Mortality Forecast: Local of Global?

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Presented to the Actuaries Institute
ASTIN, AFIR/ERM and IACA Colloquia
23-27 August 2015
Sydney

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Future mortality projection is of fundamental importance to insurance companies, pension providers and government welfare systems as accurate mortality forecasts are required for determining the right amount of insurance premiums, pension benefits and contribution rates. Hence, there is an increasing need to have a better understanding of mortality in order to increase the accuracy of future mortality forecasts.

In the literature of mortality modeling, many attempts have been made to project future mortality rates using different types of models (Booth and Tickle, 2008). Most of these models tend to identify patterns in age, time or cohort dimensions in the mortality data and extract these patterns to make projections on future mortality rates (see for example Lee and Carter, 1992; Cairns et al., 2006). There have also been a number of studies on the comparison of forecasting performances of different models (Cairns et al., 2011; Haberman and Renshaw, 2009; Hyndman and Ullah, 2007), and the quantitative and qualitative criteria used include: the overall accuracy; allowance for cohort effect; biological reasonableness; and the robustness of forecast. However, as far as we know, very few studies actually consider the differences in the nature of these mortality forecasts. For example, whether they treat past and recent mortality experience equally in the forecasting process.

We argue that mortality forecast can be divided into two categories: one uses local information (i.e. give more weights to recent mortality data) and the other uses global information (i.e. give equal weights to past and recent mortality data). Examples of models falls in the first category include the $P$-splines model by Currie et al. (2004), the two-dimensional thin plate model (Dokumentov and Hyndman, 2014) and the two-dimensional kernel smoothing (2-D KS) model (Li et al., 2015c). Examples of models using local information to produce mortality forecast include the well-known Lee-Carter model (1992), the Cairns-Blake-Dowd (CBD) model (2006) and the two-dimensional Legendre orthogonal polynomial (2-D LOP) model proposed by Li et
Therefore, one of the primary interests of this paper is to look at the question of whether local or global information is more appropriate to use and thus should be preferred in the mortality forecasting process. In this study we include two pairs of mortality models for comparison. In order to control the number of factors that will influence the forecasting performance of mortality models, in each group apart from this difference in the forecasting approach (one model mainly uses local information and the other uses global information), the design and structure of the two models bear great similarity.

In this paper, a detailed study is conducted using male mortality data of Great Britain from 1950 to 2009 for ages 50-89. Based on the empirical results from a multi-year-ahead backtesting exercise, we compare and comment on the differences in the forecasting performances across the two groups of models and conclude that local information is more relevant to produce accurate mortality forecast. Further, the robustness test shows that all four models included in the analysis appear to be reasonable robust relative the changes in the length of historical data employed in the estimation. Future work not included in this paper includes widening the countries considered and the models employed.

The plan of the paper is as follows. Section 2 reviews and provides details of the models to be compared in this study. In Section 3 we conduct a case study and comment on the forecasting performances of the models described in Section 2. Finally, Section 5 draws the conclusions and also gives future research directions.

2. MODELS FOR COMPARISON

This section begins by defining some actuarial notation in the mortality modeling literature which we are going to use throughout the paper. Models being compared in our study includes the original CBD model (Cairns et al., 2006) and its local linear
approach proposed by Li et al. (2015a), the 2-D LOP model (Li et al., 2015b) and the 2-D KS model (Li et al., 2015c). The details of the four models are then reviewed and discussed. We divide these models into two groups based on the similarity in their model designs and inclusion or not of cohort effect.

2.1. **Notation.** Recent mortality studies generally model two types of mortality rates: central mortality rates and initial mortality rates. For $x \in [a + 1, a + N]$, and $t \in [1, T]$, where $a$, $T$ and $N$ are non-negative integers, define

- $d_{x,t}$ as the observed number of deaths in calendar year $t$ aged $x$.
- $e_{x,t}$, the exposure data which measures the average population in calendar year $t$ aged $x$.
- $m_{x,t}$ as the central mortality rate, which reflects the death probability for age $x$ in the middle of the year. It is calculated by

\begin{equation}
    m_{x,t} = \frac{d_{x,t}}{e_{x,t}}.
\end{equation}

- $q_{x,t}$ as the initial mortality rate, which is the one-year death probability for a person who is aged exactly $x$ at time $t$.

