



# Regulation Risk: is there as Danger in Reducing the Volatility

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# Prudential regulation: is there a danger in reducing the volatility?

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## Abstract

Prudential regulations adopted in response to recent crises aim to reduce risks faced by financial institutions. Nevertheless, the feelings of a large number of practitioners are mixed: if these rules seem to succeed in lowering volatility, they appear to rigidify the financial structure of the economic system and, consequently, tend to increase the probability of large jumps. In other words, the volatility risk seemed to be swapped for a jump risk, producing a negative spillover, in the sense that the aimed reduction of volatility is accompanied by the increase of the intensity of jumps. Hence, the new

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This work results from a research program carried out at the Fondation Maison des sciences de l'homme (FMSH) in collaboration with the SMA insurance group since 2009. The goal of this program was to scrutinize the logic of the development of financial standards and the weakening of the financial system the new standards could induce. Colloquia *Financial standards and regulation* (2009, 2011, 2013) presented the first steps of this research. The swap between volatility risk and jump risk finds its origin in these reflections. We thank FMSH and SMA for their support in this research program and in particular: Michel Wieviorka, administrator of FMSH, Hubert Rodarie and Philippe Desurmont, respectively Deputy CEO and head of investment management of SMA group, for their encouragement to continue in this direction. The first draft of this paper was presented at the 49th Actuarial Research Conference at UCSB, July 2014. We would like to thank the conference participants for their valuable comments and discussion. We would also like to thank professors E. Brian, E. Chiapello of Ecole des hautes études en sciences sociales (EHESS) and E. Picavet of the university of Paris 1 Panthéon-Sorbonne for helpful discussions.

rules of regulators seem create a new risk. This paper discusses this idea in three ways. First, we introduce a conventionalist framework to be able to enter the puzzle of the swap. We define two conventions of quantification for the risk metrology, which allow us to introduce the risk swap effect. Second, we define precisely the two kinds of risk (volatility and intensity of jumps) and use them to document this risk swap effect by analysing a daily time series of the S&P500. We find that the recent evolution of the index indicates simultaneously a reduction in the volatility and an increase of the intensity of jumps, a result that validates the intuition of practitioners. Third, we analyse a model which allows one to appreciate a practical consequence of this swap of risks on the risk measures: using  $\alpha$ -stable motions, we find that, for a given level of Value-at-Risk (VaR), the Tail Conditional Expectation (TCE) increases with the risk swap. We conclude by challenging the main objective of regulators: we argue that concentrating on reducing the sole volatility can create a new type of risk, that we term *regulation risk*, which increases the potential losses. This risk can be revealed with a conventionalist approach of quantification.

# 1 Introduction

In the literature, “regulatory risk” has multiple definitions. A 2014 research report of the joint Risk Management Sections of Casualty Actuarial Society, Canadian Institute of Actuaries and Society of Actuaries on “Regulatory Risk and North American Insurance Organizations”[21] proposes a survey of many instances, which are related to a multitude of bodies that influence their framework. Among others, regulatory risk can refer to the exposure to financial loss arising from the probability that regulatory agencies will make changes in the current rules (or will impose new rules) that will negatively affect the already-taken trading positions. It can be also the inability to predict a regulatory outcome. More generally, the research report emphasizes the fact that the notion of regulatory risk refers to two main strands of thought: on the one hand, *the potential and actual challenges faced by insurers and regulators under a supervisory regime arising from changes to products and regulations*, and on the other hand, *the unintended results of regulations that put at risk the ability of policyholders, shareholders or regulators to achieve their legal or fiduciary objective*. Our aim in this paper is to explore, evidence and document some facts related the second meaning above of regulatory risk. In order to delineate precisely our object of study, we shall use below the term “regulation risk” to refer to the possibility that prudential regulations may have, in some circumstances, the inverse effect of amplifying risk.

In the post-financial crisis environment, the US and EU are each pursuing modernization of their regulatory frameworks [9]. Hence, the regulatory landscape is inherently more complex, and the recent regulations aiming at reducing the risk to all financial institutions lead to a fundamental redesign of the conceptual frameworks. The paths taken by the US and EU are each impacted by these conceptual frameworks. Although different, they share a same vision of the regulation building: market risk regulations are now *model based*.

The negative spillover of risk modelling in regulatory design is the topic of this paper: we argue that flaws in understanding the nature of financial uncertainty may perversely lead to the amplification of risk. Saying that, we echo the feeling of a large number of practitioners: more precisely, if the new rules achieve to reduce the volatility, they seem to rigidify the financial structure of the economic system and, consequently, tend to increase the probability of large moves, i.e. the probability of large jumps. These rules seem to swap the volatility for the intensity of jumps, in the sense that the aimed for reduction of volatility is accompanied by an unwanted increase of the intensity of jumps. The volatility risk is apparently exchanged for a jump risk. In this situation, the new rules of regulators seem

create a new risk, which is precisely the one we call regulation risk.

