Inside the Solvency 2 Black Box: Net Asset Values and Solvency Capital Requirements with a Least-Squares Monte-Carlo Approach

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Abstract

The calculation of Net Asset Values and Solvency Capital Requirements in a Solvency 2 context - and the derivation of sensitivity analyses with respect to the main financial and actuarial risk drivers - is a complex procedure at the level of a real company, where it is illusory to be able to rely on closed-form formulas. The most general approach to performing these computations is that of nested simulations. However, this method is also hardly realistic because of its huge computation resources demand. The least-squares Monte Carlo method has recently been suggested as a way to overcome these difficulties. The present paper confirms that using this method is indeed relevant for Solvency 2 computations at the level of a company.

Keywords

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Introduction

Solvency II is the new harmonized EU-wide insurance regulatory regime scheduled to start on January 1, 2016. It introduces a prospective and risk-based supervisory approach. Within this new prudential regime, insurance or reinsurance companies can use the convenient but sometimes overcharging Solvency II standard formula, or otherwise an approved Internal Model, to assess their Pillar 1 Solvency Capital Requirement (SCR). Then, the Pillar 2–ORSA is a top-down process that aims at ensuring a complete and holistic risk understanding, viewed from management or supervisory bodies. To tackle the ORSA, insurance companies should conduct their own risk and solvency assessment including the overall solvency needs taking into account their specific risk profile, approved risk tolerance limits and business strategy. Furthermore, the significance with which the risk profile of the firm deviates from the assumptions underlying the SCR should also be assessed and justified to supervisors.

Ultimately, insurance and reinsurance companies should extend their modeling tools to tackle the internal model SCR assessment and the ORSA capital calculation and justification. To achieve a compliant internal model SCR assessment or a relevant ORSA capital measurement, insurance firms need to derive their capital requirement from the stochastic distribution of own funds. The internal model SCR should correspond to the one-year maximal decrease of the insurance company’s Pillar 1 own funds (the Net Asset Value or NAV) with a 99.5 % confident level. The ORSA capital should be derived from the distribution of the firm's Pillar 2 own funds but no specific metric is imposed by the regulator for its assessment.

It is noteworthy that the Pillar 2 ORSA capital assessment can substantially differ from the Pillar 1 internal model SCR. The different recognition base (for instance the inclusion of the goodwill in the Pillar 2 own funds, which is not allowed in the Pillar 1 own funds) can explain a part of the difference. The use of a different risk metric (Tail Value At Risk instead of Value At Risk, a confidence level different from 99.5%, a time horizon superior to one year) provides another explanation of this difference. Also, the possible use in Pillar 2 calculations of a valuation base different from the market consistent one is another element in this direction.

Despite these differences, the assessment of the internal model SCR or ORSA capital should derive from the stochastic distribution of the company’s own funds no matter the differences in valuation or recognition. The straightforward approach that leads to this distribution is based on Monte Carlo simulations. In this respect, the approach based on the least-squares Monte Carlo method taken in this paper is suitable at the level of a company issuing a variety of complex insurance contracts leading to burdensome Monte Carlo simulations. It permits the assessment in a reasonable time of the internal model SCR and of the ORSA capital, and of the sensitivity analyses thereof.

For clarity, we only focus on the internal model SCR assessment (so we consider in a first calculation step the forecasting of the one-year own funds or Net Asset Value). Note that the procedure is exactly the same for the practical assessment of the ORSA capital. For illustration, we concentrate on with-profit insurance contracts. These contracts include options that benefit to policyholders (like surrender options and capital-guaranteed options). Again, the procedure followed is readily applicable to more complex contracts.
Monte Carlo nested simulations, or equivalent methods, are required to correctly estimate Net Asset Values and Solvency Capital Requirements. In this process, one first needs to simulate the one-year values of the assets for each real world scenario (also called primary, or conditional, or outer scenario). As a second step, one needs to simulate for each primary scenario the different liability values relying on market consistent scenarios (also called secondary, or conditional, or inner scenarios). Then, each NAV corresponding to a given real world scenario is obtained as the difference between the asset value and the mean value of the liabilities (the best estimate of liabilities) relying on the attached inner scenarios. Performed in a rough way, such a process needs intensive computing resources to be performed in an acceptable delay at the level of an insurance business. Further, it is illusory in a standard Monte Carlo context to be able to perform precise sensitivity analyses. However, we show in this paper that using the least-squares Monte Carlo method can considerably simplify the whole valuation and sensitivity analysis process.

The least-squares Monte Carlo method was originally devised by Carriere (1996) and Tsitsiklis and van Roy (1999, 2001). It has become popular since the work of Longstaff and Schwartz (2001) who have developed it in the context of option pricing. See also Clément, Lamberton and Protter (2002). Popinski (2000) and Stentoft (2004) give general convergence arguments for this method, and Areal, Rodrigues and Armada (2008) provide improving tools. Gobet, Lemor and Warin (2005) have applied the method for solving backward stochastic differential equations. Andreatta and Corradin (2003) were among the first to apply it in an insurance contract valuation context. Bacrinello, Biffis and Millosovich (2009, 2010) provide a comprehensive use of the least-squares Monte Carlo method in insurance theory. The articles of Cathcart and Morrison (2009) and Bauer, Reuss and Singer (2012) are fundamental references: these are the first articles applying the LSMC method in a Solvency 2 context. The method has also been used by practitioners, see for instance Koursaris (2011). Participating or with-profit contracts make up a significant part of the life insurance market in the United States, Great Britain, Japan, France, Spain, or Canada. The contracts we use for our illustrations are general participating contracts, constitute an important part of the French market, and can be easily extended to cover other contexts. For references on participating contracts, see in particular Briys and de Varenne (2001) and Ballotta, Haberman and Wang (2006). For the modeling of surrender risk using parametric forms, see, e.g., Kim (2005).

This paper shows that it is relevant to use the Least-Squares Monte Carlo method as a replacement of nested simulations for Solvency 2 calculations at the level of a general business. The article is organized as follows. In a first introductory section, we detail the financial, surrender and mortality assumptions and the dynamics of the assets and liabilities of the insurance company. We also explain how NAVs and SCRs can be computed and we recall the fundamentals of the nested simulations algorithm. In a second section, we briefly recall the principles underlying the LSMC method and its use for the computation of NAVs. In a third section, we bring direct and indirect confirmations of the relevance of using the LSMC method for computing NAVs. For this purpose, we introduce the Scenario Based and Enhanced Scenario Based methods. A fourth section presents a sensitivity analysis of NAVs and SCRs with respect to the financial (equity and rate), surrender and mortality risks.
1 Framework

We start by setting out the framework of the paper. We define the set of market dynamics and the surrender and mortality assumptions. Then, we give the characteristics of the contracts studied and we recall the main features affecting the computation of SCRs and NAVs.