Numerically, the central mortality rate and initial mortality rate are quite close in values. The approximation formula for the two types of mortality rates is as follows:

\begin{equation}
    q_{x,t} = 1 - \exp(-m_{x,t}).
\end{equation}

2.2. **CBD model and a local linear approach.** In the new era of stochastic mortality modeling, the CBD model is considered as a strong contender among different stochastic models. It was introduced by Cairns et al. (2006) and the model is of the form:

\begin{equation}
    \logit(q_{x,t}) = \log\left(\frac{q_{x,t}}{1 - q_{x,t}}\right) = \kappa_1^t + \kappa_2^t(x - \bar{x}) + \epsilon_{x,t},
\end{equation}

where $\kappa^1_t$ and $\kappa^2_t$ are time-related effects and $\bar{x}$ is the average age in the sample range.

The parameters in the CBD model can be estimated using the maximum likelihood estimation (MLE) approach proposed by Brouhns et al. (2002) based on a Poisson error structure. It provides good fitting and forecasting results for a number of countries’ mortality experience and since there are multiple factors in the model, it has a non-trivial correlation structure (Cairns et al., 2006). For the purpose of future mortality projection, $(\kappa^1_t, \kappa^2_t)$ is normally treated as a bivariate random walk with drift process. Define $\beta_t = \begin{pmatrix} \kappa^1_t \\ \kappa^2_t \end{pmatrix}$, we have

$$
\beta_t = \beta_{t-1} + \mu + C Z_t,
$$

where $\mu$ is a column vector of the drift factors and $Z_t$ is a column vector of independent standard normal random variables. $C$ is an upper triangular matrix by Cholesky decomposition of the variance-covariance matrix of $\kappa^1_t$ and $\kappa^2_t$. The drift factors and variance-covariance matrix can be estimated from the MLE estimators $\hat{\kappa}^1_t$ and $\hat{\kappa}^2_t$.

Thus the one-year-ahead forecast of $\beta_t$ is given by

$$
\beta_{T+1} = \beta_T + \hat{\mu} + \hat{C} Z_{T+1}.
$$

Li et al. (2015a) explored the possibility of $\kappa^1_t$ and $\kappa^2_t$ being smooth functions of time and introduced a local linear estimation (LLE) and forecasting approach to the model. The CBD model is re-expressed as a semi-parametric time-varying coefficient model as we define, for $x \in [a+1, a+N]$ and $t \in [1, T]$,

- $Y_{it} = \logit(q_{i,t})$ and $\epsilon_{it} = \epsilon_{x,t}$, where $i = x - a$ denotes age groups.
- $X_i = \begin{pmatrix} 1 \\ x - \bar{x} \end{pmatrix}$.
- $\beta_t = \begin{pmatrix} \kappa^1_t \\ \kappa^2_t \end{pmatrix}$ where $\kappa^1_t$ and $\kappa^2_t$ are smooth functions of $t$. 


Thus the model becomes

\[ Y_{it} = \logit(q_{x,t}) = \kappa_1^t + \kappa_2^t(x - \bar{x}) + \epsilon_{x,t} = X_i't\beta_t + \epsilon_{it}. \]

Following earlier studies of time-varying coefficients by Robinson (1989) and Cai (2007), we define \( \beta_t = \beta(\tau) \), where \( \tau = t/T \) and \( t \in [1, T] \). Thus by Taylor expansion, for any given \( \tau_0 \in [0, 1] \), the model can be approximated by

\[ Y_{it} \approx X_i'[\beta(\tau_0) + \beta^{(1)}(\tau_0)(\tau - \tau_0)], \]

where \( \beta^{(1)}(\tau_0) \) is the first order derivative at \( \tau_0 \).

