Using the market implied risk aversion to value all risk

Robert Baillie

The goal

- Propose a general approach for valuing any set of uncertain future profits (Z)
- Value = $\Sigma m.z.P(z)$ (or value = $E(mZ)$)
- m = some set of stochastic discount factors

The key criteria

- useful for enterprise risk management
- the value of any financial risk needs to be consistent with observable market values for similar risks

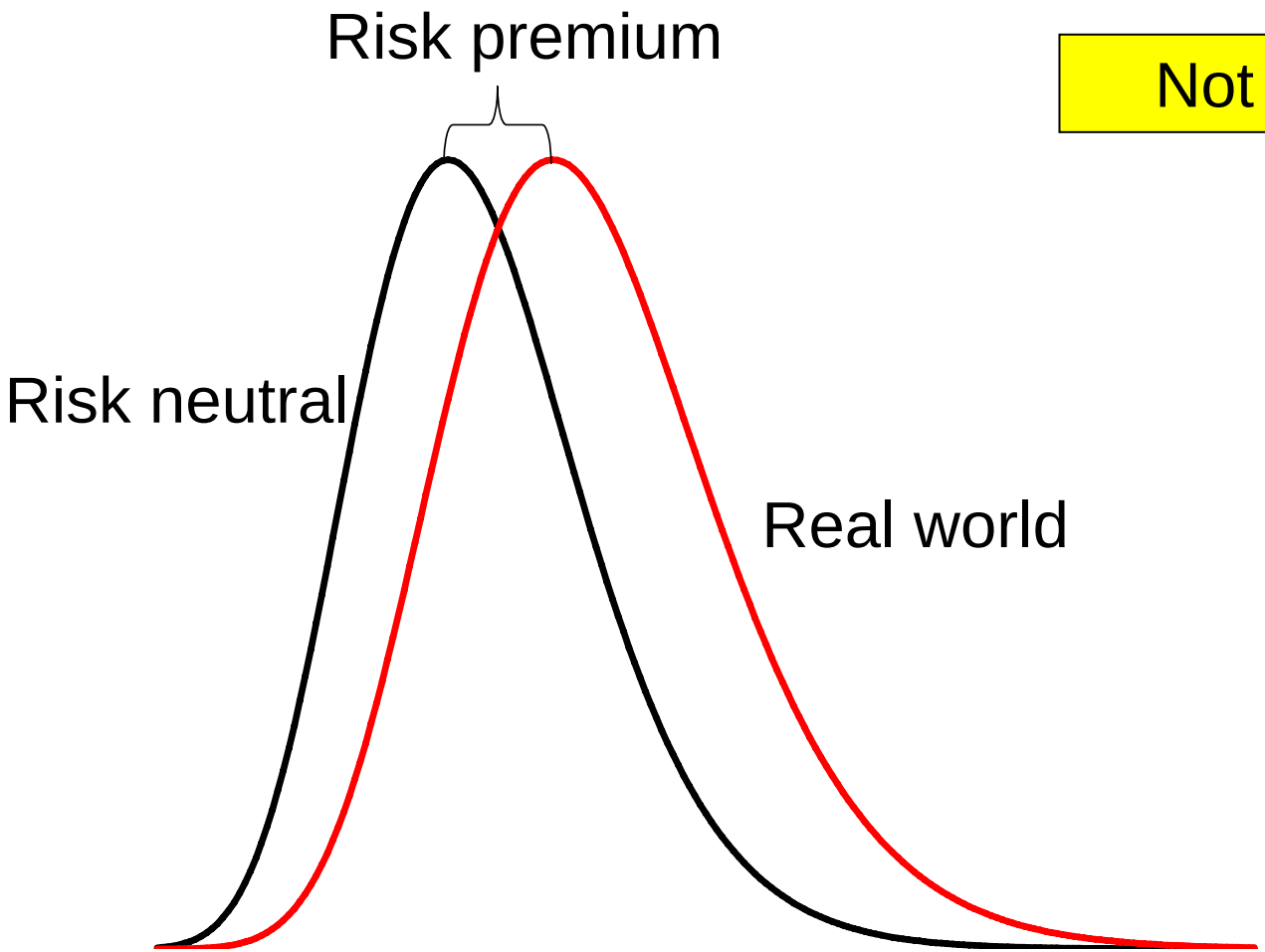
Market consistent values for financial risks

- Z_x = profit if market is at x
- Value of a replicating portfolio
- Value = $\int Z_x v Q(x) dx$