1.1 Financial market

The market considered is comprised of equities and risk-free government bonds. These two types of assets are modeled with similar stochastic dynamics both for risk assessment and market-consistent pricing, but using parameter different assumptions (real world assumptions for risk assessment versus market consistent assumptions for pricing).

The dynamics of the stocks, or of a representative index of the stock market, are modeled as follows in the real world:

\[
\frac{dS_t}{S_t} = \left( r_t + \pi^S \right) dt + \sigma^S \, d\tilde{z}^S_t
\]

(1)

In the market consistent world, the same type of dynamics are assumed to prevail, but there is no risk premium beyond the risk-free rate:

\[
\frac{dS_t}{S_t} = r_t \, dt + \sigma^S \, d\tilde{z}^S_t
\]

(2)

We assume that the bond market is driven by real-world instantaneous risk-free rate dynamics of the Cox-Ingersoll-Ross type:

\[
dr_t = \zeta \left( m - r_t \right) dt + \sigma_r \, \sqrt{r_t} \left( d\tilde{z}^r_t + \frac{\pi^r \sqrt{r_t}}{\sigma_r} \, dt \right)
\]

(3)

where the risk premium typically takes the form of \( \frac{\pi^r \sqrt{r_t}}{\sigma_r} \).

The market-consistent risk-free rate dynamics are consistently written as follows:

\[
dr_t = \zeta \left( m - r_t \right) dt + \sigma_r \, \sqrt{r_t} \, d\tilde{z}^r_t
\]

(4)

The equity and risk-free rate dynamics are assumed to be correlated, so that

\(<d\tilde{z}^S_t, d\tilde{z}^r_t> = \rho dt>

In the Solvency 2 context, the real and market-consistent worlds mainly correspond to the historical and risk-neutral worlds. Fundamentally, they correspond to choices of sets of securities and derivatives used for parameter calibration. Hopefully, a good choice of calibration instruments and an appropriate model design can ensure the identity between the real and the historical world on the one hand, and between the market-consistent and the risk-neutral world on the other hand.

1.2 Life insurance company assets

We consider a life insurance company that only issues conventional with-profit contracts. The with-profit fund is assumed to be invested in stocks, credit-risk-free Government bonds, and cash. A constant correlation of \( \rho \) between
equities and debt instruments exists. Therefore, the main risk drivers are the one-year credit-risk-free interest rate and the one-year equity yield. The quantity of debt instrument is given by the nominal $H$ invested in fixed rate bonds of maturity $M$ and coupon rate $\gamma$. The market value $V^b_t$ of such a bond at any time $t$ inferior or equal to the maturity $M$ is

$$V^b_t = \sum_{i=1}^{\lceil M-t \rceil} \gamma \cdot H \cdot P(t, M + 1 - i, r_t) + H \cdot P(t, M, r_t),$$

where $\lceil M-t \rceil$ denotes the smallest number not less than $M-t$ and $P(t, T, r_t)$ is the zero coupon bond price when the interest rate follows a CIR model:

$$P(t, T, r_t) = A(t, T)e^{-B(t, T)r_t},$$

with

$$A(t, T) = \left( \frac{2h e^{(\zeta+h)(T-t)}}{2h + (\zeta + h)(e^{(T-t)h} - 1)} \right)^{\frac{2\zeta m}{\sigma^2}},$$

and

$$B(t, T) = \left( \frac{2(e^{(T-t)h} - 1)}{2h + (\zeta + h)(e^{(T-t)h} - 1)} \right),$$

and

$$h = \sqrt{\zeta^2 + 2\sigma^2}.$$

Because the insurance company is assumed to hold a mixture of cash, bonds, and stocks, the market value of its assets at time $0$ can be written as

$$A_0 = V^b_0 + V^s_0 + c_0,$$

where the initial market value of bonds $V^b_0$ is given in Eq. (5), the initial market value of stocks satisfies:

$$V^s_0 = S_0,$$

which means that $S_0$ describes the initial value of the whole portfolio of stocks and not just of one stock, and $c_0$ is the initial cash available.

Before the maturity of the bond, the insurer receives coupons and invests them in its cash account. Because cash also receives interests at the rate $r$, the amount of cash at any coupon date $n$ inferior to the maturity is defined recursively as follows:

$$c_n = c_{n-1} \cdot e^{(r-\gamma)du} + H \cdot \gamma - F_n,$$

where $F_n$ is a liability cash-flow for the period $(n-1, n]$.

Note that liability payments are assumed to be made with cash first. If there is not enough cash, stocks, and then bonds, are sold on the market to provide the necessary liquidity.

When Government bonds mature, the insurer receives the principal and final coupon and make the liability payments of the period. Then, the insurer uses all its current cash (except a minimum amount $c_0$) to buy bonds that have
the same level of coupon and duration as before. The cash amount just before buying new bonds is

\[ c_M = c_{M-1} \cdot e^{\int M r_u \, du} + H(1 + \gamma) - F_M, \]

so that the new bonds have a total nominal \( H' \) that satisfies

\[ V^b_M = c_M - c_0 = \sum_{i=1}^{M} \gamma \cdot N' \cdot P(0, M + 1 - i, r_M) + H' \cdot P(0, M, r_M), \]

and the cash becomes \( c_{M+} = c_0 \).

At time \( 2M \), these bonds mature and new bonds are bought for another total nominal amount of \( H'' \), and so on. Finally, the evolution of the stock value is described in Eqs (1) or (2). The asset return \( R_n \) for the period between times \( n-1 \) and \( n \) can be calculated with

\[ R_n = \ln \left( \frac{V^b_n + V^s_n + c_n + F_n}{V^b_{n-1} + V^s_{n-1} + c_{n-1}} \right), \]  

(7)

where the liability cash-flow \( F_n \) is reincorporated to produce a pure financial indicator that is used later on in this text for specifying the dynamics of liabilities.

