

# **Investment Horizon, Risk Drivers and Portfolio Construction**

**Institute of Actuaries Australia – Insights Seminar**  
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# Overview

- **The key drivers of investment risk shift with horizon**
  - Short-term: changes in discount rates, cash flow expectations
  - Long-term: expected return level, reinvestment rates, cash flow delivery
- **Optimal portfolios may differ with horizon**, depending on:
  - Whether there is mean-reversion
  - Objective function: referencing a target for wealth or returns induces increasing preference for equities with horizon
- **Implications**
  - Focus investment process on drivers that matter most for your horizon
  - Consider reference-based utility functions, e.g. prospect theory
  - Mean-variance and factor paradigm is the source of some misdirection

# Motivation

- Some influences:
  - CIFR long-term investing research
  - ANU Student Managed Fund
  - MDUF project (see <http://membersdefaultutilityfunction.com.au/>)
- Mean-variance optimisation and factor analysis focused on returns over a single (short) period ... a distraction for long-term investors
- Need to better connect analysis to objectives ... especially where they involve longer-term wealth outcomes, e.g. retirement savings
- Short-term => ‘price drivers’  
Long-term => ‘value drivers’
- Utility defined over wealth places a ‘score’ on entire distribution

## Existing research

- MPT, horizon and non-iid returns (*Campbell & Viceira, etc*)
- Multi-period asset pricing (*Merton, etc*)
  - Changes in the investment opportunity set
  - Dynamic asset allocation / stochastic control
- Cash flow vs. discount rate effects (*Campbell & Shiller, etc*)
  - Cash flow innovations = permanent loss of value
  - Discount rate changes = reordering of return sequence, plus change in investment opportunity set
- Debate over time diversification and Kelly strategies
  - Kelly strategies: asset with highest geometric return ‘almost stochastically dominates’ as horizon lengthens
  - Samuelson and Merton beg to differ

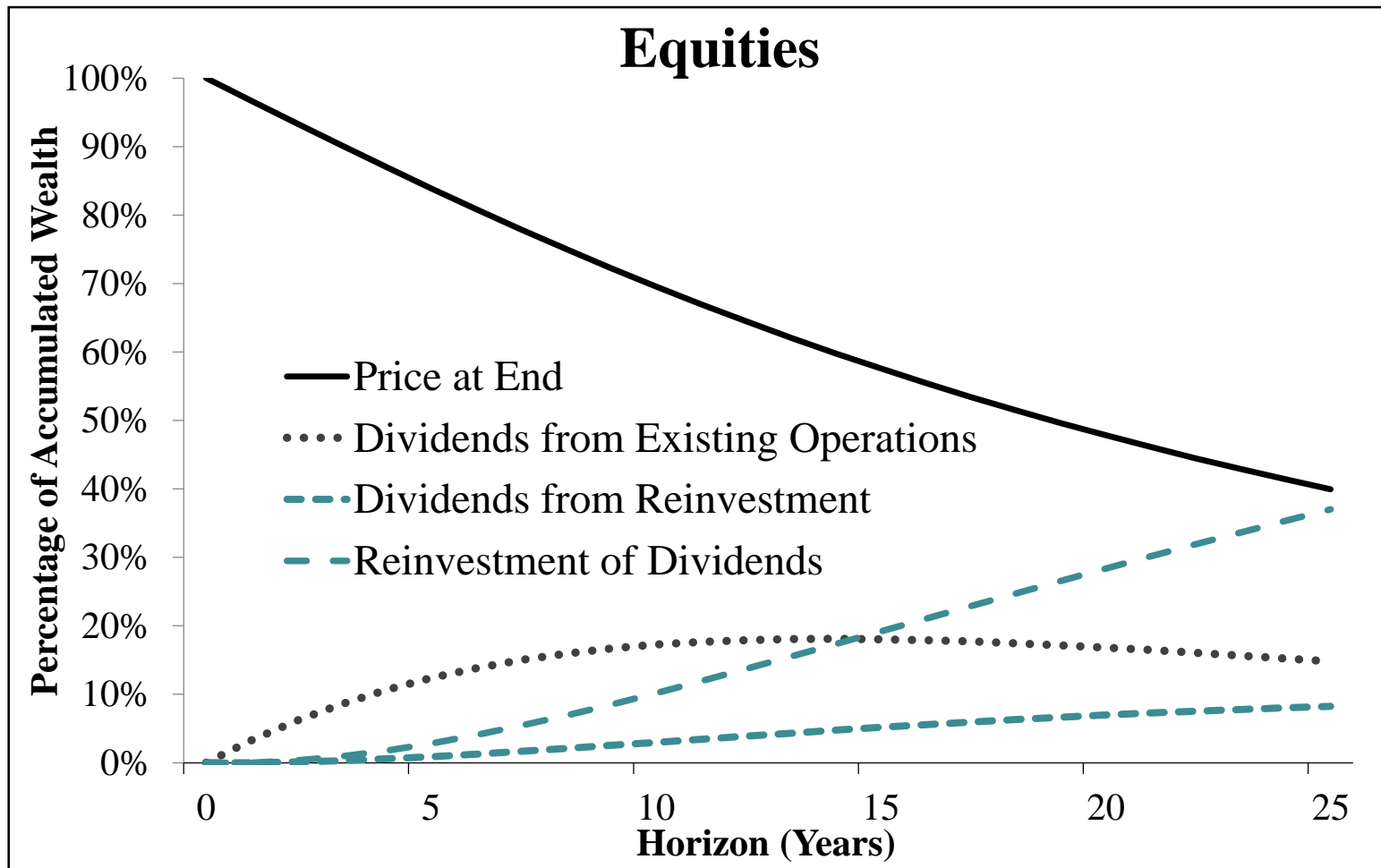
## PART A: Analysis of risk drivers and horizon

