007 [46]).

The SST regime performs well alongside Solvency II and this reflects in the results of the assessment summarized in Table 1. There are some differences but these are not major.

Australia

The current risk-based regulatory system in Australia uses a twin peaks approach with the Australian Prudential Regulation Authority (APRA) monitoring the prudential aspects of regulation and Australian Securities and Investments Commission (ASIC) taking care of advice and disclosure of products (Cooper, 2006 [23]). APRA has reviewed its standards to align itself with international systems and to improve risk sensitivity (APRA, 2011b [12]). For life insurers APRA has introduced prudential actuarial standards as well as governance and risk management standards.

Life insurers have to meet solvency and capital adequacy requirements for policyholder statutory funds and a management capital requirement for the shareholders' fund. Life insurers must calculate a solvency liability, similar to the best estimate liability but using the conservative assumptions prescribed in Prudential Standard LPS 2.04 Solvency Standard, and a capital adequacy liability, similar to the best estimate liability but using the best estimate assumption plus a margin chosen from a range prescribed in Prudential Standard LPS 3.04 Capital Adequacy Standard. A management capital requirement for the shareholders' fund is calculated in accordance with Prudential Standard LPS 6.03 Management Capital Standard. Life insurers are not currently permitted to use an internal model based approach.

The Australian prudential system also requires an Appointed Auditor to report to the Board about annual statutory accounts as well as general operations. An Appointed Actuary, unless exempted by APRA, is also required to provide an assessment of the insurer's overall financial condition as well as advice on the valuation of insurance liabilities for the Board.

APRA supports its supervision with a risk rating model called Probability and Impact Rating System (PAIRS) which allows the regulator to direct supervisory attention to entities based on their likely failure rate and on the impact should they fail (Hennessy, 2010 [38]). Through onsite visits and regulatory reporting, APRA makes an assessment based on management, risk management strategy, quantification of various risk exposures, capital coverage as well as access to excess additional capital. The scores obtained for PAIRS is then mapped to the Supervisory Oversight and Response System (SOARS), to determine the insurer's supervisory stance - normal, oversight, mandated improvement and restructure (Hennessy, 2010 [38]). These stances then dictate the level of regulator intervention necessary.

The Australian systems also performs very well alongside Solvency II and the SST.

2.1 Shortcomings of Qualitative Comparison

The assessment criteria were developed by Holzmüller (2009) [42] for application to general insurers. They have been modified to account for the inherent differences in the product characteristics and risk profiles of life annuity products in our comparison. For example, non-life insurance products, such as motor and home insurance, usually feature comparatively short terms with yearly renewals. Lifetime guaranteed annuities and other longevity insurance products, on the other hand, have very long terms to maturity and require different investment strategies.

The qualitative criteria involve a subjective assessment of the different solvency regimes. Most regimes meet the qualitative criteria. However there is a diversity in approaches across these regimes ranging from the prescribed levels of regulatory capital in Solvency II to the less prescriptive US regime. Importantly this assessment does not consider the regulatory trade-off between cost efficiency for consumers and prudential security of the insurer. Higher levels of capital require higher product loadings and prices. Higher prices reduce policyholder demand, whereas higher levels of solvency result in higher potential demand.

The US system provides a contrast with the highly prescribed levels of regulatory capital of the other regimes. The Australian and European solvency regimes have a number of similarities including the use of a minimum of 99.5% solvency level over a 1 year horizon for capital calculations. The United States solvency structure, on the other hand, is less prescriptive in the choice of solvency level for an insurer and this reflects in the insurer's corporate credit ratings, which vary considerably across insurers.

To address the shortcomings of the qualitative analysis and to quantify the trade-off between cost efficiency for consumers and prudential security of the insurer, an economic analysis of this trade-off is required. To do this a multi-period value maximization model is developed to provide a quantitative assessment of the effect of solvency capital requirements on a life annuity provider's optimal capitalization and pricing strategies. The analysis needs to allow for consumer preferences for more highly capitalized insurers, price elasticity for different product loadings and frictional costs of capital. These factors determine the trade-off between solvency levels and insurer profitability. This life insurer model for annuity business needs to include stochastic mortality and interest rate models.

3 A Life Insurer Value Maximizing Model

Value maximization models have been developed to study the effect of solvency requirements on insurers. Optimal insurer capitalization was considered by Munch and Smallwood (1981) [49] to assess the effect of solvency regulation on the property and casualty insurance industry. Optimal capitalization strategies were determined by maximizing the market value of the insurance firm. The firm value maximization model of Rees, Gravelle and Wambach (1999) [56] included the consumer's willingness to pay for insurance depending on the insurer's insolvency risk and consumers were assumed to be fully informed of insurer insolvency risk.

Imperfectly elastic demand for insurance and frictional costs of capital were incorporated in a single-period value maximization model for a multi-line non-life insurance company by Zanjani (2002) [70]. Yow and Sherris (2008) [69] used a single period model based on Zanjani (2002) [70] to assess the effects of frictional costs on a multiline non-life insurance company's pricing and capitalization strategies. The model includes frictional costs of agency, bankruptcy and taxation and consumer preferences for solvency. A single-period shareholder value maximization model for a life insurance company offering term life insurance and life annuities to insolvency-averse consumers was developed in Gründl, Post and Schulze (2006) [37] and used to study the impact of demographic risk in the optimal risk management mix of the insurer.

Product demand with respect to solvency and price is a critical component of a model that aims to assess the solvency trade-off. Estimates of consumers' reactions to insurance default risk are reported in Zimmer, Schade and Gründl (2009) [71] and Zimmer, Gründl and Schade (2011) [72] based on experiments used to elicit an individuals' willingness to pay for theft insurance contracts for differing levels of default risk. Four levels of default (0%, 1%, 2% and 3%) were used to calibrate the demand function. Monetary incentives and the secret price mechanism developed by Schade et al (2009) [57], which provides an accurate reflection of maximum willingness to pay, increased the experiment's reliability. A range of different demand functions were fitted to the data and the exponential demand function was the overall best fit. The demand curve based on Zimmer, Gründl and Schade (2011) [72] in a single-period shareholder value maximization model, as in Zanjani (2002) [70], was used to analyze the impact of consumer reactions to default risk on an insurer's optimal solvency level. This demand curve functional form will be adapted for life annuity demand.