### 1.3 Surrender and mortality risks

We assume that outflows of policyholders occur due to two types of factors: surrender and mortality. We model the surrender rate \( r_L \) as a parabolic function:

\[ r_L(\Delta) = r_{SL} + \max(0, \text{sign}(\Delta)) \times \min(\alpha \Delta^2, \beta), \]  

(8)

where \( \Delta = r - r_C \) is the difference between the market rate and the crediting rate. The parameter \( r_{SL} \) quantifies structural surrenders and the remaining of Eq. (3) measures market-contingent surrenders. See the QIS5 specifications (2010) or Kim (2005) for similar parabolic formulations. We show in Figure 1 the function \( r_L \) with respect to the values of \( \Delta \). We observe that the surrender rate parabolically increases with respect to the excess of the market rate to the crediting rate until a cap is reached.

The mortality table used in this paper is the S1PFL table produced by the Institute and Faculty of Actuaries. This table is for all pensioners (excluding dependants), females, lives.

### 1.4 Life insurance company liabilities

We consider an insurance company that provides conventional with-profits contracts for a total commitment value equal to \( L_0 \). Participating contracts offer to policyholders a profit sharing rate \( \delta \) of the annual financial profits. All policyholders liabilities are guaranteed at the rate \( r_g \) and their value cannot decrease with time. The contracts all have the same maturity of \( N \) years.
We set ourselves in a run-off situation: the company does not issue additional contracts after time zero. The contracts can either be surrendered or terminated upon deaths of policyholders. In the first case, the policyholder receives the book value of the liabilities associated with his contract, minus a penalty. In the second case, the full book value of the liabilities associated with the contract is transferred to the beneficiaries of the policyholder. At the run-off date, which is also the maturity date of insurance policies, all the contracts have either been surrendered, or terminated upon death of the policyholder, or terminated at their natural maturity.

We denote as $L_n$ the book value of liabilities at time $n$. The market value of liabilities, which is equal to the market value of the assets that constitute the fund associated with the contract, is denoted as $\mathcal{L}_n$ at time $n$. The book value of liabilities evolves as follows between any given years $n - 1$ and $n$:

$$L_n = L_{n-1} e^{r_g} + \delta \max (\mathcal{L}_{n-1} - L_{n-1} e^{r_g}, 0) - F_n$$  \hspace{1cm} (9)

where time $n^-$ is the time immediately before the surrender and mortality cash-outs $F_n$ are incurred.

In Eq. (9), the first term is the book value of liabilities increased at the guaranteed rate. The second term is the participation bonus and is computed as the proportion $\delta$ of the excess of the market value of liabilities $\mathcal{L}_{n-1}$ to the book value of liabilities $L_{n-1}$ inflated at the guaranteed rate, when this difference is positive. The third term, denoted as $F_n$, describes the surrender and mortality cash-outs at time $n$:

$$F_n = L_n \min(1, 1q_{x+n-1} + \psi r_L(\Delta))$$  \hspace{1cm} (10)

where $1q_{x+n-1}$ and $r_L(\Delta)$ are the proportions of policyholders (aged $x$ at inception) respectively dying and lapsing between times $n - 1$ and $n$, and $\psi \leq 1$ is a penalty imposed on the policyholders that surrender.

The market value of liabilities is computed as follows:

$$\mathcal{L}_n = \mathcal{L}_{n-1} - F_n$$  \hspace{1cm} (11)
with
\[ L_n = \max (L_{n-1} (1 + \delta R_n^a), L_{n-1} e^{r_g}) \] (12)
where the asset return \( R_n^a \) is defined in Eq. (7).

Therefore, in Eq. (12), a proportion \( \delta \) of the performance of the portfolio is allocated yearly to the market value of liabilities (the complementary proportion \( 1-\delta \) thus remains with the insurer). Also, the insurer guarantees that the market value of liabilities remains superior to the book value of liabilities compounded at the rate \( r_g \).

Note that Eq. (9) can also be cast in the form of
\[ L_n = L_{n-1} (1 + r_C(n)) - F_n \] (13)
where \( r_C(n) \) is the crediting rate at time \( n \) defined as follows:
\[ r_C(n) = e^{r_g} + \delta \max \left( \frac{L_n}{L_{n-1}} - e^{r_g}, 0 \right) - 1. \] (14)
For simplicity, we assume throughout the text that \( L_0 = L_0 \).

### 1.5 Net Asset Values

We first examine the definitions and components of the net asset value before turning to the definition of a solvency capital requirement. We define the “net net asset value”, or net NAV, as follows:
\[ \text{NAV} = A - \text{BE} - \text{DT} \] (15)
where \( A \) is the market value of the assets of the insurance company, \( DT \) are the deferred taxes, and \( BE \) the best estimate value of liabilities (called ‘best estimate’ hereafter). These quantities constitute the company’s balance sheet, as exposed in Table 1.

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A )</td>
<td>NAV</td>
</tr>
<tr>
<td></td>
<td>BE</td>
</tr>
<tr>
<td></td>
<td>DT</td>
</tr>
</tbody>
</table>

Table 1 – Balance sheet of the Life Insurance Company.

The best estimate can be written, for a specific scenario, as
\[ \text{BE} = \sum_{n=1}^{N} F_n e^{-\int_0^u r_u \, du} + L_N e^{-\int_0^N r_u \, du}, \] (16)

or, incorporating \( L_N \) into the cash-flows, as
\[ \text{BE} = \sum_{n=1}^{N} \hat{F}_n e^{-\int_0^u r_u \, du}. \] (17)
Therefore, the best estimate can be interpreted as the discounted sum of the mortality and surrender cash-outs and of the terminal liability book value. Note that the best estimate can also be decomposed into

\[ \text{BE} = \text{GBE} + \text{FDB}, \]  

where \( \text{GBE} \) is the guaranteed best estimate and \( \text{FDB} \) is the value of future discretionary benefits. In this decomposition, we have separated the guaranteed part of liabilities that is easy to compute from the value of liabilities that is related to profit-sharing and that is of an option-like nature and more difficult to compute. Thus, we have:

\[ \text{NAV} = \mathcal{A} - \text{GBE} - \text{FDB} - \text{DT}. \]  

Then, the “gross net asset value”, or gross \( \text{NAV} \), which is denoted as \( \text{NAV}' \), is defined as follows:

\[ \text{NAV}' = \text{NAV} + \text{FDB}, \]  

or, using Eq. (19), as

\[ \text{NAV}' = \mathcal{A} - \text{GBE} - \text{DT}. \]

Thus, the gross \( \text{NAV} \) does not include the \( \text{FDB} \) and is easier to compute than the net \( \text{NAV} \). We use this property in the rest of the paper. Also, for simplicity, we assume that DT is null.