The remaining of this paper is organized as follows. Section 2 introduces a conventionalist framework to emphasize the importance of the dichotomy continuity / discontinuity for the quantification of financial uncertainty and its implication for the policymaking about risk supervision. We build on the Comprehensive Actuarial Risk Evaluation (CARE) 2010 report to explain how the fundamental market risk methodologies are embedded in the regulatory process. Section 3 presents an empirical study of the S&P 500 based on a modelling with stable motion. We find that, for the recent period, a reduction of the volatility has come along with an increase of the local intensity of the jumps. By analysing how these evolutions echo on measures of risk as the VaR and TCE, we document the conjecture of [12] that the choice of the VaR by the regulator could lead to very large increases of the market risk and provoke crashes. Finally, section 4 explicits how this effect is translated into a regulation risk which, combined with a model risk, produces a market risk.

Let us emphasize that this article does not try to develop new financial or mathematical models. All those used here are well known. Our contribution lies in: 1) the introduction of the notion of regulation risk by using a conventionalist approach of quantification; 2) an empirical analysis showing the simultaneity of the reduction of the volatility and in the increase of the local intensity of the jumps; 3) the quantification of a negative spillover illustrated with a simple example on the calculation of the capital needs using the VaR and TCE measures with stable motions.

## **2 A conventionalist approach to risk metrology**

Building on the approach taken in several works in sociology (for example [10, 14]), this article rests on the idea that a connection can be established between quantification conventions and regulation framework. When one decides to use a mathematical model to quantify risk, an extremely large number of choices must be made. These choices are not what we refer to here in the concept of the "quantification convention". A quantification convention is more like a meta-convention: its name covers a configuration or a coherent set of operations both cognitive and normative, including selection of the items to take into account, relevant judgment criteria, choices of mathematical schemas, etc.

The Comprehensive Actuarial Risk Evaluation (CARE) 2010 report [22] emphasizes that the use of convenient mathematical models to quantify risk can be

like looking for your lost keys under the nearest lamp post. To go beyond the metaphor, we consider here the lamp post as a meta-convention of quantification and we understand the warning of the CARE report in the light of the conventionalist approach of models. The interest of this approach is that it allows the risk managers and the regulators to be equipped with a valuable toolkit coming from the social sciences, and also permits to address the issue of the “nearest lamp post” and to avoid the negative spillovers due to a flawed regulation.

Every quantification convention has an epistemic, a pragmatic and a political dimension. The *epistemic dimension* is built on a set of assumptions regarding uncertainty; the *pragmatic dimension* makes certain actions possible, such as trading, arbitraging and managing risk; the *political dimension* of each convention authorizes specific actors – not necessarily the same actors for each convention. Each arrival of a new convention enables the growth of certain practices and reconfigurations of some professions. Part of the epistemic dimension is the selection of relevant predictive factors drawn from today’s world that can be used to construct a decision, *i.e.* selection of what is true. Financial data, representations of financial theory and the conceptualizations of the regulator joint to co-construct a financial “reality”. In this sense, statistics must be conceived as simultaneously conventional and real [15].

We now present the two main meta-conventions of quantification of uncertainty and the resulting risk metrology.

## **2.1 The two quantification conventions of uncertainty**

Risk management and financial regulation seek to grasp a future which, by definition, is uncertain. It therefore requires assumptions concerning what “shape” of uncertainty the future will exhibit, which is in practice an assumption regarding the stochastic dynamic of financial and economic variables.

There are two fundamentally different ways of viewing uncertainty in finance, each of them being at the roots of a quantification convention. One assumes the principle of continuity, the other does not. According to the first view, following Bachelier (1900)’s legacy, price movements are modelled by continuous diffusion processes, as for instance Brownian motion. According to the other view, following Mandelbrot (1963)’s legacy, price movements are modelled by discontinuous processes, as for instance Lévy processes [30]. The nature of the risk that we highlight in this paper can essentially be described in one statement: some prudential rules based on the first quantification convention picture the risk with only a single dimension, as if the regulator’s belief was embedded in a Brownian (continuous)

representation of stock market fluctuations. In doing so, these rules leave aside many factors which are nevertheless crucial to ensure a proper taming of risk in a broader context.

To keep the discussion at an elementary level, we will focus here on the fact that, in a discontinuous paradigm, price changes are due to two factors: the instantaneous variance (or an equivalent scale parameter if the variance is infinite), and the local intensity of jumps (see Section 3.1 for precise definitions). A crucial point is that, by restricting the risk measurement to the sole “volatility”, the regulator aggregates these two factors. This is a consequence of the belief that prices essentially evolve in a continuous fashion. Under such an assumption, the variance may be identified to volatility and it provides all the relevant information needed to assess risk. But, as we shall quantify, ignoring the jumps, or at least the fact that they produce variations non-reducible to variance, has damaging consequences on both the measurement of risk and its perception. Therefore, the volatility (understood as the variance) must be supplemented by another factor. We meet again here the “nearest lamp post” syndrome (quantification convention) emphasized in the CARE report.

## **2.2 Risk metrology**

We now define the consequences of the choice of a quantification convention for the risk metrology. Firstly, we recall that the volatility appears to be sufficient in the continuity representation. Secondly, we address the issue of the jumps risks.