3.1 A Value Maximization Model for a Life Annuity Provider

The model is used to assess the optimal capital and pricing strategy for a life insurer offering life annuities maximizes expected profit allowing for capital subscriptions over a one-year horizon for different default levels. A one-year horizon is chosen to reflect international solvency requirements (including Solvency II). Although a one-year horizon is used, cash-flows are multi-period and stochastic.

The model in Yow and Sherris (2008) [69] was extended to allow for multiple period cash flows depending on stochastic mortality. A term structure model was also included to value future expected annuity cash flows. The insurer model includes stochastic investment returns and a price-default risk demand curve in the form estimated by Zimmer, Gründl and Schade (2011) [72]. A single cohort of males aged 65 is simulated in the model. Stochastic values of assets and liabilities are determined at the end of the period using simulation of stochastic future mortality rates and yield curves. These are used to determine stochastic profit realizations over the year.

Enterprise Value Added (EVA), as in Yow and Sherris (2008) [69], is used as the measure of economic value added by the insurance business for shareholders. This is determined by the expected profit over the first year after allowing for the establishment of reserves for future labilities for survivors at the end of the first year. To allow for the initial capital subscribed an allowance is also made for the cost of capital (CoC) to determine an EVA adjusted for CoC. Both EVA and EVA adjusted for CoC are measures of the economic value of the insurer.

Premiums: Single premiums include a loading on the best estimate annuity prices determined using the future expected mortality survival rates and the current market yield curve. Policyholder demand is assumed to depend on the price per contract and the solvency, or default risk, of the insurance company. Total premium income P at time 0 is the number of policies sold by the per policy single premium:

$$P = Q(\pi, d) \cdot \pi, \tag{1}$$

where Q is the number of annuities sold at time 0, d is the default probability, and π is the per policy single premium. For each policy

$$\pi = (1 + k) \cdot A \cdot \sum_{t=1}^{45} {}_tp_{65,0} \cdot \nu(0, t), \tag{2}$$

where k is the premium loading, A is the fixed annual payment, ${}_tp_{65,0}$ is the expected probability of a male aged 65 to survive another t years at time 0, and $\nu(0, t)$ is the discount factor for a payment at time t. The discount factor is from the expected yield curve at time 0, fitted with a Vasicek model described below.

Stochastic Mortality Model: Survival probabilities are derived from the stochastic mortality model described in Wills and Sherris (2008) [68]. This model extends the traditional Lee-Carter mortality model to incorporate age and cohort effects as well as multiple risk factors. The model provides a very good fit to Australian mortality data for lives aged 50 to 99 and includes a simulation procedure for projecting future mortality rates incorporating cohort longevity improvements over time. The model was re-calibrated to incorporate ages up to 110.

The force of mortality $\mu(x, t)$ for age x at time t is modeled as a discrete approximation to a stochastic diffusion process:

$$d\mu(x, t) = (a(x_0 + t) + b)\mu(x, t)dt + \sigma\mu(x, t)dW(x, t), \tag{3}$$

where a, b and σ are constants and $dW(x, t)$ is a multivariate Wiener process. x_0 is the initial age at the start of the contract.

Demand Curve: The price-default risk demand curve from Zimmer, Gründl and Schade (2011) [72] captures both the default risk aversion and the price sensitivity of policyholders. The functional form is given in the equation below.

$$\phi(\pi, d) = e^{(\alpha \cdot d + \beta \cdot \pi + \gamma)}, \tag{4}$$

where $\phi(\pi, d)$ represents the percentage of individuals willing to buy annuities at price π from an insurer with default probability d, α is the default sensitivity parameter ($\alpha < 0$), β is the price sensitivity factor ($\beta < 0$) and γ is a constant.

Given the demand function and the maximum potential market size, M, the number of annuities Q sold at time 0 is:

$$Q(\pi, d) = M \cdot \phi(\pi, d). \tag{5}$$

Initial Capital: Initial capital, R, is subscribed from the shareholders at time 0 in order to achieve a target solvency level d promised to the policyholders.

Expenses: Expenses are assumed to be a fixed percentage of the single annuity premium π and are a one-off, paid at the end of the first year. Total expenses are given by:

$$c = P \cdot expense\ factor. \tag{6}$$

In practice there are other expenses. Including these does not change the conclusions.

Claims and Reserves: Claims at the end of the first period are random. They are given by:

$$\tilde{L}_1 = Q(\pi, d) \cdot A \cdot {}_1\tilde{p}_{65}, \tag{7}$$

where Q is the number of annuities sold at time 0, A is the fixed annual payment per annuity contract, and ${}_1\tilde{p}_{65}$ the random probability of a 65-year old to survive to age 66.

The policy liability reserves are set up to include the net present value of future liabilities as well as premium loadings weighted by the proportion of survivors, valued using the government bond yields to maturity. The reserve at time 0 is given by:

$$Reserve_0 \quad = \quad Q(\pi, d) \cdot (1 + k) \cdot A \cdot \left[\sum_{t=1}^{45} {}_t p_{65,0} \cdot \nu(0, t) \right]. \tag{8}$$

The reserve at time 1 is calculated using random time 1 survival probabilities ${}_t\tilde{p}_{66,1}$ and discount factors $\tilde{\nu}(1, t)$. These differ across simulation scenarios. The reserve is established for the random number of survivors in each scenario ${}_1\tilde{p}_{65} \cdot Q(\pi, d)$:

$$\widetilde{Reserve}_1 = {}_1\tilde{p}_{65} \cdot Q(\pi, d) \cdot (1 + k) \cdot A \left[\sum_{t=1}^{44} {}_t\tilde{p}_{66,1} \cdot \tilde{\nu}(1, t) \right]. \tag{9}$$

The reserves include the value of the loadings in the single premium. Since expenses are assumed to be incurred at the end of year 1 as a single up front payment, the premium loadings are included in the determination of economic profit to offset the initial expenses in the first period.

Assets and Investment Returns: Assets at time 0 comprise the total premium income P and the capital R subscribed from the shareholders: $V_0 = P + R$. Returns on the assets are random and denoted by $\widetilde{return}_t$, a weighted average of the per period returns from different investments. Asset classes included in the model are bonds, stocks and a cash account.