### 1.6 Nested Simulations

We show here how net asset values can be computed using nested simulations. The first step is the projection over one year in the real world of the risk-factors, which we denote by \( X_{k=1,\ldots,K} \). In this paper, there are \( K = 2 \) risk-factors, the dynamics of stocks and interest rates. We assume that \( N_p \) primary real-world simulations of the risk factors are performed. For each of these simulations, we want to compute \( \text{NAV} \). Therefore, our goal is to obtain the following set:

\[ \{ \text{NAV}(\omega_p, t=1) : p = 1, \ldots, N_p \} \]  

where for each primary simulation \( p \) the state of the world \( \omega_p \) describes the values \( X_{k=1,\ldots,K} \) of the risk drivers.

Each value \( \text{NAV}(\omega_p, t=1) \) in the set (22) is estimated by averaging over secondary market-consistent simulations. In fact, the \( \text{NAV} \) one year after the valuation date is a random variable and, as such, can be written as a conditional expectation. In a Solvency 2 context, this conditional expectation is computed in the market-consistent world by projecting cash-flows over a century (in this paper until the run-off date). Using Eqs (15) and (17), we have:

\[ \text{NAV}(\omega_p, t=1) = \mathcal{A}_1^p - E_Q \left( \sum_{n=2}^{N_p} \hat{F}_n^p e^{-\int_0^t r^n_u du} \right) \]  

where \( \mathcal{A}_1^p \) is the market value of assets at time 1 conditional on \( \omega_p \) and \( \hat{F}_n^p \) is the best estimate cash-flow that occurs at time \( n \) conditional on \( \omega_p \). Similarly, \( r^n_u \) is the market-consistent interest rate at time \( u \) conditional on \( \omega_p \). Now, let \( N_s \) be
the number of market-consistent secondary simulations. The nested simulations algorithm amounts to computing

$$\text{NAV}(\omega_p, t = 1) = \mathcal A_{1}^{p} = \frac{1}{N_s} \sum_{s=1}^{N_s} \sum_{n=2}^{N} \hat F_{s,n}^{p} e^{-\int_{0}^{1} r_{x,n}^p du} \quad (24)$$

for each \( p \), where the cash-flow \( \hat F_{s,n}^{p} \) and the interest rate \( r_{x,n}^p \) stem from the secondary simulation \( s \).

The algorithm is as follows. First, simulate \( N_p \) real-world values of the risk-drivers. Here, we obtain \( N_p \) pairs \((S_{p1}, r_{p1})\) of stock and interest rate values by simulating Eqs (1) and (3) until time 1. For each pair \((S_{p1}, r_{p1})\), we derive an asset market value \( A_{1}^{p} \) at time 1. Then, starting from each pair \((S_{p1}, r_{p1})\), we perform \( N_s \) secondary simulations. For each secondary simulation \( s \), we generate values of the stock and interest rate in the market-consistent world using Eqs (2) and (4) until the run-off date \( N \). Then, the book and market values of liabilities and the cash-flows respectively stem from Eqs. (9), (12), and (10). It remains to estimate Eq. (24) for each primary state of the world \( \omega_p \) by averaging over secondary simulations.

The numbers of primary and secondary simulations, \( N_p \) and \( N_s \), are typically equal to 10,000 and 2,500. With only two risk-drivers and a maturity \( N \) of thirty years, the quantity of random numbers that need to be generated for the company studied in this paper is approximately equal to 1.5 billion, assuming a rough yearly discretization. Therefore, because of the huge quantity of random numbers involved, NAV cannot be estimated in a reasonable computational time for a real business using the nested simulations approach. We show in the following section how it is possible to perform similar computations without making use of nested simulations.

1.7 Solvency Capital Requirements

We can now come to the definition of Solvency Capital Requirements. An SCR is given as the difference of two NAVs as follows:

$$\text{SCR} = \text{NAV}_0 - \text{NAV}_{99.5\%} e^{-r_1} \quad (25)$$

In this equation, \( \text{NAV}_0 \) is the current - or central - NAV. It is the net asset value that is computed using the current market conditions, with no primary simulation, and directly performing secondary market-consistent simulations. This simply amounts to estimating Eq. (24) at time 0 by simulating \( S \) and \( r \) in the market-consistent world.

In order to compute \( \text{NAV}_{99.5\%} \), we rank the \( N_p \) values of the set (22), where each value corresponds to a specific primary simulation and to \( N_s \) secondary simulations. Then, \( \text{NAV}_{99.5\%} \) is the 99.5% worst value among these \( N_p \) net asset values. For example, when there are 10,000 primary simulations, \( \text{NAV}_{99.5\%} \) is the fiftieth worst simulated NAV. The discount rate \( r_1 \) is the zero-one year risk-free interest rate from the initial yield curve.

2 The least-squares Monte Carlo approach

We first recall the main ideas concerning the least-squares Monte Carlo approach. Then, we explain how it can be used to compute Net Asset Values, and
therefore also Solvency Capital Requirements. Finally, we conduct an illustration of the choice of polynomials.

2.1 The method

Let us first describe the main features of the least-squares Monte Carlo (hereafter LSMC) method. This method is used when American options need to be computed, more generally when optimal choices need to be determined, or when successive simulations need to be performed (the so-called nested simulations approach), within a forward Monte Carlo approach.

The insurance balance sheet problem of this paper is akin to that of pricing complex exotic options that include both an American component and a path-dependent component, and depend on multivariate underlyings. While path-dependent and multivariate programs are most easily tackled with Monte Carlo simulations, the pricing of American options requires a backward approach and has been considered for several decades to be impossible to price within a Monte Carlo framework.

Indeed, the price $C^a$ of an American put option can be computed as follows:

$$C^a = \sup_{\tau} E_Q \left( (K - S_\tau)^+ e^{-r\tau} \right)$$  \hspace{1cm} (26)

where $S$ is the underlying, $K$ is the strike, and the supremum is taken over admissible optimization strategies (corresponding to stopping times $\tau$).

The difficulty of applying the Monte Carlo method to compute Equation (26) can be explained as follows. At each step of any given trajectory, simulated starting from time 0, the optimizer has to decide to either early exercise the option or to keep it in her book. This amounts to computing the value of early exercise and the value of continuation and to, rationally, choosing the biggest one. While the value of stopping is straightforward to calculate anywhere in the trajectory, the value of continuing is a conditional expectation and requires, in turn, the simulation of additional trajectories. The problem very quickly explodes in terms of the number of simulated paths and is not solvable in this specific form.