Properties of Q

- Q is a risk neutral distribution
- vQdx = value of an asset that pays 1 when the market is between x and x+dx
- this asset can be constructed using options
- vQ(x) is the second derivative of the option price with respect to the strike price

Risk neutral (Q) and real world (P)



Stochastic discount factors

Risk neutral

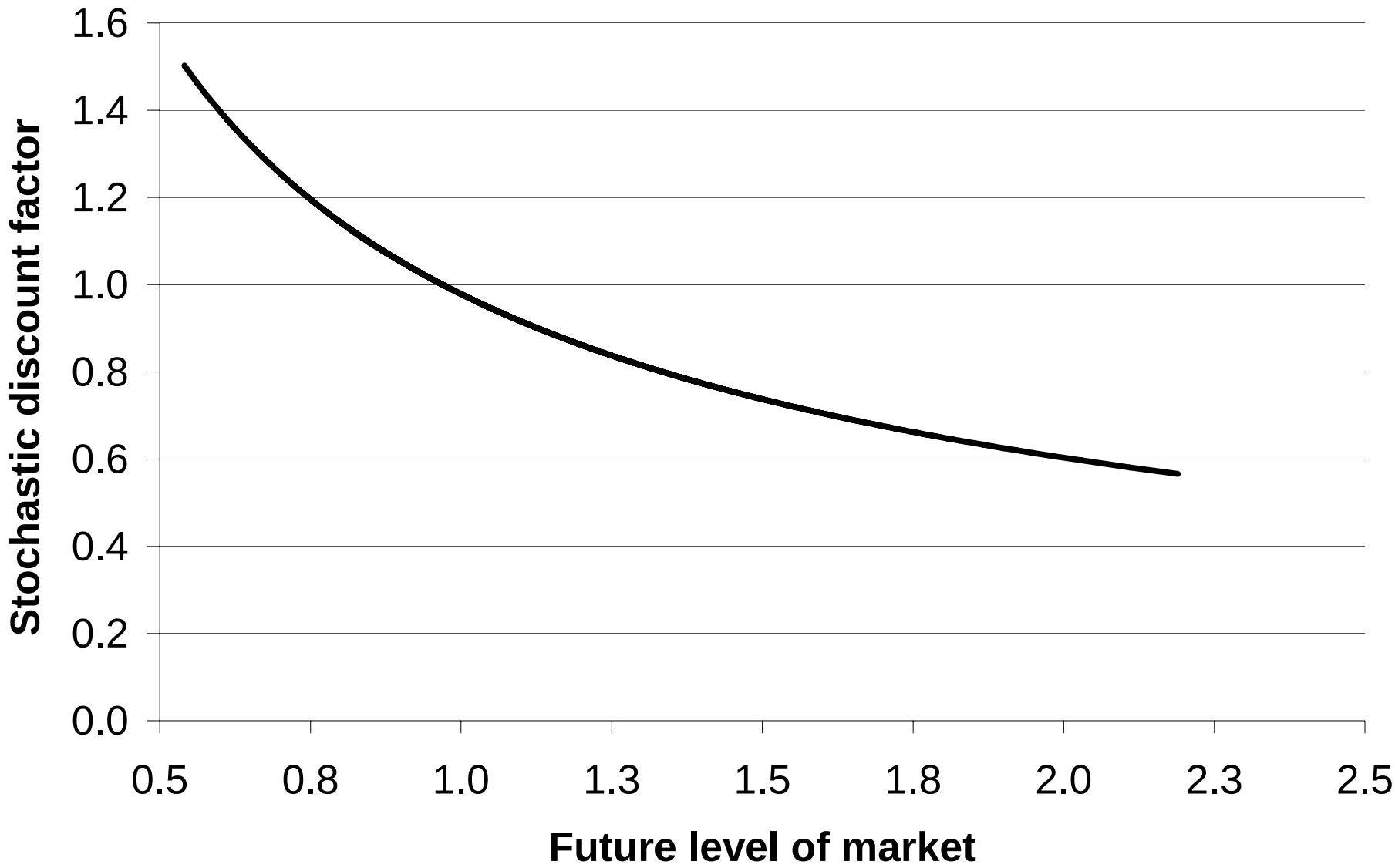
$$\text{value} = \int Z_x v Q(x) dx$$

Real world

$$\text{value} = \int Z_x m_x P(x) dx$$

$$vQ(x) = m_x P(x)$$

$$m_x = \frac{Q(x)v}{P(x)}$$


Stochastic discount factor
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
0.5
0.8
1.0
1.3
1.5
1.8
2.0
2.3
2.5
Future level of market

Link with utility theory

- For financial risks, stochastic discount factors are proportional to the marginal utility of the optimal market portfolio.