The local linear estimator of \( \beta(\tau_0) \) can be obtained by minimizing the following weighted sum of squares with respect to \( (\beta(\tau_0), \beta^{(1)}(\tau_0))^2 \):

\[ \sum_{i=1}^{N} \sum_{t=1}^{T} \{Y_{it} - X_i'[\beta(\tau_0) + \beta^{(1)}(\tau_0)(\tau - \tau_0)]\}^2 K_h(\tau - \tau_0), \]

where \( K \) is the kernel function which determines the shape of kernel weights and \( K_h(u) = h^{-1}K(u/h) \). \( h \) is the bandwidth which determines the size of the weights. Following Li et al.’s investigation, in this paper we use leave-one-out cross-validation as the bandwidth selection algorithm and adopt the Epanechnikov kernel function as follows

\[ K(u) = 0.75(1 - u^2)I(|u| \leq 1). \]

One of the strengths of this approach relies on the fact that the projection of future mortality rates can be done simultaneously with fitting in the estimation process. In order to produce a one-year-ahead mortality forecast, we redefine \( \beta_t = \beta(\tau) \), where \( \tau = t/(T + 1) \) and let \( \tau_0 = \frac{T+1}{T+1} = 1 \). Solve the minimization problem in equation (8) and we then obtain the one-year-ahead forecast for \( \beta_{T+1} \).

\[ \text{For detailed explanation of the method and matrix expression of the local linear estimator please refer to: Li, H., O’Hare, C., Zhang, X., 2014. A semiparametric panel approach to mortality modeling. Insurance: Mathematics and Economics 61, 264-270.} \]
2.3. **2-D LOP model and 2-D KS model.** Including the CBD model, most of the recent mortality models make certain assumptions on the age, time or cohort structure of the mortality surface. Li et al. (2015b) proposed a flexible functional form approach to mortality modeling through the introduction of 2-D LOP model. The model is defined as follows:

\[
\log(m_{x,t}) = \sum_{m=0}^{6} \sum_{n=0}^{6} \beta_{m,n} \varphi_{m,n}(x, t) + \epsilon_{x,t},
\]

where \( \varphi_{m,n} \) is the two-dimensional Legendre polynomial as described in Mádi-Nagy (2012) and \( \beta_{m,n} \) is the coefficient for the \( (m, n)^{th} \) polynomial. The maximum order of polynomials used in this paper is 6 which is consistent with Li et al.’s study (2015b). As the interval of orthogonality for Legendre polynomials is \([-1, 1]\), in this model we need to normalize the \( x \) and \( t \) indexes into the range of \([-1, 1]\) first. Least absolute shrinkage and selection operator (Lasso) is used as the regularization tool in the model selection procedure. For the selection of tuning parameter in Lasso estimation, we follow the study of Li et al. (2015b) and use 10-fold cross-validation method.

The one-year-ahead mortality forecast can then be obtained as

\[
\log(m_{x,T+1}) = \sum_{m=0}^{6} \sum_{n=0}^{6} \hat{\beta}_{m,n} \varphi_{m,n}(x, T + 1),
\]

where \( \hat{\beta}_{m,n} \) is the Lasso estimator of \( \beta_{m,n} \).

Unlike some existing stochastic mortality models, the 2-D LOP model does not impose any restrictions on the functional form of the model and thus allows us to tailor our model design according to different countries’ specific mortality experience. Based on the empirical study by Li et al. (2015b), it is concluded that the model provides comparable fit quality and better forecasting performance for a range of developed countries comparing to some widely used mortality models in the literature.
Moreover, it has been shown that the 2-D LOP model can capture cohort effect adequately by including interactions between age and time effect into the model via the use of two-dimensional polynomials of \( x \) and \( t \).

According to Härdle (1990), any modeling process involving the use of prespecified parametric functions is subject to the problem of “misspecification” which may result in high model bias. Nonparametric techniques, on the other hand, are more flexible and data-driven and thus could provide a more general approach to mortality modeling (Currie et al., 2004). Li et al. (2015c) proposed a mortality model which implements two-dimensional kernel smoothing techniques to mortality surfaces. The 2-D KS model is of the form:

\[
\log(m_{x,t}) = \beta(x, t) + \epsilon_{x,t},
\]

where \( \beta(x, t) \) is any unknown functions of \( x \) and \( t \). Without loss of generality we normalize the \( x \) and \( t \) indexes into interval \([0, 1]\) (Härdle, 1990). The kernel smoother at any given \( x_0 \in [0, 1] \) and \( t_0 \in [0, 1] \) can be obtained by solving the minimization problem as follows,