The LSMC approach, as explained by Longstaff and Schwartz (2001) in the case of a standard American put option, first amounts to simulating a set of trajectories. Then, after computing the payoffs at the maturity, the algorithm proceeds recursively starting from the penultimate discretization date. At this date, it computes a least-squares fit of the value of continuing with respect to the value of the underlying. This least-squares fit being achieved, the value of continuing can be estimated for each trajectory at the penultimate date: the comparison of this value with respect to the value of early exercise concludes the analysis of the penultimate period. Then, the same procedure is performed at the antepenultimate date, and so on until an estimation of the option price is obtained at time zero.

Therefore, the idea of the LSMC approach is the computation of conditional expectations with a least-square fit performed on trajectories. Once the least-square functional has been estimated, the local values of the quantities related to particular paths are introduced in it to obtain individual estimations of the conditional expectation for each particular path. The next subsection illustrates how this method can be used for the computation of Net Asset Values.
2.2 Computation of \( \text{NAV} \)

We now come to the computation of \( \text{NAV} \) using the least-squares Monte Carlo method. Because any square integrable random variable can be decomposed onto an orthonormal basis of \( L^2 \) (see for instance Royden (1988)), we can express the market-consistent conditional expectation of Equation (23) as a series:

\[
A_p^1 - E_Q \left( \sum_{n=2}^{N} \hat{F}^p_n e^{-\int_0^n r^p du} \right) = \sum_{l=1}^{+\infty} a_l \phi_l \left( X^{k=1,\ldots,K}_p \right)
\]

(27)

where \( \phi_{l=1,\ldots,\infty} \) is an orthonormal basis of \( L^2 \) that depends on the risk drivers and \( a_{l=1,\ldots,\infty} \) are the coefficients along this basis.

In practice, a finite (and small) dimension \( d \) of the basis is sufficient. The market-consistent conditional expectation can then be approximated, for each primary simulation \( p \), as follows:

\[
A_p^1 - E_Q \left( \sum_{n=2}^{N} \hat{F}^p_n e^{-\int_0^n r^p du} \right) = \sum_{l=1}^{d} a_l \phi_l \left( X^{k=1,\ldots,K}_p \right)
\]

(28)

From a practical viewpoint, the coefficients \( a_{l=1,\ldots,d} \) are estimated by performing a least-squares fit with respect to the conditional expectation computed with one or a very small number \( n_s << N_s \) of secondary simulations. The estimated coefficients \( \hat{a}_{l=1,\ldots,d} \) satisfy:

\[
\hat{a}_{l=1,\ldots,d} = \arg \min_{a_{l=1,\ldots,d}} \sum_{p=1}^{N_p} \left( A_p^1 - \frac{1}{n_s} \sum_{s=1}^{n_s} \sum_{n=2}^{N} \hat{F}^p_{n,s} e^{-\int_0^n r^p_{n,s} du} - \sum_{l=1}^{d} a_l \phi_l \left( X^{k=1,\ldots,K}_p \right) \right)^2
\]

(29)

See Stentoft (2004) for a study of the convergence of such estimators to the conditional expectations they approximate. The LSMC method therefore yields the \( N_p \) estimates of \( \text{NAV} \):

\[
\text{NAV}(\omega_p, t = 1) = \sum_{l=1}^{d} \hat{a}_l \phi_l \left( X^{k=1,\ldots,K}_p \right)
\]

(30)

In order to compute \( \text{NAV}_{99.5\%} \), we rank the values shown above and we seek for the 0.5%-quantile. The simulation \( p \) giving the parameters \( X^{u=1,\ldots,U}_p \) that correspond to the quantile is denoted by \( q \). Therefore, we have:

\[
\text{NAV}_{99.5\%} = \sum_{l=1}^{d} \hat{a}_l \phi_l \left( X^{k=1,\ldots,K}_{q} \right)
\]

(31)

Then, it is possible to incorporate this quantity into Equation (25) in order to obtain the value of the SCR.

2.3 Least-squares fit in practice

Before the detailed study of the computation of NAVs and SCRs with the LSMC method and other approaches in the next section, we conduct a simple illustration of the choice of an orthonormal basis. Several bases have been proposed in the literature, amongst which that of Laguerre polynomials. However,
nothing compares with simplicity: we illustrate here the choice of a basis of power functions. We compute the price of a simple American put (that measures, e.g., the value of early defaulting a guarantee in the balance sheet of an insurer) using the Least-Squares Monte Carlo method and relying on power-like polynomials for the least-squares fit.

In Table 2 we show the set of parameters retained for our experiment. This parameter set is the same as that used by Longstaff and Schwartz (2001). \( S_0 \) is the initial stock price, \( K \) is the strike, \( r \) is the risk-free rate, \( \sigma \) is the stock volatility and \( T \) is the option maturity. \( N \) represents the number of stock paths, whereas \( M \) is the discretization step.

\[
\begin{array}{ccccccc}
S_0 & K & r & \sigma & T & N & M \\
36 & 40 & 0.06 & 0.2 & 1 & 100000 & 50
\end{array}
\]

Table 2 – Dataset

In Table 3 we display the results of the simulations. The price denoted by ‘PDE’ is the true price, obtained with a sufficiently discretized finite difference method. The results denoted by \( n = 2, \ldots, 4 \) correspond to least-squares fits performed with power functions of order two to four. It is clear from this illustration that a least-squares fit performed with simple polynomials such as power functions is precise enough to avoid the use of more complex bases such as that of Laguerre polynomials. Also, it is clear from our illustration that polynomials of order \( 3 \) can be sufficient to reach reasonably accurate pricing, whereas polynomials of order \( 2 \) are not reliable in our example.

\[
\begin{array}{cccc}
n = 2 & n = 3 & n = 4 & \text{PDE} \\
3.952 & 4.482 & 4.479 & 4.478
\end{array}
\]

Table 3 – Results

Let us now apply the LSMC method to the more complex setting of the computation of NAVs and SCRs for insurance businesses.

3 Ranking net asset values

This section is devoted to the study of the ranking of NAVs. We first show how the LSMC approach can be used for this purpose. Then, we introduce the scenario-based method, before examining a mixed approach.

3.1 The least-squares Monte Carlo approach

The least-squares Monte Carlo approach is a powerful tool to compute net asset values at the aggregate level of an insurance company, which can hardly be achieved with the traditional nested simulations method. We now illustrate this fact.