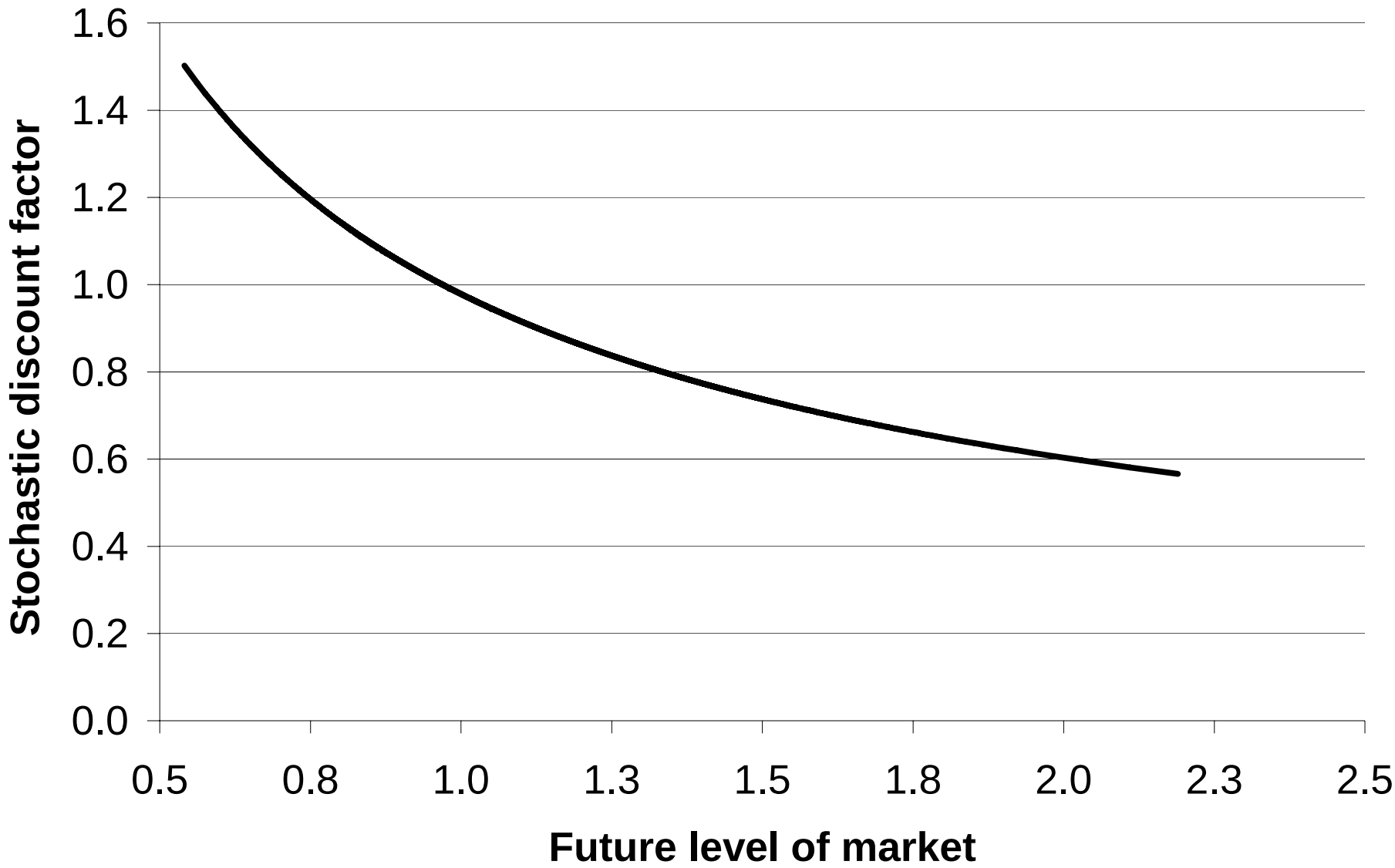
Risk aversion

- Absolute risk aversion

$$-\frac{U''(w)}{U'(w)} \quad \left(\text{or } -\frac{m'(x)}{m(x)}\right)$$

- Relative risk aversion

$$-\frac{U''(w)}{U'(w)}.w \quad \left(\text{or } -\frac{m'(x)}{m(x)}.x\right)$$


Stochastic discount factor
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
0.5
0.8
1.0
1.3
1.5
1.8
2.0
2.3
2.5
Future level of market