\[
\hat{\beta}(x_0, t_0) = \arg \min_{\beta(x_0, t_0)} \sum_x \sum_t [\log(m_{x,t}) - \beta(x_0, t_0)]^2 K_{h_1, h_2}(x - x_0, t - t_0),
\]

where kernel function \( K \) determines the shape of kernel weights and \( K_{h_1, h_2}(x - x_0, t - t_0) = K\left(\frac{x-x_0}{h_1}, \frac{t-t_0}{h_2}\right) \). While the size of the weights is set by \( h_1 \) and \( h_2 \), which are bandwidths in age and time dimension respectively. Again, following Li et al. (2015c), we adopt the bivariate standard normal kernel function with correlation \( \rho \in (0, 1] \) in order to capture the dependence in age and time dimensions, in other words, the cohort effect. The bandwidths and correlation parameter \( \rho \) will be selected based on the out-of-sample mean square forecast error\(^3\).

It has been shown that the model produces satisfactory fitting and forecasting results and it also incorporates cohort effect into both estimation and forecasting processes. To obtain one-year-ahead mortality forecast, since the time index extends to $T + 1$, we first adjust $t$ back into $[0, 1]$ and set $t_0 = 1$. Then the kernel estimator at $T + 1$ can be computed by solving the minimization problem in equation (12).

2.4. **A discussion on the two groups of mortality models.** A common feature in this two groups of models is that, in each group the two models are of similar designs in the structure but one mainly uses local information in the forecasting procedure and the other uses global information. In the first group, apart from different estimation and forecasting methodologies, the age and time structures of the two models are exactly the same. The random walk (RW) forecast uses global information to produce mortality forecast since when estimating the drift factor $\mu$, past and recent mortality experience are equally treated. While on the other hand, based on the weight function described in Section 2.2, the local linear (LL) forecasting method will set greater weights on most recent mortality data and lighter weights on historical mortality data. Similarly, in the second group, the two models show clear similarities in the model design as they both assume smoothness in age and time dimensions. The difference is: the parameters in the 2-D LOP model are estimated based on the entire mortality surface and thus the model produces mortality forecast using global information; while the 2-D KS model mainly uses local information to project future mortality rates since the kernel function assigns greater weights on recent mortality experience in the forecasting process.

The main differences between the two groups of models are: firstly, both the 2-D LOP model and the 2-D KS model are data-driven without prior assumptions on age, time or cohort structure of the underlying data; however, the CBD model in the first group imposes restrictions on structures in age and time dimensions. Secondly, the CBD model does not allow for cohort effect in its model design while both of the two models in the second group tend to capture cohort effect using either a parametric or
Finally, it is worth noting that, the 2-D KS model is the only model that is free of the “misspecification” problem mentioned earlier in this section as it does not involve any parametric functions in the modeling process and uses a pure nonparametric approach. While both the CBD model and the 2-D LOP model are subject to the problem of “misspecification” since both of them contain parametric components.


This section starts with a description of the mortality data used in the case study. Then we will assess the fit quality of the four models based on both statistical measures and the randomness in the residual plots. To comment on the short, medium and long term forecasting performances of the models, properly constituted backtesting has been carried out and the forecasting results for various forecast horizons are presented and compared. The robustness of the mortality forecast relative to the period of data employed is also tested for each of the four models. We choose different investigation period to see how sensitive the forecasting performance of each model is to the length of historical data used to fit the model.

3.1. Data. The data set used in this study is: male mortality data of Great Britain (GB) during 1950-2009 for age range 50-89. Even though longer historical data is also available, we choose to use mortality data in this post-war time period because we believe that the data is of good quality and more reliable. Since our primary interest is to improve the forecast accuracy for older ages - to which longevity risk is more exposed, in this study we use age range from 50 to 89. This is consistent and in line with other studies in the literature (see for example Cairns et al., 2009, 2011; Dowd et al., 2010). The deaths and exposures data used to calculate central mortality rates and initial mortality rates is downloaded from the Human Mortality Database (HMD)\(^4\).