### **2.2.1 The first convention : continuity and volatility risk**

In physics, the so-called principle of continuity states that change in nature is continuous rather than discrete. Leibniz and Newton, inventors of differential calculus, stated the maxim: “*Natura non facit saltus*” (nature does not make jumps). This same principle underpinned not only the works of Linnaeus on the classification of species and later Charles Darwin’s theory of evolution (1859). It is also a crucial assumption in Alfred Marshall’s 1890 *Principles of Economics*, allowing the use of differential calculus in economics and the subsequent development of neoclassical economic theory. Modern financial theory grew out of neoclassical economics and naturally assumes the same principle of continuity. One of the great success stories of modern financial theory is the valuation of derivatives. Examples include the formulas of Fisher Black, Myron Scholes, and Robert Merton (1973) for valuing options, and the subsequent fundamental theorem of asset

pricing that emerged from the work of Michael Harrison, Daniel Kreps, and Stanley Pliska between 1979 and 1981. These success stories rest on the principle of continuity.

Early in the 20th century, physics and genetics abrogated the principle of continuity in favour of discontinuity: it is now widely recognized that nature does make jumps. Quantum mechanics postulated discrete energy levels while genetics took discontinuities into account. But economics, including modern financial theory, stood back from this intellectual revolution. The early attempt by Mandelbrot in 1962 to take explicit into account discontinuities at all scales in stock market prices led to huge controversies in the profession [40]. But, by the 1980's, despite the repeated financial crises following the 1987 stock market crash and despite rather overwhelming empirical counterevidence, continuity maintained popularity and the academic consensus reaffirmed the principle of continuity. Many popular financial techniques, such as portfolio insurance or the calculation of capital requirements in the insurance industry assume that (financial) nature does not make jumps and therefore promote continuity. Most statistical descriptions of time series in finance assume continuity. This is the *epistemic dimension* of the quantification convention.

It follows that Brownian representation became the standard model, part and parcel of finance curricula across the globe. It is the point of reference of most top journals in the field of finance; it is the dominant view in the financial industry itself. This is the *pragmatic dimension* of the quantification convention. And it underlies almost all prudential regulation worldwide: for instance, the so-called square-root-of-time-rule underlying the regulatory requirements (Basel III and Solvency II) for calculating minimum capital is a very narrow subset of time scaling rules of risk, and comes directly from the hypothesis that returns are independent and stationary normal, *i.e.* a Brownian framework. In other words, the principle of continuity was adopted not only in purely academic circles, but also in policymaking. This is the *political dimension* of the quantification convention.

But if the volatility perfectly summarizes the risk in a Brownian universe, that is to say with a continuous representation of financial fluctuations, this is no longer the case as soon as we consider another representation of financial uncertainty where discontinuities of all sizes exist.

### **2.2.2 The second convention : discontinuity and jump risk**

There exist large sudden variations in the markets. This is a known and documented phenomenon. The first response of regulator to the presence of these large



variations was to prescribe the use of internal models. A look at the professional practices for the implementation of internal models reveals the “come-back” of a well-established statistical theory, the so-called extreme value theory. This theory focuses on the most important variations of a random phenomenon, whether continuous or discontinuous. Without distinguishing between the two representations of uncertainty (continuous or discontinuous), prudential regulations attempted to deal with large variations in isolating them, that is to say by splitting the markets into two regimes, in effect adding large variations to smooth and small variations. The dissociation between these two market regimes is an intellectual consequence of the adoption of a paradigm of continuity. On the contrary, [25] shows that, with a discontinuous paradigm, it is not necessary to split market movements into two regimes: this paradigm allows one to unify in the same mindset quiet periods and periods of strong movements.

The first convention of quantification (continuous representation of stock market fluctuations) is dangerous for this very precise reason: discontinuities occur out of the blue, and only at large scales. Adopting this convention explains the words of Alan Greenspan, former chairman of the Federal Reserve of the United States: ‘We will never be able to anticipate all discontinuities in financial markets. Discontinuities are, of necessity, a surprise’ (*Financial Times*, March 16, 2008). With the second convention of quantification (discontinuous representation of stock market fluctuations), however, discontinuity can not be reduced to large variations, but remains present at small scales: discontinuity is a general property of the stock paths [30], not just a consequence of liquidity or other crises. It is then best to not separate large and small variations and to consider “quiet” periods, not as phases of continuity, but as a succession of micro-discontinuities (see Section 3.1 for details). Analysing discontinuities at small scales helps understanding the intrinsic fragility of a market and the occurrence of significant financial risks [19, 25]. The financial large scale risk management can be improved by extrapolating large discontinuities (large risks) from small moves (small discontinuities due to partial lack of liquidity, etc..).

In this framework, it is necessary to consider at least two independent factors governing price movements: in addition to volatility, which is a scale parameter, it is crucial to take into account the intensity of jumps, or *erraticity*, a factor which calibrates the statistical distribution of the sizes of jumps. In addition, because of the non-stationarity of the markets, it is essential to consider instantaneous versions of these two variates: an adequate understanding of risk requires measuring the local volatility and local intensity jumps. Empirical studies confirm the relevance of this view: typically, price changes exhibit jumps, and even an infinite

number of jumps [1]; the intensity of these jumps and the volatility change over time; and all these market characteristics impact risk measures such as VaR [16].