Term Structure Model: The Vasicek model (Vasicek, 1977 [66]) is used for the term structure model. This is a one-factor short rate model that incorporates mean reversion of interest rates.[2] The stochastic process for the short rate r_t is:

$$dr_t = \alpha(\mu_r - r_t)dt + \sigma_r dW_t, \tag{10}$$

where α, μ_r and σ_r, together with the initial condition r_0, characterize the dynamics of the instantaneous interest rate. Discount factors $\nu(t, T)$ for the value at time t of a payment at time T assuming continuously compounded zero-coupon bond yields are given by (see Van Deventer, Imai, and Mesler, 2005 [67], pp. 209-212):

$$\nu(t, T) = e^{-F(t,T)r_t - G(t,T)}, \tag{11}$$

where (using τ to denote the maturity):

$$F(t, T) = F(\tau) = \frac{1}{\alpha}(1 - e^{-\alpha\tau}) \tag{12}$$

$$G(t, T) = G(\tau) = \left[\mu + \frac{\lambda\sigma}{\alpha} - \frac{\sigma^2}{2\alpha^2}\right][\tau - F(\tau)] + \frac{\sigma^2}{4\alpha}F^2(\tau). \tag{13}$$

When calibrating the model's parameter, a market price of risk λ is calibrated to observed market yield curve data (see Van Deventer, Imai, and Mesler, 2005 [67], pp. 209-212 and 222-225).

The short rates generated by the Vasicek model are used for single period bond returns. Stock prices and cash rates are modeled as Geometric Brownian motions with drift terms μ_s and μ_c, and volatility parameters σ_s and σ_c, respectively (see, e.g., Hull, 2009 [43]).

[2] The Vasicek model can generate negative interest rates which are unrealistic. This problem is dealt with by eliminating simulation runs where the short rate becomes negative and the same number of simulations that attain the highest returns are also removed to avoid bias.

Balance Sheet	
Assets	Liabilities
Stocks	*Reserve*
Bonds	*Equity*
Cash	*Capital subscribed*

Profit and Loss Account	
Income	Expenses
Premiums	*Claims*
Investment income	*Expenses*
	Frictional costs
	Change in the reserve

Figure 1: Insurer balance sheet and profit and loss account over the first year.

Frictional Costs: Three types of frictional costs are included in Yow and Sherris (2008) [69]: taxation, agency and bankruptcy costs. The taxation rate is denoted as τ_1. Total agency costs are assumed to be proportional to the initial capital subscribed, $\tau_2 R$. Bankruptcy costs, f, reflecting financial distress, are included whenever profit is negative and are larger in absolute amount for larger losses.

Insurer Profit and Enterprise Value Added: Insurer profit is calculated using an economic valuation approach. Balance sheet amounts are market value based in accordance with International Financial Reporting Standards.[3] The balance sheet and the profit and loss account, used to determine economic profit, are shown in Figure 1.

Enterprise Value Added (EVA) is defined as the expected present value of profits to shareholders in excess of the initial capital subscribed. This is determined based on the random profit over the first year:

$$\widetilde{Profit}_1 = \begin{cases} Reserve_0 + \tilde{K} + (P+R)\cdot\widetilde{return}_1\cdot(1-\tau_1) - \tilde{L}_1 - c - \tau_2\cdot R - Reserve_1 \quad if\ \widetilde{Profit}_1 > 0 \\ \Big(Reserve_0 + \tilde{K} + (P+R)\cdot\widetilde{return}_1\cdot(1-\tau_1) - \tilde{L}_1 - c - \tau_2\cdot R - Reserve_1\Big)(1+f) \\ if\ \widetilde{Profit}_1 < 0. \end{cases} \tag{14}$$

The profit is determined as the premium, which is used to establish the time 0 initial reserve, plus the present value of future premium loadings for the survivors, $\tilde{K} = {}_1\tilde{p}_{65}\cdot\frac{k}{1+k}\cdot P$, less claims, expenses, frictional costs as well as the amount required to establish a reserve at the end of the year for the survivors. The simulated EVA then depends on the amount of capital subscribed and, where this is positive, the extent to which it offsets losses. For lower levels of solvency and higher levels of premium loadings it is possible that the amount of capital required to establish the target solvency is negative. This reflects the fact that the premium loadings alone are sufficient to ensure the target solvency level. For these reasons the simulated EVA

[3] Balance sheet amounts are also in accordance with the Australian Accounting Standards set by AASB.

is:

$$\widetilde{EVA} = \begin{cases} \begin{cases} \widetilde{Profit}_1 & if\ \widetilde{Profit}_1 + R \geq 0 \\ -R & if\ \widetilde{Profit}_1 + R < 0 \end{cases} & if R \geq 0 \\ \begin{cases} \widetilde{Profit}_1 & if\ \widetilde{Profit}_1 \geq 0 \\ 0 & if\ \widetilde{Profit}_1 < 0 \end{cases} & if R < 0 \end{cases} \tag{15}$$

In the first two cases, a positive amount of capital R is subscribed from shareholders to establish the target default probability d. In the first case, shareholders receive a profit. In the second case, the losses are so large the shareholders lose their initial capital (EVA $= -R$). In the third and the fourth case, shareholders can extract capital at time 0 because the total premium income is more than enough to establish the target default probability. In the fourth case, the company defaults but EVA $= 0$ because shareholders did not have to invest capital at time 0.[4]

The EVA is the expected value of this simulated EVA across all the simulation scenarios. The EVA adjusted for cost of capital is the EVA minus the initial subscribed capital times the cost of capital.

3.2 Calibration of the Life Insurer Model

The model was calibrated to market and other relevant Australian data for yield curves, asset returns, expenses, mortality and frictional costs. The demand function and the frictional costs are the most challenging to calibrate to market data. There are no studies or industry information that can be readily used for this purpose. We follow a similar process as in Yow and Sherris (2008) [69] to do this calibration. The robustness of these calibrations are assessed in the results.

Stochastic Mortality Model: The stochastic mortality model was estimated using Australian male mortality rates for ages 50 to 110 from 1971 to 2007 obtained from the Human Mortality Database (2011), [44]. The maximum likelihood parameter estimates are shown in Table 2.

Figure 2 shows the age-specific (standardized) model residuals. The model fits Australian mortality data well. Prior to 1960, mortality data in the Human Mortality Database for older ages

[4] In this model, reserves are calculated assuming no default of the insurer as required by accounting and solvency requirements. As a result, the reserves include the default put option (DPO) value (which is given by the expected value of the payments policyholders will not receive in the case of insolvency, that is if $R + Profit < 0$).