\(^4\)The HMD mortality database can be found at [http://www.mortality.org](http://www.mortality.org)
3.2. **Fit quality and residual plots.** Following the investigations of O’Hare and Li (2012) and Li *et al.* (2015a), we define the following statistical measures:

- The average error ($E_1$), which is a measure of overall bias, is calculated as
  \[
  E_1 = \frac{1}{NT} \sum_x \sum_t \frac{\hat{m}_{x,t} - m_{x,t}}{m_{x,t}}.
  \]

- The absolute average error ($E_2$), which measures the absolute size of the deviance, is calculated as
  \[
  E_2 = \frac{1}{NT} \sum_x \sum_t \frac{|\hat{m}_{x,t} - m_{x,t}|}{m_{x,t}}.
  \]

- The standard deviation of error ($E_3$), which is an indicator for large deviance, is calculated as
  \[
  E_3 = \sqrt{\frac{1}{NT} \sum_x \sum_t (\frac{\hat{m}_{x,t} - m_{x,t}}{m_{x,t}})^2}.
  \]

Table 1 illustrates fitting results of the four models using the mortality data described in Section 3.1. It can be concluded that, the 2-D KS model gives the best fitting results among the four models on all three measures for GB male mortality data for the period 1950-2009, and ages 50-89. The 2-D LOP provides a slightly worse quality of fit but the results are still comparable with the 2-D KS model. Overall, the fitting results from the second group are better than those from the CBD model for both the MLE method and the LLE method. On average, the fit quality in the second group is between 1-1.5% points better on $E_1$, $E_2$ and $E_3$ measures. This result is not surprising as the structure of the CBD model is relatively simple and it does not incorporate cohort effect.

<table>
<thead>
<tr>
<th></th>
<th>CBD model: MLE</th>
<th>CBD model: LLE</th>
<th>2-D LOP model</th>
<th>2-D KS model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_1(%)$</td>
<td>1.01</td>
<td>-0.16</td>
<td>0.06</td>
<td>0.08</td>
</tr>
<tr>
<td>$E_2(%)$</td>
<td>3.95</td>
<td>4.22</td>
<td>2.54</td>
<td>2.21</td>
</tr>
<tr>
<td>$E_3(%)$</td>
<td>5.40</td>
<td>5.27</td>
<td>3.32</td>
<td>2.98</td>
</tr>
</tbody>
</table>

**Table 1.** Fitting results for GB male mortality data from 1950 to 2009, ages 50-89.
Whilst all the models provide a reasonably good fit, it is worth noting, as in a recent study on mortality model comparisons by Cairns et al. (2011) that a good fit to historical data does not ensure good forecasting performances. In addition, Li et al. (2015b) argued that the “cleanness” of residual plots should also be taken into account when assessing the fitting quality of mortality models since we need to make sure that no useful information in the mortality surface is left unexplained. A check of the residual plots for the four models included in this analysis is done first before we move onto future mortality projection. The residuals are plotted below.

![Residual plots for (a) CBD model: MLE, (b) CBD model: LLE, (c) 2-D LOP model, (d) 2-D KS model based on GB male mortality data from 1950 to 2009, ages 50-89.](image)

**Figure 1.** Residual plots for (a) CBD model: MLE, (b) CBD model: LLE, (c) 2-D LOP model, (d) 2-D KS model based on GB male mortality data from 1950 to 2009, ages 50-89.

It is clear in Figure 1 that, several cohort trends can be observed from the residual plots of the first group of models. Both the MLE approach and LLE approach to the
CBD model produce a residual plot with certain diagonals exhibiting strong clusterings of positives and negatives. Also, we can see that the cohort patterns seem to be stronger in the residual plot from the LLE approach. While on the other hand, the residual plots of the second group look sufficiently random compared to the first group. In particular, the residual plot of the 2-D KS model seems to be free of diagonal patterns except for one or two cohorts with systematically higher or lower mortality rates. Further, the colormaps shown in these residual plots agree with the fitting results on $E_3$ measure which indicate that there are more large deviances from the CBD model than the 2-D LOP model and the 2-D KS model. In the next section we will see how these patterns in the residual plots will affect the forecast ability of mortality models.