### **3 Uncertainty modelling and risk of risk measures**

In order to put the above elements in a form amenable to computations, we need to precise the characteristics of the quantification convention with a class of models for prices variations. Our aim here is not to determine the most adequate model but rather to evidence the impact of the presence of jumps on market risk. For reasons given in [1], we will choose infinite activity additive processes (that is, processes with infinite number of jumps) and we will base our discussion on the simplest class of these processes, which is the one of stable motions. Note however that more complex processes with infinite activity, such as CGMY ones, could be considered instead.

#### **3.1 Recalls on stable motions**

We briefly recall some basic notions on stable motions. These will be used in the empirical study of Section 3.2 as well as in Section 3.3. They will allow us to define and estimate the instantaneous scale parameter and stability exponent, which are respectively our local volatility and jump intensity measures.

Stable motions were introduced in financial modelling in [18, 30]. They have been since the subject of numerous studies, for instance in relation with pricing issues [7, 32, 36] or risk management [24, 31].

The main feature of interest to us is that paths of non-Gaussian stable motions almost surely display jumps. In fact, they are so-called “pure jump” processes, that is, they only move through jumps, and they possess almost surely a countably infinite number of jumps on any time interval. This allows one to account for the activity on markets in a way very different from the one provided by Brownian motion, or, more generally, continuous diffusions. More precisely, one may argue that in practice prices evolve in a discontinuous fashion, since their movements are quantized. Now, all pure jumps processes considered in financial modelling are such that “most” jumps are “small” (see below for a more precise statement), and thus akin to account for price changes even in “quiet” periods. For this reason, most of what is developed below would remain valid with other infinite activity pure jump processes.

A stable motion is a stochastic process with stationary and independent increments (as is Brownian motion), whose increments follow an  $\alpha$ -stable law. Such laws are described by their characteristic function, which takes the following form ( $\text{sign}(u)$  denotes the sign of  $u$ ):

$$\varphi(u) = \begin{cases} \exp \left\{ i\mu u - \sigma^\alpha |u|^\alpha \left[ 1 - i\beta \text{sign}(u) \tan\left(\frac{\alpha\pi}{2}\right) \right] \right\} & \text{if } \alpha \neq 1 \\ \exp \left\{ i\mu u - \sigma |u| \left[ 1 + i\beta \text{sign}(u) \frac{2}{\pi} \ln(|u|) \right] \right\} & \text{if } \alpha = 1 \end{cases} \quad (1)$$

Choosing  $\alpha = 2$  above yields the characteristic function of a Gaussian RV, a case we exclude from now on. As the definition of  $\varphi$  shows, stable laws are characterized by four parameters:

1. The number  $\alpha$  ranges between 0 and 2, and it quantifies the distribution of the size of jumps: within a given period of time, and for any integer  $j$ , the mean number of jumps of a stable motion whose increments follow a stable law with parameter  $\alpha$  with size of order  $2^j$  is proportional to  $2^{-j\alpha}$ . In particular, the mean number of jumps larger than any non-zero threshold is always finite: “most” jumps are “small” as announced above. Besides, when  $\alpha$  is large (close to 2), the mean number of jumps decreases fast when their size increases, while, when  $\alpha$  is close to 0, it decreases slowly: a large  $\alpha$  corresponds to a small jump intensity, and vice-versa.
2. The positive real  $\sigma$  is a scale parameter: multiplying the RV by  $a > 0$  transforms  $\sigma$  into  $a\sigma$  (in the Gaussian case,  $2\sigma^2$  is the variance). One may thus identify  $\sigma$  as governing volatility.
3. The real number  $\mu$  is a location parameter: adding  $a$  to the RV transforms  $\mu$  into  $\mu + a$ . Also, when  $\alpha > 1$ ,  $\mu$  coincides with the expectation of the RV.
4. The real number  $\beta$ , which ranges in  $[-1, 1]$ , is a skewness parameter. A distribution that is symmetric around  $\mu$  has  $\beta = 0$ .

Our focus in this work is on the parameters  $\alpha$  et  $\sigma$ , which we recall account respectively for the jump intensity and the volatility. As there is no reason to believe that these numbers remain constant in time, we will consider local versions, denoted  $\alpha(t)$  and  $\sigma(t)$ .

## 3.2 Empirical study

Figure 1 displays the local volatilities and jump intensities on daily data of the S&P 500, between 01/03/1928 and 02/01/2012. Volatilities and jump intensities were estimated using two classical statistical methods, namely the McCulloch and Koutrouvelis ones.

We first note that both methods give very similar results as far as  $\sigma$  is concerned. The difference between the estimated  $\alpha$  values is somewhat larger. This is however of little consequence for us, since both estimators yield roughly parallel curves: as our aim is to compare the evolutions of  $\sigma$  and  $\alpha$ , a constant shift in the stability exponent does not modify our conclusions.

A second fact is that, since 1960 or so, the jump intensity and the volatility evolve in opposite directions: when the volatility (that is,  $\sigma$ ) increases, the jump intensity decreases (since  $\alpha$  increases), and vice versa. In other words,  $\alpha$  and  $\sigma$  almost always move in the same way. Thus, as was announced above, when the market is less “rough”, or less “nervous”, it is more prone to large jumps.