- Focus on end-of-horizon wealth
  - More general than first appears
  - Investors typically don't think dynamically, they react to circumstances
- Estimate expected accumulated wealth over time
  - Includes wealth generated from reinvestment by either:
    - (a) The investor, at prevailing discount rates each period
    - (b) An agent, e.g. companies ... possibly at a different rate
  - DCF principles, e.g. at any time  $t$ , Price = NPV of future cash flows at the discount rate prevailing at that time
- Equities, 10-year bond, 5-year bond and 1-year bond (cash proxy)
- Establish baseline expected wealth given cash flows, discount rates and reinvestment rates; then investigate impact of change in inputs

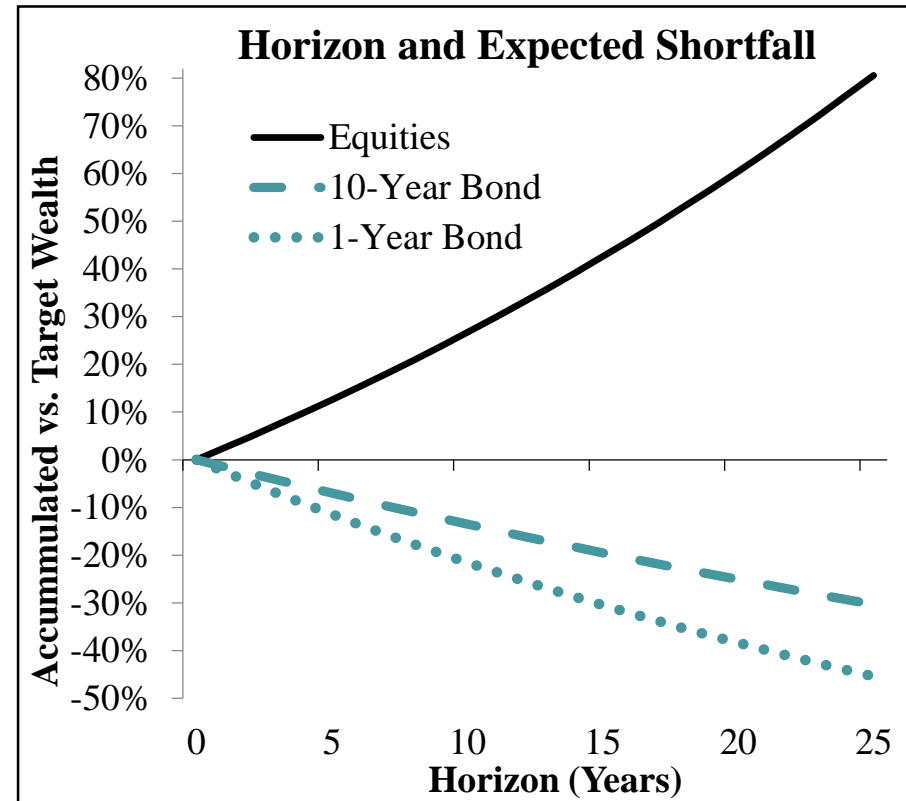
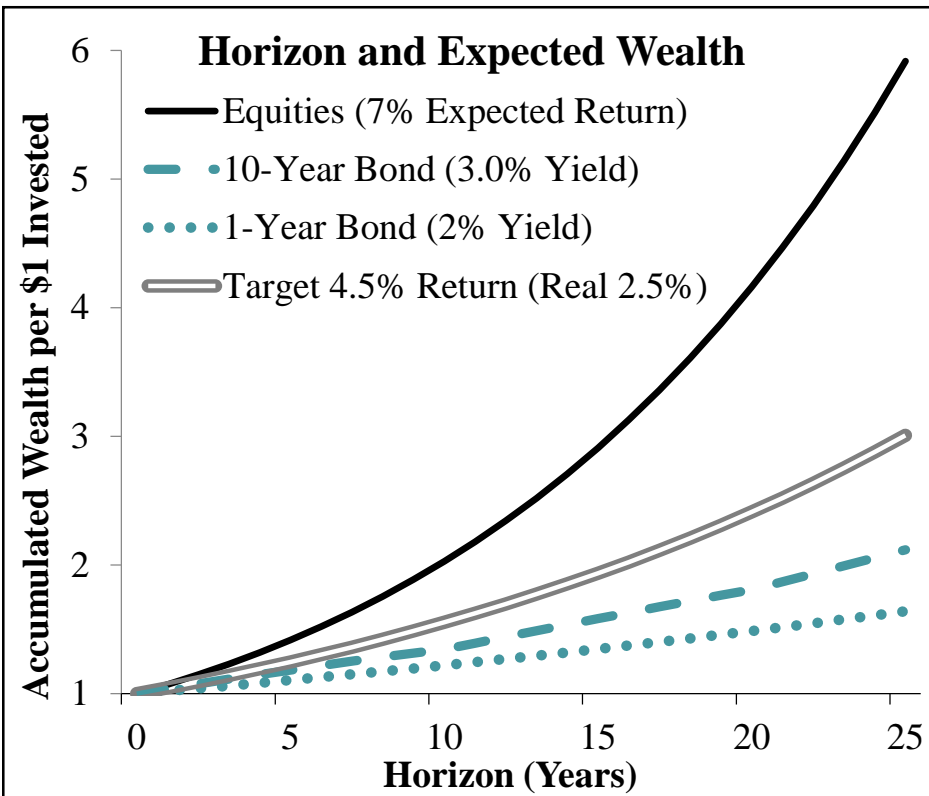
# Drivers of wealth over time

Driver	Nature	Horizon effect on wealth
1) Expected return	Foundation of baseline expected wealth at end of horizon	Impact builds with horizon due to compounding
2) Discount rate innovations	Causes immediate price change; but level of expected return adjusts thereafter	Relation negative in short term (rise in discount rate => lower price); but impact reduces and may reverse over time
3a) Reinvestment rates – Distributions	Distributions reinvested by investor at different rate than expected due to change in discount rates	Impact increases with horizon
3b) Reinvestment rates – Retention	Retained cash flows reinvested at different rate than expected due to changing investment opportunities, or agency effects	Impact increases with horizon
4) Cash flow innovations	Price and hence wealth impacted by changes in cash flow expectations	Impact felt across all horizons: permanent loss of value