What it takes to apply market implied risk aversion to all risk

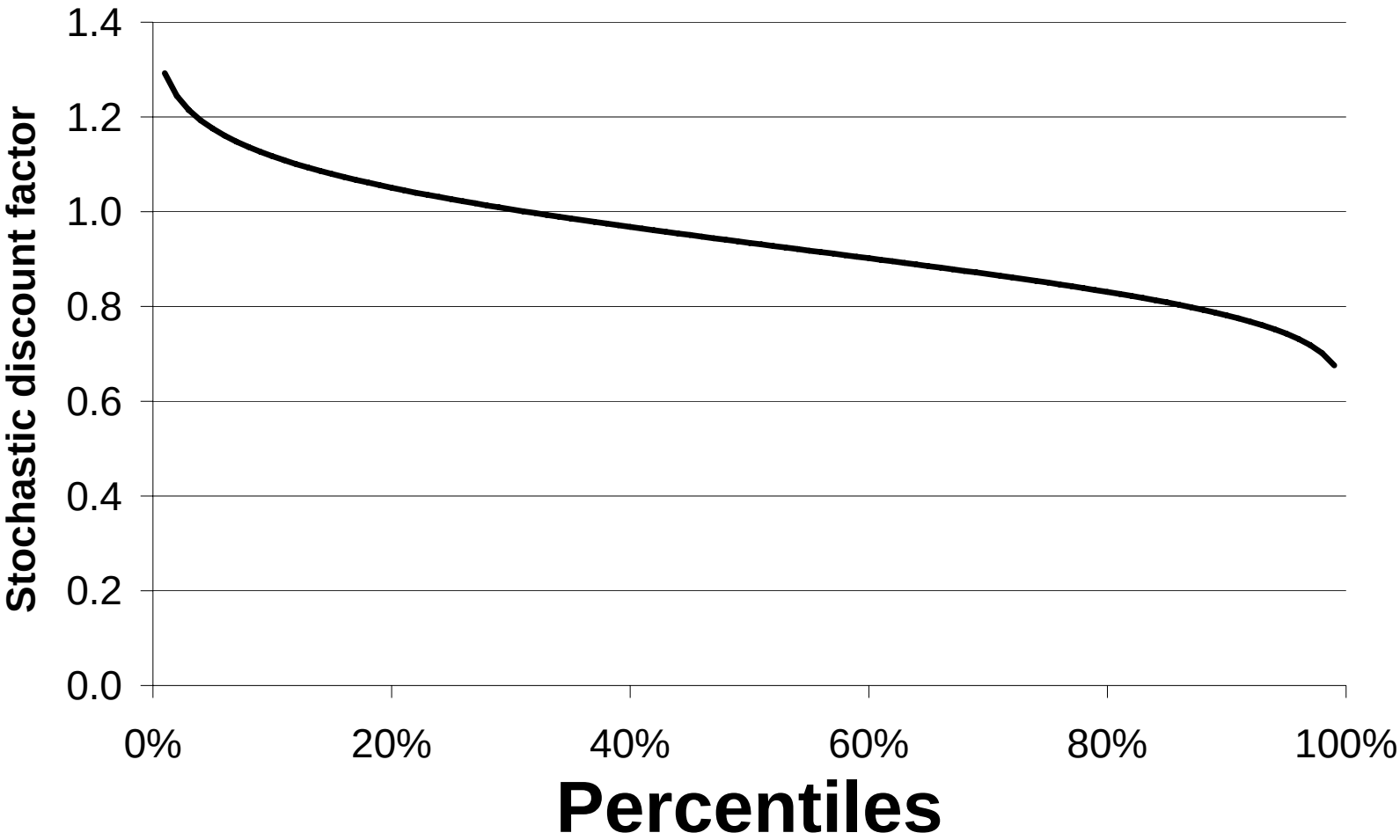
- Operating within uncertainty
- MCVs and market capitalisations
- Enterprise risk management

Enterprise risk management

The Casualty Actuarial Society defines ERM as

"the process by which organizations in all industries assess, control, exploit, finance, and monitor risks from all sources for the purpose of increasing the organization's short and long term value to its stakeholders."

Company level factors



Applications

- Risk adjusted values = E(mZ)
- Under simple assumptions: Cost of capital and recovering Black Scholes
- Company valuations / value of all risk
- ERM / diversification
- Risk adjusting historical returns
- Equivalent risk portfolios

Limitations

- Agency costs
- Not arbitrage free
- Deriving the market's view of the real world distribution
- Market and company risk aversion
- Recovering the value of financial risks
- Extension to multi-periods
- Not good for extreme risks

Next steps