A final noteworthy fact is that the evolution of the last years does confirm the general feeling of practitioners: volatility has significantly decreased, but at the expense of a notable increase of jump intensity. The market appears to be most of the time “under control”, but the risk that it produces large jumps has increased. In [27], we emphasized this as follows: “the market is very quiet, except when it moves a lot”.

A legitimate question is then the following one: could it be the case that these coordinated evolutions of  $\sigma$  and  $\alpha$  be themselves, at least in part, consequences of the constraints imposed by certain prudential regulations under a given quantification convention? Indeed, these rules concentrate on the volatility, aiming at reducing it. This effect is confirmed by our empirical study. However, it could well be that the very mechanism that reduces volatility also leads to a decrease of  $\alpha$ . In such a scenario, we would have an example where regulation risk directly translates into a market risk, without being mediated by a model risk as in the situation considered in the next section. A study of the potential influence of prudential rules on the evolution of jump intensity and volatility is presented in [26].

## 3.3 Consequences on risk measures

We now turn our attention to the risk measures. According to the International Actuarial Association (IAA) note on the use of models for risk management [23],

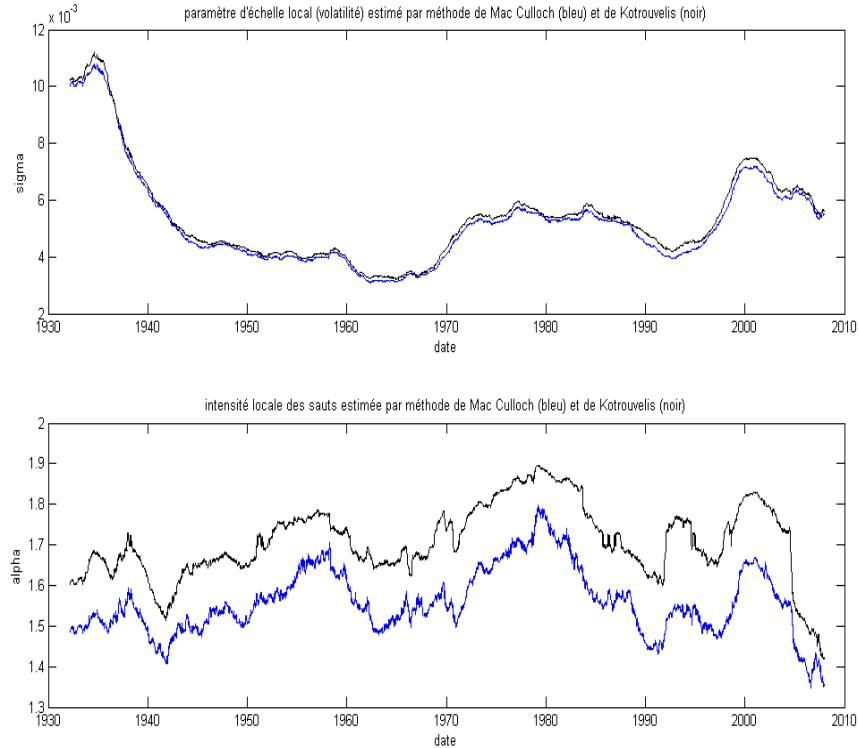


Figure 1: Compared evolutions of the local volatilities (top) and jump intensities (bottom). Estimations performed with the McCulloch and Koutrouvelis methods on daily data of the S&P 500 between 01/03/1928 and 02/01/2012. Each value is estimated on a centred sliding window of size 2000 data points.

a risk measure is a numeric evaluator that can be used to help to determine the solvency capital requirement for an insurer. In this context, VaR is the main risk measure put forward by the regulator.

### 3.3.1 The coherency of risk measures

Recall that VaR at confidence level  $1 - p$  and horizon  $T$  is simply the amount such that the probability that losses at time  $T$  be smaller than VaR is equal to  $p$ , so that

$$\mathbb{P}(X_T < -\text{VaR}) = 1 - p.$$

VaR is simple to understand, and it reflects adequately certain aspects of risk. There is a huge quantity of works devoted to its study, that cannot be reviewed here. We just recall here some well-known shortcomings of VaR [12, 17, 23]. First, it is not a coherent measure of risk, in the sense that it is in particular not sub-additive: in general, the VaR of  $X_T + Y_T$  is not necessarily smaller than or equal to the sums of the VaRs of  $X_T$  and of  $Y_T$ . This is counter-intuitive for instance because it does not account for the reduction in risk typically entailed by diversification. In this respect, we note however that, in our frame where  $X_T$  follows an  $\alpha$ -stable distribution, this problem does not arise: as shown in [39], VaR is a coherent risk measure for sums of independent stable random variables when  $\alpha \geq 1$ .

Other limitations of VaR remain. For instance, it does not give any indication of what happens beyond VaR, although this is a crucial information. This is why various other risk measures are considered to complement it. We shall be interested in the quantity called *Tail Conditional Expectation* (TCE), which was recommended by the IAA report [23] and has recently been adopted by the Basel regulation [4]. TCE at confidence level  $1 - p$  and horizon  $T$  is defined as:

$$\text{TCE} = \mathbb{E}(X_T \mid X_T < -\text{VaR}).$$

In words, TCE gives the mean loss beyond VaR. It is a coherent risk measure.