The LSMC approach to computing NAVs can be described as follows. In a first step, which is identical to the first step of the nested simulations method, we generate a sample containing \( N = 10000 \) one-year real-world values, or
outer simulations, of the risk drivers: the risk-free rate and the equity yield. In a second step, which differs from that of the nested simulations method, we generate only two (instead of 2500) ten-year market-consistent trajectories, or inner simulations, for each of the 10,000 sets of risk drivers. An imprecise NAV is then computed from these two inner simulations for each of the outer simulations. In a third step, we fit a NAV proxy function by means of a polynomial regression performed on the imprecise NAVs. In a fourth and terminal step, we estimate the NAV distribution by applying the NAV proxy function to each of the N one-year real-world values of the risk drivers.

Table 4 – Parameter set (contract and fund specification)

<table>
<thead>
<tr>
<th>$r_g$</th>
<th>$\delta$</th>
<th>$c_0$</th>
<th>$\gamma$</th>
<th>$M$</th>
<th>$H$</th>
<th>$V_0^b$</th>
<th>$V_0^s$</th>
<th>$L_0$</th>
<th>$T$</th>
<th>$x$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1%</td>
<td>80%</td>
<td>30%</td>
<td>3%</td>
<td>10%</td>
<td>105</td>
<td>96</td>
<td>45</td>
<td>120</td>
<td>30</td>
<td>75</td>
</tr>
</tbody>
</table>

In order to illustrate this method, we use the parameter set shown in Tables 4, 5, and 6. In Table 4 we first give the guaranteed rate $r_g$ and the participation rate $\delta$. Then, $c_0$ is the initial amount of cash. The parameters $\gamma$, $M$, $H$ and $V_0^b$ respectively describe the coupon rate, maturity, nominal and market value of the bond portfolio. Next, $V_0^s$ is the market value of the portfolio of stocks. $L_0$ is the initial book value of liabilities. $T$ is the time horizon under which the study is performed, and $x$ is the age of the insured at time 0. In Table 5 we show volatility of rate $\sigma_r$ and the volatility of stocks $\sigma_s$. $r_0$ is the initial value of the interest rate, while $\zeta$, $m$ and $\rho$ are the values of the parameters in the CIR process describing rates. Finally, $\rho$ is the correlation between stocks and interest rates. In Table 6 $r_{SL}$ gives the structural surrender rate, while $\alpha$ is the market-consistent surrender rate and $\beta$ provides a maximum market-consistent surrender rate. All of these parameter values are used until the end of the present section dedicated to the ranking of NAVs.

Table 5 – Parameter set (market dynamics)

<table>
<thead>
<tr>
<th>$\sigma_r$</th>
<th>$\sigma_s$</th>
<th>$r_0$</th>
<th>$\zeta$</th>
<th>$m$</th>
<th>$\rho$</th>
</tr>
</thead>
<tbody>
<tr>
<td>8%</td>
<td>30%</td>
<td>3%</td>
<td>20%</td>
<td>5%</td>
<td>50%</td>
</tr>
</tbody>
</table>

Table 6 – Parameter set (surrender features)

<table>
<thead>
<tr>
<th>$r_{SL}$</th>
<th>$\alpha$</th>
<th>$\beta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5%</td>
<td>50</td>
<td>0.4</td>
</tr>
</tbody>
</table>

We conduct a first illustration of the LSMC method with a fourth degree polynomial regression with no interaction term. Therefore, we consider terms such as $x^{1-4}$ and $y^{1-4}$, but not terms such as $xy$, $xy^3$, or $x^2y^2$ in the proxy function. We present in Figure 2 the results of this experiment. In this figure, the cloud of black points represents the NAVs associated with outer simulations. Each of them is computed by averaging over two inner simulations. The shaded surface is the NAV proxy function calibrated on the cloud of points.
In order to check that the fourth degree polynomial proxy function correctly approximates the NAV surface, we compare it in Figure 3 with the ‘true' surface obtained with nested simulations. In this figure, the cloud of points is made of the NAVs associated with each outer simulation but now computed by averaging over 2,500 inner simulations. The shape of the cloud is flat and approximately matches the surface of the proxy function. However, in Figure 3, the flat cloud is not exactly matched in the tails by the proxy surface: this first LSMC calibration can be improved. For this purpose, we conduct another experiment by now taking into account interaction terms in the LSMC regression.

In Figure 3, we represent a cloud of black points similar to that of Figure 2, representing the NAVs associated with each outer simulation and computed

Figure 2 – LSMC regression surface (no interaction terms).

Figure 3 – Control of the LSMC regression accuracy (no interaction terms).
by averaging over two inner simulations. However, the shaded surface is now the NAV proxy function incorporating interaction terms and calibrated on this cloud of points. Also, in order to reach a higher degree of precision, we rely on \( N = 40000 \) outer simulations.

![Figure 4 – LSMC regression surface (with interaction terms).](image)

We confront in Figure 5 the new proxy surface shown in Figure 4 with the relevant NAVs generated with nested simulations (relying on 2500 inner simulations). Comparing with Figure 3, it immediately appears that the proxy surface obtained by incorporating all the terms (including the interactions) up to order four matches the true NAVs obtained with nested simulations in all the parts of the graph, especially also in the high quantiles zones.

![Figure 5 – Control of the LSMC regression accuracy (with interaction terms).](image)

Another interesting way of checking the validity of the LSMC regression
consists in representing the ranked NAVs. In Figure 6, we plot the smallest NAVs (10% of all the points) shown in Figure 3. The plain curve represents the true values, obtained with nested simulations. The dashed curve shows the NAVs stemming from the LSMC proxy function.

This graph clearly confirms that neglecting the interaction terms in the LSMC approach does not allow us to perform a good fit of the true NAVs in the high quantile zone. This is problematic because we are specifically interested in the values of NAV at the 99.5% quantile. Hopefully, the plain implementation of the LSMC approach does not share this drawback, as we now show.

We plot in Figure 7 the smallest NAVs (10% of all the points) shown in Figure 5 so taking interaction terms into account. Again, the plain curve represents the true values, obtained with nested simulations, and the dashed curve shows the NAVs stemming from the LSMC proxy function. We observe that the LSMC proxy function performs very well in reproducing the true NAVs.
when the interaction terms are not neglected. This visual conclusion is confirmed quantitatively by the value of the root mean square error, which is equal to 5.7\% for the data of Figure 7 and to 18.7\% for the data of Figure 6.

It stems from this study that the LSMC approach, calibrated with polynomials of a sufficiently high order and taking into account interaction terms can be a substitute to the nested simulations method, which cannot be implemented in practice at the level of a company. In addition, the LSMC approach provides an overall estimation of the NAV distribution. Therefore, sensitivity analyzes of NAV with respect to each risk driver, or to a combination of risk drivers, can be performed instantaneously. We now come to the study of an alternative method that allows us to compute net asset values even more quickly than the LSMC approach, but in a more constrained way.