3.3. **Comparison of forecasting performance.** In this section a series of backtesting exercises are conducted for different forecast horizons based on mortality data since 1950. Since we want to ensure that there is sufficient length of historical data to fit the models, in this paper we consider 3, 5 and 8 years forecasting horizons reflecting short, medium and long term mortality forecast for both groups of models. The forecasting results are presented in Tables 2 and 3.

We first compare the performances of the two forecasting approaches to the CBD model. From Table 2, it can be seen that overall, the accuracy of the local linear forecast is better than the random walk forecast on all three statistical measures and for all different forecast horizons. Smaller absolute values of $E_1$ also indicate that the local linear forecast is less unbiased. The local linear approach performs particularly well on medium to long term forecast horizons: the 5-year-ahead and 8-year-ahead mortality forecast is around 2% points better than the forecast from random walk approach on $E_1$, $E_2$ and $E_3$ measures.

Table 3 shows the forecasting results from the 2-D LOP model and the 2-D KS model. It can be seen from these results that the 2-D KS model provides more accurate mortality forecast for all three forecast horizons compared to the 2-D LOP model. One
interesting fact is that, the 2-D KS also performs particularly well on longer forecast horizons. For example, the 8-year-ahead forecast error from the 2-D KS model is only about half of the forecast error from the 2-D LOP model.

As both the local linear approach and the kernel smoothing approach give greater weight to the most recent mortality data, it can be summarized from the two sets of forecasting results that local information is more relevant and appropriate to use when making future mortality projections. Intuitively, this is not difficult to understand as recent mortality experience would obviously have great predictive power than historical mortality experience. Since both the random walk approach and the 2-D LOP approach give equal weight to past and recent mortality data and assume the long-term pattern found in the past will continue in the future, there is an increased risk that less relevant or “out of date” information will be taken into account and thus affect the overall accuracy of the mortality projection.

Moreover, when comparing the forecasting performance across the two groups, we observed that the forecasting results from the second group are generally better. As mentioned earlier, both the 2-D LOP model and the 2-D KS model are data-driven and impose no restrictions on age, time and cohort structures of the mortality data.
Also, both of the models incorporate cohort effect in their model design which is reflected in the clear residual plots shown in Section 3.2. This could possibly explain why the forecast from the second group outperforms that from the first group.

A comparison of the 3, 5 and 8-year-ahead mortality forecasts for the four models against the real mortality experience for GB male aged 50, 60, 70 and 80 are illustrated in Figure 2-4. It can be seen from these plots that the 2-D KS model outperforms the other three models in the majority of circumstances. This is consistent with conclusions we drawn earlier based on the statistical measures.

**Figure 2.** 3-year-ahead forecast from 2007-2009 for GB male aged (a) 50, (b) 60, (c) 70 and (d) 80 based on mortality data since 1950.

**Figure 3.** 5-year-ahead forecast from 2005-2009 for GB male aged (a) 50, (b) 60, (c) 70 and (d) 80 based on mortality data since 1950.
3.4. **Robustness of projections.** It has been claimed that in some cases mortality forecast can be sensitive to the length of historical data employed in the modeling process (Denuit and Goderniaux, 2005). According to Cairns *et al.* (2011), robustness of the forecast relative to the sample period used to calibrate the model is one of the desirable properties of mortality models. Therefore, in this section we examine the robustness of forecast for all four models by changing the starting time of the investigation period to 1970. Same backtesting techniques have been applied and comparison and comments are made based on the differences in forecasting results. We illustrate these results in Tables 4 and 5.

It can be seen from Table 4 that, for both random walk approach and local linear approach, the change in the length of historical data employed in estimation process seems to have a minor influence on the mortality projection. Compared to Table 2, the differences in the short and medium term forecasting results are modest. While the results show that the 8-year-ahead mortality forecast for both approaches improve when we truncate the period of historical data used. It is worthwhile to undertake some further research to investigate possible reasons for this finding. Further, it can be argued that the random walk approach is more exposed to the influence of changes in the length of historical data employed than the local linear approach. On
the other hand, the local linear approach seems to be more robust. This should be expected since the bandwidth selection in local linear approach gives more weight to recent mortality experience and thus the method will be less affected by the change in starting time of the investigation. Overall, we conclude that both of the forecasting approaches to CBD model appear to be relative robust even if the local approach is better in this regard.