### 3.3.2 Risk measures and quantification conventions

We now turn our attention to the quantification conventions. In the second quantification convention (discontinuous representation), when  $X_T$  follows an  $\alpha$ -stable distribution, one can compute explicit asymptotic values of VaR and TCE when  $p$  tends to 1, that is, for very small risk levels. We note that, for given asset and time horizon, it may well be the case that, even for a high confidence level, say  $p = 99.5\%$ , one is still far from the region where the asymptotic holds. As a consequence, the computations presented below should be considered as indications of a general behaviour rather than as exact values.

Let:

$$C_\alpha = \frac{1 - \alpha}{\Gamma(2 - \alpha) \cos(\pi\alpha/2)}$$

where we assume for simplicity that  $\alpha \neq 1$ . Then, if  $X_T$  follows a stable distribution with parameters  $\alpha < 2, \sigma, \mu, \beta = 0$ , one has [37]:

$$\lim_{\lambda \rightarrow \infty} \lambda^\alpha \mathbb{P}(X_T < -\lambda) = \frac{C_\alpha}{2} \sigma^\alpha.$$

A similar formula holds in the non-symmetric case ( $\beta \neq 0$ ), but we restrict to  $\beta = 0$  for simplicity. Assuming that  $p$  is close enough to 1, and thus that VaR is sufficiently large, one thus has, by definition,

$$\text{VaR}^\alpha(1-p) \simeq \frac{C_\alpha}{2} \sigma^\alpha,$$

or

$$\text{VaR} \simeq \sigma \left( \frac{C_\alpha}{2(1-p)} \right)^{\frac{1}{\alpha}}.$$

One checks that, as is intuitively clear, VaR increases linearly with volatility. How it varies as a function of  $\alpha$  is less obvious from the formula. Figure 2 displays the evolution of VaR when  $\alpha$  varies between 1.1 et 1.9. As one can see, VaR decreases when  $\alpha$  increases, a fact that fits with intuition: a larger jump intensity corresponds to a riskier market and thus a larger VaR.

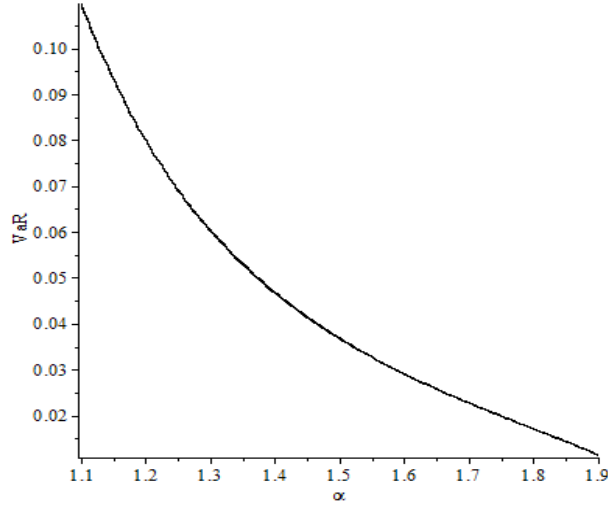


Figure 2: VaR as a function of  $\alpha$  for  $\sigma = 0.005$  and  $p = 0.99$ .

Let us now deal with TCE. Assuming  $\alpha > 1$ , one can show [25, 42] that, when  $p$  is sufficiently close to 1,

$$\text{TCE} \simeq \frac{\alpha}{\alpha - 1} \text{VaR}. \quad (2)$$

The important point for us in the above formulas is the following one: assume that the market evolves in such a way that the volatility decreases while the jump

intensity increases (whether this joint evolution is the result of prudential regulations or not). This is what we have observed for recent years in our empirical study. Then, at constant VaR<sup>2</sup>, one deduces from (2) and from the fact that the function  $\alpha \rightarrow \frac{\alpha}{\alpha-1}$  is decreasing, that TCE will increase. If, for instance,  $\alpha$  moves from 1.75 to 1.4, as is observed empirically on Figure 1, then, although the VaR risk measure does not change, TCE is multiplied by 1.5. This is indeed a strong negative impact on financial companies. This simple mechanism highlights how model risk and regulation risk may combine to create a market risk.

## 4 Regulation risk in a conventionalist approach

We now conclude with the notion of regulation risk we introduced at the beginning of this work.

In the financial and statistical literature, model risk is traditionally understood as the existence of a discrepancy between a given probabilistic model and the ‘true’ nature of the financial phenomenon. This approach belongs to a classical epistemology scheme in which idealized models are facing the ‘true’ world, either with a descriptive or a prescriptive role. For instance, the following definition explicitly displays this understanding: a model is “an idealized representation of reality that highlights some aspects and ignores others” [34]. With such an epistemology, it is possible to distinguish model risk (designer-based risk) and market risk (reality-based risk), because one can consider separately the model and the market, and thus the associated risks. This approach neglects an important component of model risk: the risk that the model itself can induce by influencing professional practices, either directly (tool-based influences) or through prudential rules (regulatory-based influences). This so called reflexive effect of models is crucial and must be taken into account in the management of financial risks [41]. For risk management issues, the ‘representational idiom’ of [34] has to be replaced by a ‘performative idiom’ [35]: it advocates for a move from an understanding of science as an attempt to represent ‘nature’ to an understanding of science as a mechanism creating ‘reality’.