### 3.2 The scenario-based method

We now introduce a new method that allows us to readily obtain net NAVs at given quantiles, based on gross NAVs at the same quantiles and on related risk drivers.

![Figure 8 – Mitigation effect of the profit sharing mechanism](image)

We start by representing gross and net NAVs with respect to risk drivers. In Figure 8 we represent, with respect to the risk free rate, gross NAVs obtained by simulation and net NAVs obtained with nested simulations. In Figure 9 a similar study is conducted, where the graph is now plotted with respect to the equity yield.

From these graphs, we observe that both the gross and the net NAVs decrease with respect to the risk-free rate and increase with respect to the equity yield. We also note that, for each fixed set of risk drivers, gross NAVs are always sharply larger than their net counterparts, with a typical ratio of the two indicators ranging between 1.2 and 1.7.

Figures 8 and 9 hint at the following empirical law: for typical values of the main risk drivers (at least those considered in this paper), the net NAV should increase with the gross NAV. Put differently, the ranked values of net NAV can be associated with the same sets of risk drivers as the ranked values of gross
NAV. By this assertion we mean that if we rank net and gross NAVs, a net NAV and a gross NAV taken at a given common rank correspond to the same values of the risk drivers. For example, the ranked gross and net NAVs that are represented in Figure 10 should correspond at each rank to the same set of risk drivers.

The above-mentioned effect amounts to assuming that the impact of the FDB linking NAV' and NAV cannot lead to a modification of the ranks when going from NAV' to NAV for given sets of risk drivers. In relation with this feature, the scenario-based (hereafter SB) method can be described as follows.

When ordering the set of \( N_p \) values:

\[
\{ \text{NAV}_p = \text{NAV}(\omega_p, t = 1) : p = 1, \ldots, N_p \}
\]

we construct a permutation function \( \phi : [1, \ldots, N_p] \to [1, \ldots, N_p] \) defining the
ordered set:
\[ \{ \mathcal{N} \mathcal{A} \mathcal{V}_{\phi(p)} \} \]
where \( \phi(p_1) < \phi(p_2) \Rightarrow \mathcal{N} \mathcal{A} \mathcal{V}_{\phi(p_1)} < \mathcal{N} \mathcal{A} \mathcal{V}_{\phi(p_2)} \)

Then, we perform the same operation with gross NAVs. We order the set of \( N_p \) values:
\[ \{ \text{NAV}'_p = \text{NAV}'(\omega, t = 1) ; p = 1, \ldots, N_p \} \]
by constructing the permutation function \( \psi : [1, \ldots, N_p] \rightarrow [1, \ldots, N_p] \) that defines the ordered set:
\[ \{ \mathcal{N} \mathcal{A} \mathcal{V}'_{\psi(p)} \} \]
such that \( \psi(p_1) < \psi(p_2) \Rightarrow \mathcal{N} \mathcal{A} \mathcal{V}'_{\psi(p_1)} < \mathcal{N} \mathcal{A} \mathcal{V}'_{\psi(p_2)} \)

The SB method amounts to assuming the identity of permutation operators:
\[
\phi = \psi \tag{32}
\]

This implies the following simple algorithm for computing \( \text{NAV}_{99.5\%} \). The first step is the computation of \( \text{NAV}' \) for each of the \( N_p \) primary simulations. Note that \( \text{NAV}' \), which does not include the value of future discretionary benefits by definition, is quick to compute because it does not require secondary simulations. The second step is the ordering of the \( \text{NAV}' \) just obtained and the selection of the set of risk drivers that produces \( \text{NAV}'_{99.5\%} \). The third and final step is the computation of FDB, and therefore of NAV, using inner simulations starting from the selected set of risk drivers.

We illustrate the SB method in Figure 11. The black dots represent the net NAVs computed with nested simulations and ranked in increasing order. The gray points represent their SB counterparts. We observe that the SB method gives an approximation of ranked net NAVs but is not precise enough to generate reliable predictions. We now present an enhanced version of the SB method that is fully exploitable for obtaining precise ranked net NAVs.

![Figure 11 – NS and SB net NAVs ranked in increasing order](image)

The enhanced scenario-based (hereafter ESB) method introduces a smoothing effect that aims at reducing the dispersion of the gray points observed in
This method, compared with the SB method, adds a final reshuffling step, where for all $\phi(p)$

$$
\hat{NAV}_{\phi(p)} = \frac{\sum_{i=-M}^{M} NAV_{\phi(p)+i}}{2M + 1}
$$

so that the NAV at a given quantile $\phi(p)$ is computed as an average of the ranked NAVs located around that quantile and computed using the SB method. A small value of $M$, typically $M = 9$, yields very precise results, as confirmed by the following experience.

We illustrate the ESB method in Figure 12. As in Figure 11, the black dots represent the net NAVs computed with nested simulations and ranked in increasing order. Here, the gray points represent their ESB counterparts. We observe that the ESB method performs very well in predicting ranked net NAVs.

Among the benefits of the SB and ESB approaches, we can mention the fact that they are much faster than the LSMC approach for the assessment of ranked NAVs, and therefore of the SCR. They need few complete inner simulations (about 9 for the enhanced scenario-based version) to assess the 99.5% NAV quantile. However, these methods are not devoid of drawbacks. They do not give an overall estimation of the NAV distribution. As a consequence, analyzes of the NAV sensitivity to each risk driver or to a combination of them are not possible. In the next subsection, we show how the ESB method can be used as a powerful tool for the confirmation of the validity of the LSMC approach.

3.3 Validation of the LSMC approach

We now come to the use of the ESB method as a way of validating the LSMC approach. The advantage of using the ESB method, instead of nested simulations, is that this method is applicable at the scale of a large enterprise. Therefore, it can provide a broader confirmation of the relevance of the LSMC method.
In Figure 13, we compare the ranked net NAVs obtained using both the ESB method and the LSMC approach, where in the latter case the proxy function is calculated based on 10,000 primary simulations and neglecting the interaction terms. The figure zooms in on the left tail, where the 99.5% quantile is located. This graph brings a clear confirmation that the LSMC approach implemented without interaction terms is not able to correctly predict net NAVs at high quantiles.