Parameter	MLE Value
$\hat{a}$	$4.5089E-04$
$\hat{b}$	-0.1011
$\hat{\sigma}$	0.0605

Table 2: Parameter estimates for the mortality model using MLE techniques.

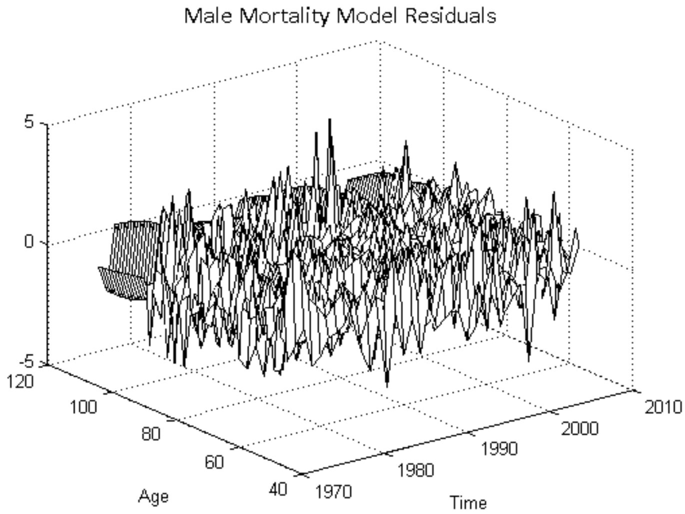


Figure 2: Standardized residuals from the mortality model.

was smoothed which results in the smoothing of residuals past the age of 96. Prior to this age, the model residuals fluctuate randomly around a mean of zero without age or time trends.

Table 3 gives the descriptive statistics of the standardized residuals. The standard error of the mean estimate is small and the standard deviation is very close to one.

Parameter	Value
Mean	$4.5797e-016$
Standard Error	0.0215
Standard Deviation	1.0002
Minimum	-4.3935
Maximum	4.6108

Table 3: Residuals descriptive statistics for the standardized residuals from the mortality model.

Pearson's chi-square goodness of fit test between observed mortality rates and expected mortality rates has a value of $\chi^2 = 124.42$. This statistic approximately has a chi-square distribution with 326 degrees of freedom.[5] The critical value is $\chi^2_{326}\ \ at\ \ 99\% = 269.55$ and the test statistic is less than the critical value, confirming the model provides a good fit to the data.

Mortality scenarios are simulated using the procedure in Wills and Sherris (2008) [68]. Figure

[5] There are 2160 observations, 3 parameters in the main model and 1830 parameters in the 60×60 correlation matrix of $dW(x,t)$: $df = number\ of\ observations - number\ of independent\ parameters - 1 = 326$.

3 plots the expected survival curve for a 65 year old at time $t = 1$ together with confidence intervals.

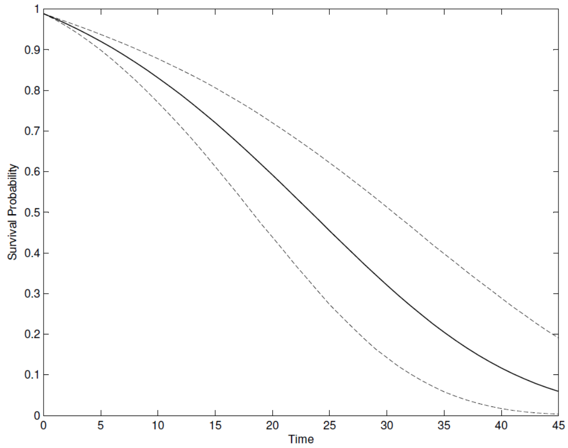


Figure 3: Simulated survival probabilities at time $t = 1$, mean values and 95% confidence intervals.

Premiums: With a zero loading, the average annual payment of $A = \$5,149.70$ was based on a single premium of \$70,000 using expected survival rates and the fitted initia yield curve. This is consistent with the average retirement savings of approximately \$71,000 for a 65 year old (Australian Bureau of Statistics, 2010, [2]).

Expenses: Expenses were assumed to be 3.3% of the total annuity premium (including the loading). Challenger Life Company Ltd is the major company writing lifetime guaranteed annuities in Australia. Their product disclosure statement reports an upfront adviser service fee of up to 3.3% of the purchase price (Challenger, 2011 [21]).

Assets Allocation: The insurer's asset mix was based on investment strategies of insurers offering annuities with longevity risk. The Australian Prudential Regulation Authority (APRA) publishes assets backing policy liabilities in their Half Yearly Life Insurance Bulletin. An allocation of 5.5% in cash, 86.8% in bonds, and 7.7% in stocks was used (APRA, 2010a [8]). A portfolio consisting largely of bonds provides a matching investment strategy for a life insurer issuing life annuities. However the maturity of available government bonds in Australia are not long enough for full asset liability matching. As a result interest rate risk is captured in the model over the one year horizon with a stochastic yield curve model.

Yield Curve and Asset Returns: The data used to calibrate the yield curve and asset return models came from 'Australian Government Bonds Yields and Interest Rates' obtained from Bloomberg (accessed September 2011), time series for the period 1990-2010 for the 'Cash

Rate - Interbank Rate' (accessed August 2011) and capital market yields of 10-year Australian Government Bonds (accessed July 2011) from the Reserve Bank of Australia.

Least squares was used to estimate the parameters of the Vasicek model. The fitted initial yield curve and simulated yield curves at time 1 are shown in Figure 4. The initial curve fits the current Australian yield curve well apart from the very short maturity. The Vasicek model parameters are shown in Table 4.

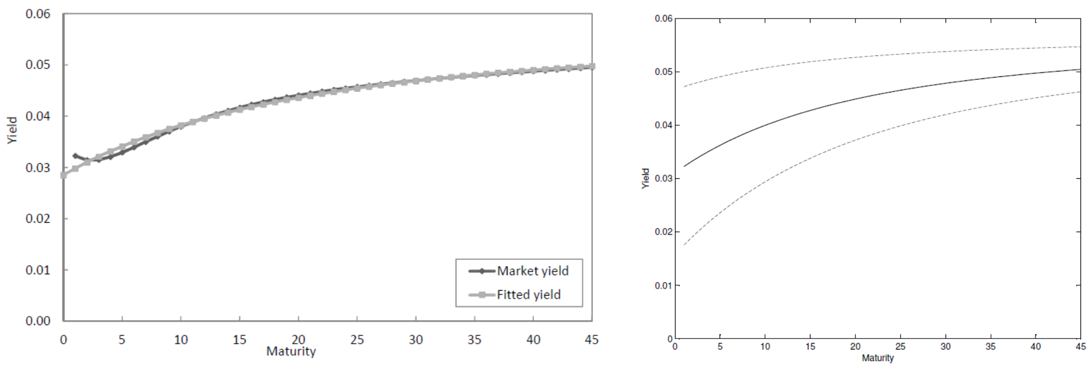


Figure 4: Fitted yield curve at time $t = 0$ and simulated yield curves at time $t = 1$, mean values and 95% confidence intervals.