The CARE report [22] definition of model specification risk is that the structure of the model itself is incorrect. The CARE report illustrates this model risk with the example of the erroneous use of the lognormal distribution when a Pareto

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<sup>2</sup>The assumption of constant VaR is justified by the fact that the regulator imposes solvency capital requirements which are increasing functions of VaR. In order to maintain these requirements at a reasonable level, companies wish to control their VaR as much as possible



distribution would have been a better representation of the underlying process. This is the sense in which we use the notion of model specification risk: the erroneous use of a continuity assumption when a discontinuity one would have led to a better representation of the underlying process. But we also say something more. Using the taxonomy of risks introduced in the CARE report, we argue that the model specification risk combines with the regulatory risk to create a new type of market risk we term regulation risk, a new risk which can be revealed with a conventionalist approach.

In section 2, we highlighted how financial data and quantification conventions of uncertainty join to co-construct a ‘reality’ of financial markets. We are now in a position to understand more precisely what this co-construction carries: a particular morphology of financial uncertainty. The so-called ‘observations’ are nothing but chains of mediation and human choices: for example, a chart of market path is the result of several steps of successive specific choices related to the relevant variables, the time scale, the length duration etc. In the same manner, risk metrology is the result of successive specific choices related to the convention of quantification that underpins prudential regulations. The first convention considers that prices movements are only due to volatility<sup>3</sup>, while an adequate model should distinguish at least two irreducible dimensions of risk, namely volatility and jump intensity.

To summarize, the non trivial relationships between the calculations and the institutions give birth to a morphology of uncertainty which may be different from the one that is desired. This fact was conjectured in [5] and our study is an example that makes precise the way it works. This negative spillover is exactly what we call regulation risk: by conveying the first quantification convention with prudential standards, economic incentives or regulatory constraints in order to reduce financial uncertainty, regulators create the uncertainty they seek to curb. This policymaking corresponds exactly to the political dimension of the quantification convention we introduced above. Specifically, regulators can provoke the very accidents that they want to avoid: here, in a market with both large and small jumps, imposing a VaR level increases TCE.

Furthermore, given its specific conventionalist characteristics, backtesting is a poor way to capture regulation risk. This new risk can be analysed either from an economic or statistical perspective [3, 11, 13] or from a sociological perspective

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<sup>3</sup>Note that the danger of such a reduction has been emphasized rather early and has been studied in a stream of research that tries to evaluate volatility risk model. For instance, the fact that volatility alone is not sufficient for characterizing the dynamics of returns and how it may be complemented is analysed in [2, 20].

[6, 28, 29, 33]. Both approaches yield the same findings: model risk has an effect on the phenomenon they seek to understand because they shape professional practices through regulations and technical tools that rely on the conventions of quantification they carry. In this situation, our work shows that risk will also come from a bad shaping of business practices by an inadequate regulation, which itself rests on an erroneous representation of uncertainty, embedded in a particular quantification convention.

We suggest that the effect evidenced in this article is a special case of a class of regulation risks that remains to be explored.

## References

- [1] Y. Aït-Sahalia and J. Jacod (2011). Testing whether jumps have finite or infinite activity, *Ann. Statist.* **39**, (3), pp. 1689-1719.
- [2] A. Alami and E. Renault (2000), “Risque de modèle de volatilité”, *Journal de la société française de statistique*, 141 (1-2), p. 103-136.
- [3] S. Auray, C. Gouriéroux (2014) “The procyclicality of financial markets regulation”, *Opinion and Debates*, Labex Louis Bachelier, 4.
- [4] Basel Committee on Banking Supervision (2003), *Basel II. Significant Progress on Major Issues*.
- [5] E. Brian (2009), “Aléas, normes sociales et limite de la performativité”, in C. Walter, *Nouvelles normes financières. S’organiser face à la crise*, Paris, Springer, 2009, p. 190-219.
- [6] M. Callon, Y. Millo, F. Muniesa (eds.) (2007), *Market Devices*, Blackwell.
- [7] P. Carr and L. Wu (2002). The finite moment log stable process and option pricing. *CRIF Working Paper series*, paper 19.
- [8] P. Carr, H. Geman, D. Madan and M. Yor (2002). The fine structure of asset returns: an empirical investigation. *Journal of Business*, **75**, p. 305-332.