A second experiment is conducted in Figure 14, where we again plot the ranked net NAVs obtained using the ESB method and where the LSMC approach is implemented with a proxy function calculated based on 40,000 primary simulations and taking into account interaction terms. This second experiment distinctly confirms that the LSMC approach implemented with interaction terms on a sufficiently high number of primary simulations is adequate for the estimation of ranked NAVs, and therefore of SCRs, in a business context. The next section offers a sensitivity analysis of the NAVs and SCR computed with this method with respect to the main financial and actuarial variables.

4 Sensitivity analysis

The following sensitivity analysis is conducted using the parameters given in Tables 4 to 6. We test the dependence of NAV and of the ratio NAV/SCR on the main financial and actuarial parameters. For all of the graphs shown below, a lower NAV indicates a lower financial strength of the insurance company in absolute terms, whereas a lower NAV/SCR ratio indicates a lower solvency measured in relative terms: ideally, NAV should be equal to a high multiple of SCR.

4.1 Sensitivity to interest rate risk

In Figure 15, we examine the sensitivity of NAV and NAV/SCR with respect to the level of interest rates $r_0$. As shown in the left panel of the figure, a high
Figure 14 – Comparison of the LSMC (with interactions) and ESB methods

Figure 15 – Sensitivities w.r.t. Initial Interest Rate Level
level of interest rate is detrimental to the absolute level of NAV. However, and as illustrated by the right panel of the figure, decreasing interest rates mean levels of SCR that decrease more quickly than NAV, yielding a ratio NAV/SCR that increases (at a larger rate than NAV decreases). To sum up, when interest rates are high, the best estimate value of liabilities increases in such a way that NAV diminishes, but the solvency of the company is not threatened because SCR decreases much more quickly, and in fact NAV becomes a higher multiple of SCR. For the company considered in this article, a high value of interest rates is therefore desirable.

We then observe from Figure 16 that the levels of NAV and NAV/SCR decrease with respect to $\sigma_r$, the volatility of interest rates. Indeed, with a higher value of the volatility of interest rates, a higher number of scenarios exist where the level of interest rates is high, yielding a smaller value of the bond portfolio that is part of the assets of the firm. The value of these assets diminishes and is not compensated by an equivalent decrease of the best estimate value of liabilities, inducing a decrease of NAV at 99.5%. The overall effect of the volatility of interest rates is therefore detrimental to the insurance firm.

4.2 Sensitivity to equity risk

In Figure 17 we examine the sensitivity of NAV and NAV/SCR with respect to the volatility $\sigma_S$ of the equity portfolio. The results are very similar to those obtained with respect to the volatility of interest rates. A higher volatility of
equity yields a higher risk for the insurance company, whether it is measured in absolute or in relative terms. The ratio NAV/SCR decreases from 10 to about 6 for an increase of stock volatility from 20 to 50%. Again, this is similar to the decrease of this ratio from 10 to 5 for an increase of interest rate volatility from 4 to 16%, which can be considered a rise equivalent to that of equity volatility from 20 to 50%.

4.3 Sensitivity to mortality risk

We now look at Figure 18 that shows the dependence of NAV and the ratio NAV/SCR on the value of the parameter $\eta$, which is a proportionality parameter multiplied to the whole mortality table (the higher $\eta$, the higher mortality rates). The results obtained are not surprising because mortality risk affects only the liabilities part of the balance sheet (contrary to interest rate and equity risks that affect both parts). The higher the mortality multiplier, the lower NAV and the ratio NAV/SCR. Note however that the effect is very small compared to that of equity and interest rate volatilities. Doubling mortality rates only produces a marginal decrease of NAV/SCR. The conclusion is nevertheless non-ambiguous: the smaller the mortality rate, the better the financial situation of the insurance company.

4.4 Sensitivity to surrender risk

We now look at Figure 19 that shows the dependence of NAV and the ratio NAV/SCR on the value of the parameter $r$, which is a proportionality parameter multiplied to the whole surrender risk. The results obtained are not surprising because surrender risk affects only the liabilities part of the balance sheet (contrary to interest rate and equity risks that affect both parts). The higher the surrender risk, the lower NAV and the ratio NAV/SCR. Note however that the effect is very small compared to that of equity and interest rate volatilities. Doubling surrender rates only produces a marginal decrease of NAV/SCR. The conclusion is nevertheless non-ambiguous: the smaller the surrender rate, the better the financial situation of the insurance company.
We plot in Figures 19 and 20 the sensitivities of NAV and NAV/SCR for varying levels of the structural and market-contingent surrender parameters \( r_{SL} \) and \( \alpha \). Similar to our remark on the mortality multiplier, we observe that these parameters affect only the liabilities part of the balance sheet under scrutiny. The conclusion reached from this figures is clear: the higher surrender risk (whether structural or market-contingent), the lower NAV and NAV/SCR and therefore the riskier the insurance firm.

![Figure 20 – Sensitivities w.r.t. Market-Contingent Surrenders](image)

4.5 Sensitivity to contract parameters

We conclude this sensitivity study by examining in Figures 21 and 22 the dependence of NAV and NAV/SCR on the contract parameters, namely the guaranteed rate \( r_g \) and the bonus rate \( \delta \). The effects of these two parameters should be distinguished. Increasing the rate guaranteed to policyholders has a clear negative impact on the solvency of the insurance company, with both NAV and NAV/SCR monotonously decreasing. The behavior of NAV with respect to the bonus rate is the same as that of NAV with respect to guaranteed rate. This is not so for the behavior of NAV/SCR with respect to the bonus rate. Contrary to what could be expected, the NAV/SCR ratio is not monotonously decreasing but presents a hump. This can be explained by the fact that \( NAV_0 \) decreases more quickly than \( NAV_{99.5\%} \) for small values of the participation rate, implying an increasing value of SCR (see Eq. 25). This can nevertheless be considered as an artefact, because typical values of bonus rates offered by the industry

![Figure 21 – Sensitivities w.r.t. Guaranteed Rate](image)
are typically larger than 70%, so belong to a range where indeed NAV/SCR decreases with the participation rate.

![Graph](image_url)

**Figure 22 – Sensitivities w.r.t. Bonus Rate**

**Conclusion**

This article illustrated how the Least-Squares Monte Carlo method can be efficiently used for computing the risk budgets of an insurance company in a Solvency 2 context. We leave to another study the extension of the framework to the use of more complicated stochastic processes for asset modeling and to the management of funds that include defaultable bonds. A distinct extension that is conducted in another study makes use of non risk-neutral valuation methods, namely utility certainty equivalent methods, for pricing in an ORSA context.

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