Parameters of the stochastic processes for the stock returns and the cash rate were estimated based on 1990-2010 data for the 'S&P/ASX 200 Accumulation Index' and for 'Cash Rate - Interbank Rate' provided by the Reserve Bank of Australia and are shown in Table 4. The time series used for the 'S&P/ASX 200 Accumulation Index' covering this period was constructed by linking 2006-2010 data from the Reserve Bank of Australia and with a longer time series from a free internet resource[6] (both accessed June 2011).

The correlation matrix between cash, bond, and stock returns over the period 1990-2010 is shown in Table 5.

Yield Curve		Stock Returns		Cash Rate	
Parameter	Value	Parameter	Value	Parameter	Value
$\hat{\alpha}_r$	0.0790	$\hat{\mu}_s$	0.0981	$\hat{\mu}_c$	0.0613
$\hat{\mu}_r$	0.0608	$\hat{\sigma}_s$	0.1239	$\hat{\sigma}_c$	0.0222
$\hat{\sigma}_r$	0.0079				
$\hat{r}_0$	0.0285				
$\hat{\lambda}$	0.0102				

Table 4: Parameter estimations for yield curve and asset return models, annual data for the period 1990-2010.

[6] http://www.economagic.com/em-cgi/data.exe/rba/fsmspasx2ai

	Cash	Bond	Stock
Cash	1.0000	0.9490	0.0167
Bond	0.9490	1.0000	0.0013
Stock	0.0167	0.0013	1.0000

Table 5: Correlation matrix between asset returns, annual data for the period 1990-2010.

Frictional costs: For the shareholder value we assume a tax rate of 0% allowing for the benefits from the imputation credit system in Australia.[7] The taxation rate is denoted by τ_1. Tax for life annuities owned as superannuation is normally 15% and applies to investment income. The shareholder agency cost of capital, τ_2, was assumed to be 2% based on Swiss Re (2005) [63]. The bankruptcy cost factor, f, was assumed to be 15%.

Cost of capital: The one year bond rate is used for the cost of capital. This is 3.3% from the market yield curve used to calibrate the yield curve model.

Market size: A maximum potential market size of $M = 25,000$ was assumed for the representative life annuity provider. The current Australian male population aged 65 is 102,857 (Australian Bureau of Statistics, 2010 [2]). In 2010, the largest life insurer in Australia had a market share of 25-30% (APRA, 2011a [11]).

Demand curve: There is no empirical study that we are aware of that provides estimates of the price and default risk sensitivity of the demand for life annuities.[8] Annuity markets sizes and premium loadings observed internationally provide only limited insight because of differences in solvency regulations and social security systems.

The price-default risk life annuity demand curve was calibrated based on studies of the Australian annuity market along with informed judgement. The sensitivity of the analysis to this assumption is assessed in Section 4.2. The assumed demand curve is:

$$\phi(\pi, d) = e^{(\alpha \cdot d + \beta \cdot \pi + \gamma)} \tag{16}$$

$$= e^{(-100 \cdot d + -0.00015 \cdot \pi + 10)}, \tag{17}$$

[7]The Australian imputation system allows corporate tax entities to distribute to their members franking credits for taxes paid by these corporations in order to avoid double taxation of the same income earned. These franking credits act as a tax offset on assessable income for the shareholders of these corporations (Australian Tax Office, 2011 [13]).

[8]Babbel and Merrill (2006) [16] employ a multi-period utility maximization framework to study the impact of an insurer's default risk on annuity demand. The results of this theoretical study suggest that for moderate levels of risk aversion annuity demand is not very sensitive towards premium loadings of up to 30%. However, optimal annuitization levels are shown to drop sharply when the default risk of the annuity provider is stepwise increased from riskless to a 'AAA', 'AA', and 'A' rating.

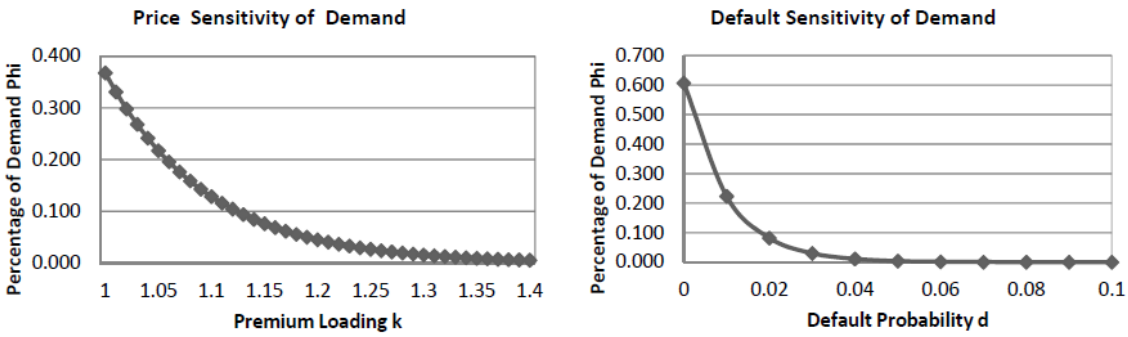


Figure 5: Assumed price and default sensitivity of the demand for annuities.

where $\phi(\pi, d)$ represents the percentage of individuals willing to buy the annuity contract at the premium of π from an insurer with default probability d. The default sensitivity parameter is set to $\alpha = -100$, the price sensitivity factor is $\beta = -0.00015$, and the constant is $\gamma = 10$.