- [9] Committee on Finance Research and CAS, CIA, SOA Joint Risk Management Section, Life Insurance Regulatory Structures and Strategy: EU compared with US.
- [10] E. Chiapello (2014), “Financialisation of Valuation”, *Human Studies*, on line 09 Dec 2014, DOI 10.1007/s10746-014-9337-x.
- [11] J. Danielsson (2002), “The emperor has no clothes: Limits to risk modelling”, *Journal of Banking & Finance*, 26, 1273-1296.
- [12] J. Danielsson, P. Embrechts, C. Goodhart, C. Keating, F. Muennich, O. Renault and H. S. Shin (2001) “An Academic Response to Basel II”, LSE Financial Market Group.
- [13] J. Danielsson, K. James, M. Valenzuela and I. Zer (2014), “Model Risk of Risk Models”, Federal Reserve Board Finance and Economics Discussion Series, 2014-34.
- [14] A. Desrosières (2003), “Managing the economy”, in T. Porter, D. Ross (eds.) *The modern social sciences, The Cambridge History of Science*, vol. 7, Cambridge, Cambridge University Press.
- [15] A. Desrosières (2009a), “How to be real and conventional: A discussion of the quality criteria of official statistics”, *Minerva*, 47 (3), Special Issue: Governance of and Through Science and Numbers (September), p. 307-322.
- [16] H. El Mekeddem and J. Lévy Véhel (2013). Value at Risk with tempered multistable motions. *30th International French Finance Association Conference*, May 2013, Lyon, France.
- [17] P. Embrechts, A. McNeill and D. Strautman (1999), “Correlation and Dependence in Risk Management: Properties and Pitfalls”, ETH Zurich.
- [18] E. Fama (1965). The behaviour of stock market prices. *Journal of Business*, **38**, pp. 34-105.
- [19] H. Geman (2002), “Pure jump Lévy process for asset price modelling”, *Journal of Banking & Finance*, 26, p. 1297-1316.
- [20] C. Gouriéroux and G. Le Fol (1997), “Volatilité et mesures du risque”, *Journal de la Société française de statistique*, 138, n°4, p. 7-32.

- [21] T. Herget and D. Sandberg (2014), Regulatory Risk and North American Insurance Organizations, Joint report from Casualty Actuarial Society, Canadian Institute of Actuaries, Society of Actuaries.
- [22] International Actuarial Association (2010a), Comprehensive Actuarial Risk Evaluation (CARE), IAA Papers, May.
- [23] International Actuarial Association (2010b), Note on the use of Internal Models for Risk and Capital Management Purposes by Insurers, IAA Papers, November.
- [24] I. Khindanova, S. Rachev and E. Schwartz, (2001). Stable Modelling of Value at Risk. *Math. Comput. Model.*, **34** (9-11), pp. 1223-1259.
- [25] O. Le Courtois and C. Walter (2014). *Extreme Financial Risks and Asset Allocation*, London, Imperial College Press.
- [26] J. Lévy Véhel (2015). *A simple isochore model evidencing regulation risk*, preprint.
- [27] J. Lévy Véhel and C. Walter (2002). *Les marchés fractals. Efficience, rupture et tendances sur les marchés financiers [Fractal markets: Efficiency, crashes and trends on financial markets]*, Paris, PUF, coll. "Finance".
- [28] D. MacKenzie (2006), *An Engine, Not a Camera. How Financial Models Shape Markets*, London, MIT Press.
- [29] D. MacKenzie, F. Muniesa, L. Siu (eds.) (2007), *Do Economists Make Markets? On the Performativity of Economics*, Princeton, Princeton University Press.
- [30] B. Mandelbrot, (1963). The variation of certain speculative prices. *J. of Business*, **36**, pp. 394-419.
- [31] S. Mittnik, S. Rachev and E. Schwartz (2002). Value-at-Risk and asset allocation with stable return distributions, *Advances in Statistical Analysis*, **86** (1), pp. 53-67.
- [32] Y. Miyahara and N. Moriwaki (2009). Option Pricing Based on Geometric Stable Processes and Minimal Entropy Martingale Measures.

*Recent Advances in Financial Engineering*, Proceedings of the 2008 Daiwa International Workshop on Financial Engineering, World Scientific, pp.119-133.

- [33] F. Muniesa (2014), *The Provoked Economy: Economic Reality and the Performative Turn*, London: Routledge.
- [34] J. Pearl (2000). *Causality: Models, reasoning, and inference*. Cambridge, England: Cambridge University Press.
- [35] A. Pickering (1995), *The Mangle of Practice: Time, Agency and Science*, Chicago: University of Chicago Press.
- [36] I. Popovalina and P. Ritchken (1998), “On bounding option prices in Paretian stable markets”, *Journal of Derivatives*, Summer, pp. 32?43.
- [37] G. Samorodnitsky and M.S. Taqqu, M.S. (1994), *Stable Non-Gaussian Random Processes*, Chapman and Hall.
- [38] K.I. Sato, (1999). *Lévy Processes and Infinitely Divisible Distributions*, Cambridge: Cambridge University Press.
- [39] W. Sy (2006). On the Coherence of VaR Risk Measures for Levy Stable Distributions, APRA working paper, <http://www.apra.gov.au/AboutAPRA/Research/Documents/On-the-Coherence-of-VaR-Risk-Measure-for-Levy-Stable-Distributions-May-2006.pdf>
- [40] C. Walter (2009), “Research of scaling laws on stock market variations”, in P. Abry, P. Gonçalves, J. Lévy Vehel (éds.), *Scaling, Fractals and Wavelets*, London, Wiley, p. 437-464.
- [41] C. Walter (2011), “Performance et surveillance du système financier [Performance and supervision of the financial system]”, *Revue d'économie financière*, 101, p. 105-16.
- [42] L. Zhu and H. Li (2010), Asymptotic Analysis of Tail Conditional Expectations, preprint.  
[http://www.math.wsu.edu/math/faculty/lih/TCE-Zhu\\_Li-May2010.pdf](http://www.math.wsu.edu/math/faculty/lih/TCE-Zhu_Li-May2010.pdf)