<table>
<thead>
<tr>
<th>Forecast horizon</th>
<th>CBD model: RW forecast</th>
<th>CBD model: LLE forecast</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>-1.10 5.64 7.20</td>
<td>-1.90 5.26 6.70</td>
</tr>
<tr>
<td>5</td>
<td>0.66 6.26 7.68</td>
<td>0.23 4.76 5.68</td>
</tr>
<tr>
<td>8</td>
<td>3.35 6.79 8.34</td>
<td>-4.30 5.22 6.76</td>
</tr>
</tbody>
</table>

**Table 4.** Forecasting results of the CBD model for GB male aged 50-89 based on mortality data since 1970.

An important finding from the results in Table 5 is that the forecasting results from the 2-D KS model remain unchanged when different historical data periods are used. As described in Section 2.3, the optimum bandwidths and correlation parameter selected based on the out-of-sample forecasting performance of the model. Therefore, the validation data set is still the same and the bandwidths would still give more weight to the most recent mortality experience. This could possibly explain the reason why we have the forecasting results remain unchanged. In contrast, compared to Table 3, even though the short to medium term forecasting results of the 2-D LOP model do not change much, the model seems to be a bit unstable in long-term forecasting performance relative to the changes in starting time of the investigation period. However, we can still argue that the level of robustness in this two models is overall satisfactory.

<table>
<thead>
<tr>
<th>Forecast horizon</th>
<th>2-D LOP model</th>
<th>2-D KS model</th>
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<tbody>
<tr>
<td></td>
<td>$E_1(%)$  $E_2(%)$  $E_3(%)$</td>
<td>$E_1(%)$  $E_2(%)$  $E_3(%)$</td>
</tr>
<tr>
<td>3</td>
<td>-0.23 3.50 4.64</td>
<td>-3.53 4.37 5.75</td>
</tr>
<tr>
<td>5</td>
<td>2.20 4.65 5.63</td>
<td>0.98 3.74 4.83</td>
</tr>
<tr>
<td>8</td>
<td>8.83 10.07 12.50</td>
<td>1.78 4.39 5.68</td>
</tr>
</tbody>
</table>

**Table 5.** Forecasting results of the 2-D LOP model and the 2-D KS model for GB male aged 50-89 based on mortality data since 1970.
Our conclusions made in Section 3.3 on the preference to use local information in mortality projection still holds based on the results shown in Table 4 and Table 5 as the performances of local linear forecast and kernel smoothing forecast are still better than their comparators which use global information in the forecasting process. Moreover, it can also be concluded that the forecasts from the local approaches are more robust than the forecasts from the global approaches. Plots comparing the 3, 5 and 8-year-ahead mortality forecasts for the four models for GB male aged 50, 60, 70 and 80 based on mortality experience from 1970 are illustrated in Figure 5-7.

**Figure 5.** 3-year-ahead forecast from 2007-2009 for GB male aged (a) 50, (b) 60, (c) 70 and (d) 80 based on mortality data since 1970.

**Figure 6.** 5-year-ahead forecast from 2005-2009 for GB male aged (a) 50, (b) 60, (c) 70 and (d) 80 based on mortality data since 1970.
4. CONCLUSION

In this paper we make a formal comparison between two sets of mortality models on their forecasting performance. The two models in each group have similar design in their structure but one projects future mortality rates using local information and the other uses global information. One main conclusion made from the case study conducted based on GB male mortality experience is that, in the forecasting process, local information seems to have greater predictive power and thus it should be given more weight when doing future mortality projection. The study also includes a test on the robustness of the forecast relative to different historical period used in estimation. We conclude that overall, the four models included in the analysis have a satisfactory level of robustness and are suitable for the purpose of future mortality projection. However, local modeling does perform slightly better in this respect.

Since we have only considered the mortality experience in GB, future research will consider a wider range of countries and potentially male and female data. Moreover, the analysis carried out in this study could also be extended to include a broader range of models with different characteristics such as the Lee-Carter model (1992) and the $P$-splines model (Currie et al., 2004).
REFERENCES