Figure 5 shows the reduction in demand due to changes in price and the demand curve's responses to changes in default risk. The assumed fair annuity premium is \$70,000. The left graph plots annuity demand $\phi(\pi, d)$ against different levels of the premium loading k, assuming a default probability of $d = 0.5\%$. At a zero loading, 37% of individuals would be willing to purchase the annuity contract. At a loading of $k = 24\%$ a very small demand of 3% results, which is consistent with annuity demand in the Australian market.[9] The right graph plots $\phi(\pi, d)$ against the default probability d for a premium of \$70,000. Annuity demand decreases rapidly as insolvency risk increases. At a default probability of $d = 5\%$ demand is zero.

4 Optimal Solvency and Premium Loadings

4.1 The Life Insurer's Value Optimization

The life annuity insurer maximizes enterprise value added (EVA) over a one-year horizon by choosing the default probability d, the premium loading k and initial capital subscribed from shareholders R. The premium loading and the initial capital determine the default probability d, which has an indirect impact on the EVA via the price-default risk demand curve $\phi(\pi, d)$. A complex optimization problem arises. For a given default probability, the combination of premium loading and capital that results in a higher EVA or EVA adjusted for the cost of

[9] Ganegoda and Bateman (2008) [35] estimate the loading on a nominal Australian life annuity for a 65 year old male in the general population to be approximately 24%. See Evans and Sherris (2009) [33] for an assessment of annuity demand in Australia.

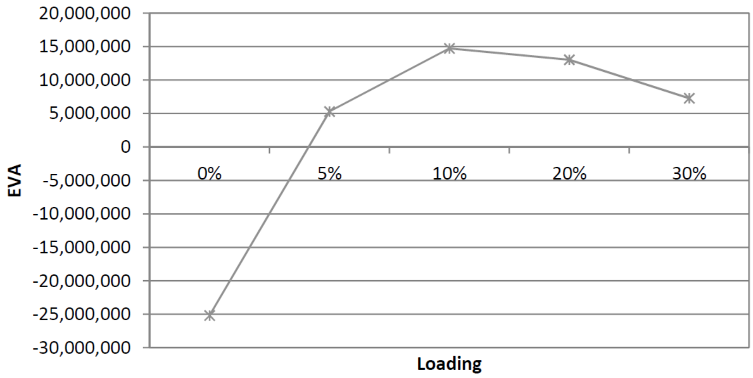


Figure 6: EVA for different combinations of premium loading and initial capital subscribed resulting in a solvency level of 99.5%.

capital is preferred by the shareholders. The optimal strategy is the combination of default probability d, premium loading k and initial capital R that gives the highest EVA or adjusted EVA value. This optimum is determined by comparing insurer profit and EVA for different combinations of d, k and R. For each combination, 100,000 simulations of the insurer model are used to estimate a profit and EVA distribution.

A given default probability, say d^*, can result from a number of different combinations of the premium loading k and initial capital R. These combinations are determined by considering different levels of the loading k in the range $k = 0, 5\%, 10\%, 20\%, 30\%$ and applying an iterative algorithm that determines for each k the initial shareholder capital R needed to achieve the target default probability d^*.

Different regulated environments are considered including a case where all insurers have the solvency probability of 99.5%, the Australian and European situation, and where insurers have flexibility to choose a target solvency level and credit rating, reflecting the situation in the US.

EVA for a Target Solvency Probability of 99.5%

Figure 6 shows the EVA for different combinations of premium loading and initial capital subscribed that provide a solvency probability of 99.5%. There is a hump-shaped relationship between premium loading and EVA reflecting the demand curve elasticity. The highest EVA value, for the loadings considered, occurs for a loading of 10%. The EVA adjusted for CoC occurs for higher loadings where the level of initial capital subscribed by shareholders is lower. At a zero loading EVA is negative because of the up-front expenses.

Table 6 shows the numerical results. The higher the premium loading k is, the less initial capital

Loading k	Capital R	Premiums P	$P/(P+R)$	Quantity Q	EVA	EVA adj CoC
0%	190.5	643.8	0.228	9,197	-25.2	-31.5
5%	90.9	399.9	0.185	5,441	5.3	2.3
10%	41.1	247.8	0.142	3,218	14.7	13.4
20%	5.9	94.6	0.059	1,126	13.0	12.8
30%	-0.9	35.9	*Note*	394	7.3	7.3

Table 6: Number of annuities sold (Quantity Q), EVA and EVA adjusted for CoC in millions for different combinations of premium loading and initial capital subscribed resulting in a solvency level of 99.5%. **Note: When the loading is high enough the solvency requirement is met wholly from policyholder loadings and no capital subscription is required from shareholders. This is the situation where the insurer would be structured as a mutual.**

R is required from the shareholders to attain the solvency level of 99.5%. At a loading of 30%, total premiums P are more than enough to ensure the target solvency probability of 99.5% and initial capital R is negative. In this case the policyholder loadings are sufficient to meet solvency requirements and no shareholder funds are required. In these cases, from a policyholder perspective, the insurer would be optimally structured as a mutual. Insurance demand, given by the number of annuities sold Q, decreases as the loading increases.

EVA for Varying Solvency Probabilities

A range of different one-year default probability is considered ($d = 0.1\%, 0.5\%, 1\%, 3\%$) and insurers are assumed to select the default level that maximises EVA. The default probabilities reflect AM Best ratings developed for the insurance sector (AM Best, 2007 [4]).

Figure 7 plots the EVA for different combinations of premium loading and initial capital subscribed resulting for each default probability. Higher solvency levels, or lower default probabilities, result in higher EVA values for positive premium loadings. For each case the highest EVA value is attained at a premium loading of 10%. At a default probability of 3% only very few annuity contracts are sold (for example, 755 at a zero loading and 447 at a loading of 10%). The adjusted EVA allowing for cost of capital implies a higher loading is optimal.

4.2 Robustness of Model Assumptions

The calibration of the demand curve and the assumption regarding the frictional costs for bankruptcy are the hardest to calibrate because of a lack of market data. These are important assumptions for our model. In order to confirm the robustness of our analysis, different assumptions are considered for their impact on maximum EVA.

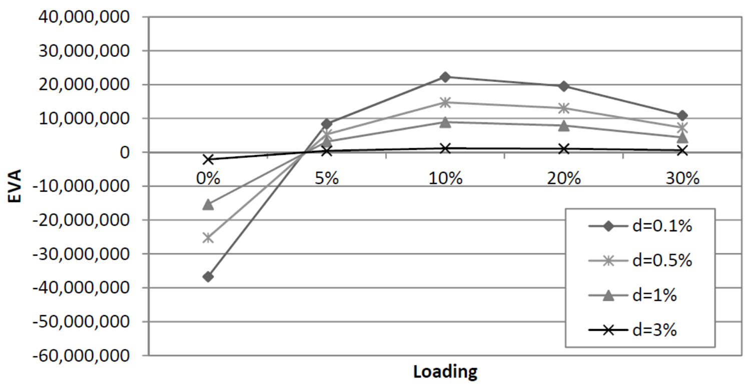


Figure 7: EVA for different combinations of premium loading and initial capital subscribed resulting for different solvency levels.

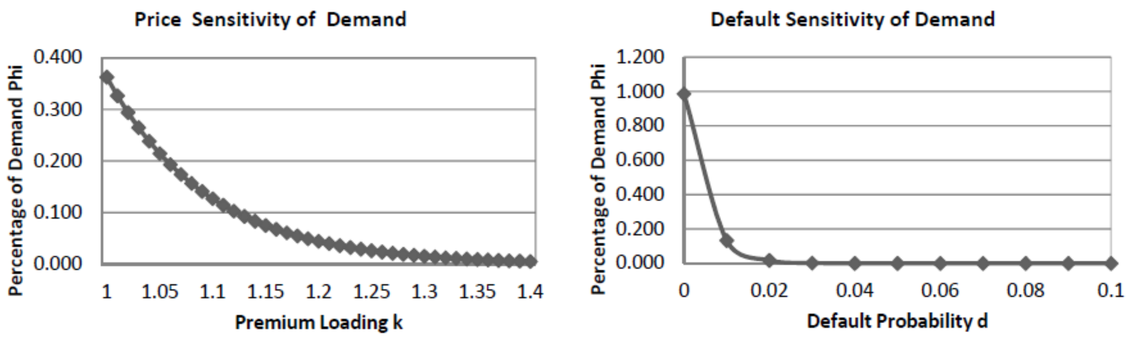
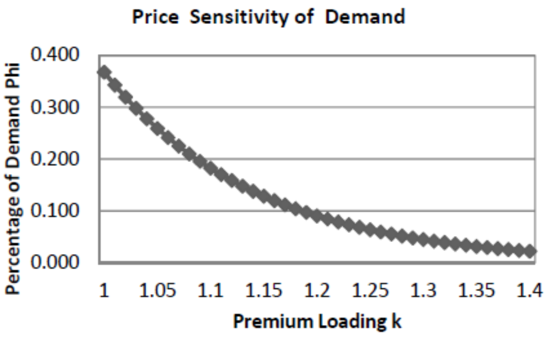


Figure 8: Sensitivity analysis: Policyholders being more default sensitive. Assumed price and default sensitivity of the demand for annuities.

Demand Elasticities

As a first case it is assumed that policyholders are more default risk averse than in the original case. The demand curve $\phi(\pi, d)$ is determined by three parameters, a default sensitivity parameter α, a price sensitivity factor β and a constant γ. The calibration for these parameters was $\alpha = -100$, $\beta = -0.00015$, $\gamma = 10$. Due to the exponential functional form of $\phi(\pi, d)$, changing the default sensitivity parameter α also affects the demand's reaction to price changes. To ensure separate effects, all three parameters are calibrated such that in the first case policyholders' demand reacts to price changes as before and only the reaction to default risk is increased. The parameters for this situation are $\alpha = -200$, $\beta = -0.00014997$, $\gamma = 10.48479301$ and the corresponding graphs for the demand function are shown in Figure 8. Annuity demand decreases more sharply with default risk than in the original case. At a default probability of 2% demand is basically zero, whereas in the original case demand was positive up until a default probability of 5%.

The second case assumes that policyholders are less price sensitive. All three parameters are



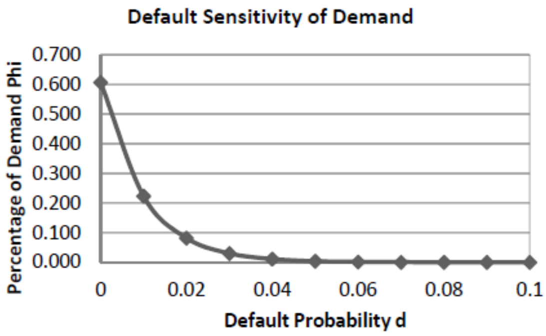


Figure 9: Sensitivity analysis: Policyholders being less price sensitive. Assumed price and default sensitivity of the demand for annuities.

calibrated such that only the demand reaction to price changes is changed (lowered) and the reaction to default is the same as in the original analysis. The corresponding parameters are $\alpha = -100.00625552$, $\beta = -0.00010000$, $\gamma = 6.49997956$. Figure 9 shows that the price sensitivity now runs flatter than in the original case and that there is a positive demand for the highest loading shown here (40%).

Figure 10 shows the EVA results for these alternative demand curves. As in Figure 7, default probabilities $d = 0.1\%, 0.5\%, 1\%, 3\%$ are compared.

The left graph is the case where policyholders are more default sensitive. As before, EVA shows a hump shape pattern for all four cases $d = 0.1\%, 0.5\%, 1\%, 3\%$ when loadings are increased from 0% to 30%. EVA is negative for a zero loading and for each default probability the highest EVA value occurs at a loading of 10%. The relative ranking is unchanged. EVA is highest for a default probability of $d = 0.1\%$. There are some differences to the original case. At a target default probability of $d = 0.1\%$, more annuities are sold and EVA is higher for loadings of 5%, 10%, 20% and 30%, and the loss at a zero loading is also higher than in the base case. At a default probability of $d = 0.5\%$, results are very similar to the original case. At $d = 1\%$ and 3%, less annuity business is sold and EVA is lower than originally.

The right graph is the case where policyholders are less price sensitive. As before, the relative ranking of the cases is the same. EVA is highest for a default probability of $d = 0.1\%$. However, the highest EVA value now occurs at a higher loading of 20%. Price sensitivity is an important determining factor for the level of the loading in the premium but not for the optimal solvency level.

The results show that the model's results are robust to assumptions regarding the policyholder's

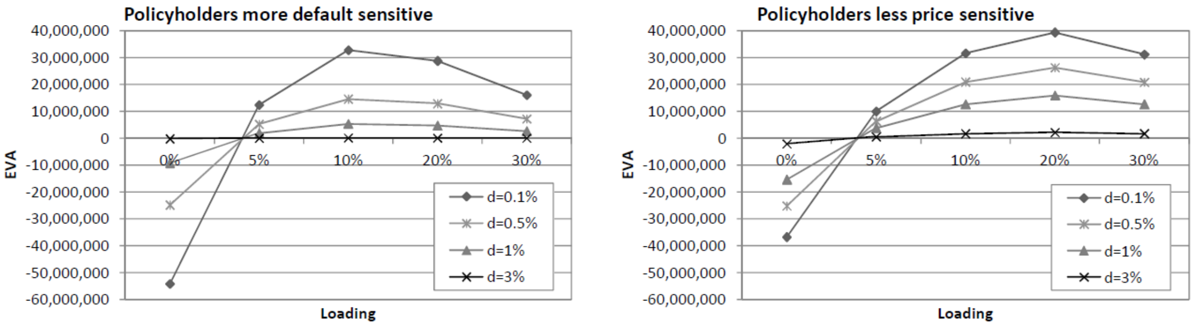


Figure 10: Sensitivity analysis: EVA levels for different assumptions about the demand curve $\phi(\pi, d)$.

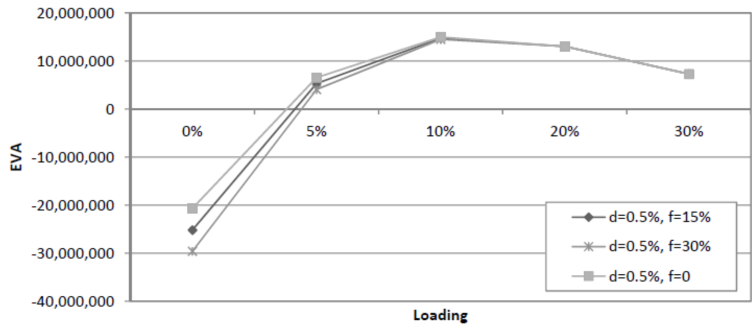


Figure 11: Sensitivity analysis: EVA levels for different frictional costs for bankruptcy f.

default sensitivity: EVA is highest for higher levels of solvency with lower default probabilities. The optimal loading depends on the policyholders' assumed price sensitivity but this does not impact on the conclusions for the solvency level. Similar conclusions follow for the adjusted EVA allowing for the cost of capital.

Frictional Costs for Financial Distress

Frictional costs for bankruptcy f were assumed to be 15% and applied to any losses. Optimal solvency levels for costs of 0% and 30% are consistent with the results for a 15% bankruptcy cost. The premium and demand are not dependent on the level of bankruptcy cost rate.

Figure 11 shows the sensitivity to different frictional costs. The percentage of these costs does not have a significant impact on the results. This is the case since financial distress only occurs with a low probability given the level of solvency assumed.

5 Discussion and Conclusion

An overview of current solvency requirements for lifetime guaranteed annuity issuers in the United States of America, the EU, the United Kingdom, Switzerland and Australia showed that the Australian and European systems are quite similar in structure, with a principles-based approach and a strong risk management focus. Regulatory regimes have moved to an insurer risk based capital system including prescribed levels of minimum solvency levels following Solvency II in Europe. The US system is less prescriptive.

The qualitative analysis of the regulatory regimes confirms how most regulatory regimes meet the assessment criteria. Principles-based approaches with a risk management focus, through risk management and governance standards, aimed at protecting policyholder interests and ensuring market stability are prominent. Minimum capital requirements and an internal model-based approach for calculating minimum capital requirements encourages an insurer to develop a better understanding of their risk profile and tolerance levels. A standard model, which is less risk sensitive, allows smaller insurers to assess capital required more efficiently and minimize regulatory compliance costs.

Market valuation of assets and liabilities provides an accurate view of the insurer's financial position from a solvency perspective. External auditors and appointed actuaries providing reports on financial conditions and general operations of the insurer are also valuable. A strong peer review process reduces the risk of misreporting. Requiring business strategies with a future focus as well as current risk management strategies to be presented to the regulator are positive factors. Onsite examinations and offsite analysis are an integral part of the supervisory process, allowing the regulator to gain an accurate perspective of an insurer's conditions, both qualitative, with respect to senior management and risk culture, as well as quantitative, regarding required capital levels and financial condition situation.

There are some limitations that must be considered. Factor-based calculations for more complex risks reduce the risk sensitivity of the system as a whole. Incorporating complicated yet risk sensitive methods of calculation may be counterproductive in these situations. Qualitative assessments of these risks, as in Solvency II, or scenario analysis, as in the SST, in addition to factor-based calculations increase risk sensitivity. The SST's scenario analysis has proven to be a very effective and efficient method of incorporating catastrophe risk into a prudential system's framework. Predetermined scenarios, such as increases and decreases in mortality alongside various market conditions, are a relatively straightforward way of doing this.

An issue that is an important limitation of the risk based solvency regimes is that all insurers use similar internal models leading to the potential for more severe impacts from systemic risks. One of the main restrictions for the increased use of internal models in industry is the need for regulatory approval. 'Own Risk Self Assessments' as used in Solvency II, would allow insurers to gain a better understanding of internal and industry wide risk management, in turn alleviating some of the impact from systemic risks.

Ensuring that product issuers will be able to deliver on long term life annuity contracts with a very high degree of certainty is very important to both regulators and consumers. This is a fundamental requirement to support the development of a viable private sector annuity market. The quantitative analysis of the optimal level of solvency, which balances the regulatory trade-off between prudential concerns and consumer attitudes towards purchasing annuities, showed that higher solvency levels maximized shareholder wealth and also satisfied consumer preferences for solvency.

The results were robust to different levels of default sensitivity. The main impact of price-default elasticity was on the optimal loading in the premium that maximized the shareholder value and not the solvency capital level. Levels of capital determined using a 99.5% confidence level for a one year time horizon were not optimal for life annuities. Higher levels of solvency were found to be optimal for a life insurer based on reasonable assumptions for consumer preferences for solvency.

Pillar 3 of the ICAS system and Solvency II focus on increasing market transparency through mandatory public disclosure of insurers' financial and prudential situations. In a number of regimes, including Australia, there is no explicit prudential regulation which mandates public disclosure of financial reports. Public disclosure of the financial health of insurers should be mandated, especially for long term business. Although high compliance costs can disadvantage smaller insurers, this will be in the interest of policyholders if it results in more confidence in life insurers when purchasing long term contracts such as life annuities.

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