# Contributions to accumulated wealth



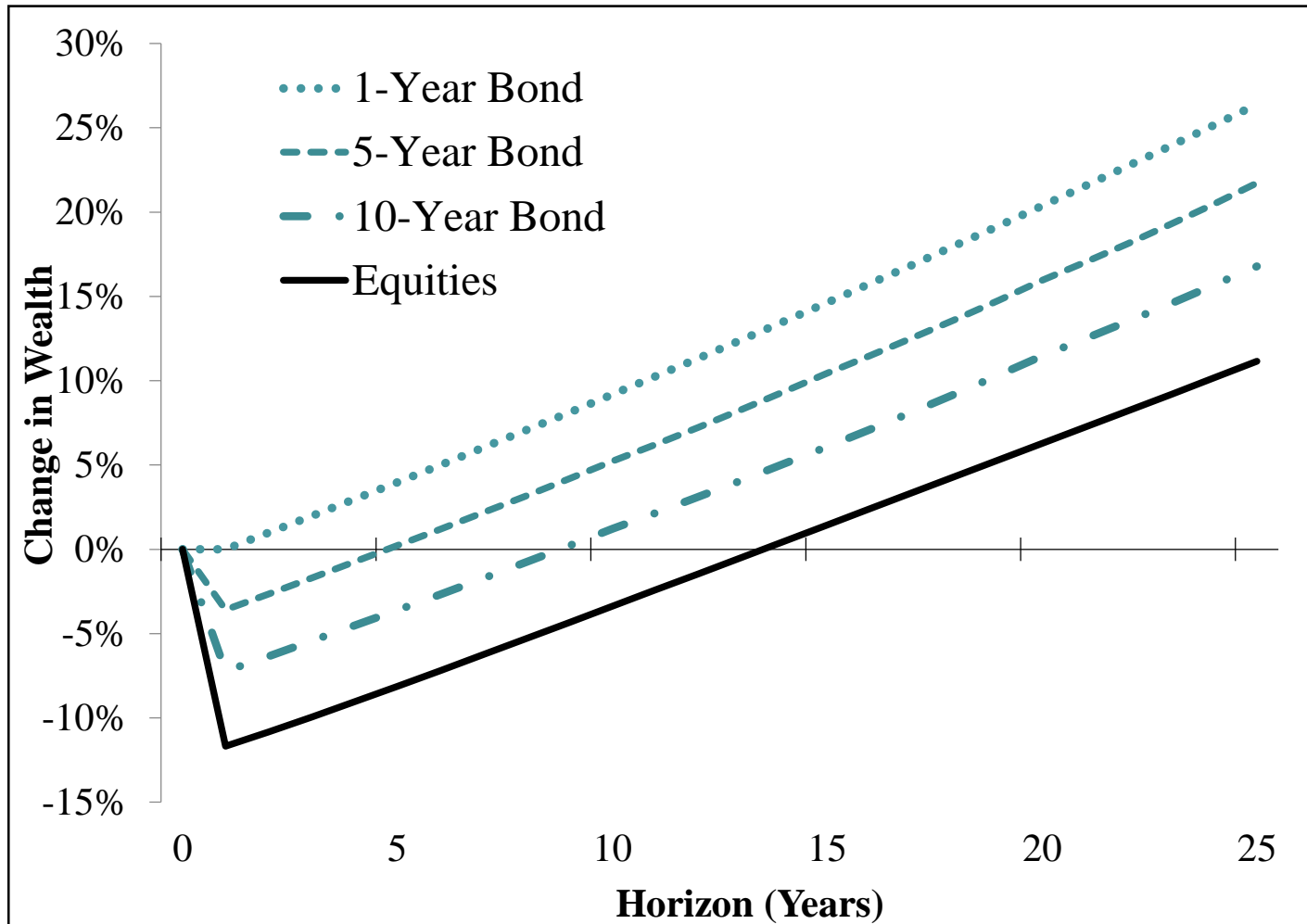
# Expected wealth, target wealth and shortfall



- Expected return may be considered ‘return on offer in the market’
- Easier to observe for bonds than equities
- Effect of mis-estimating expected return compounds with horizon

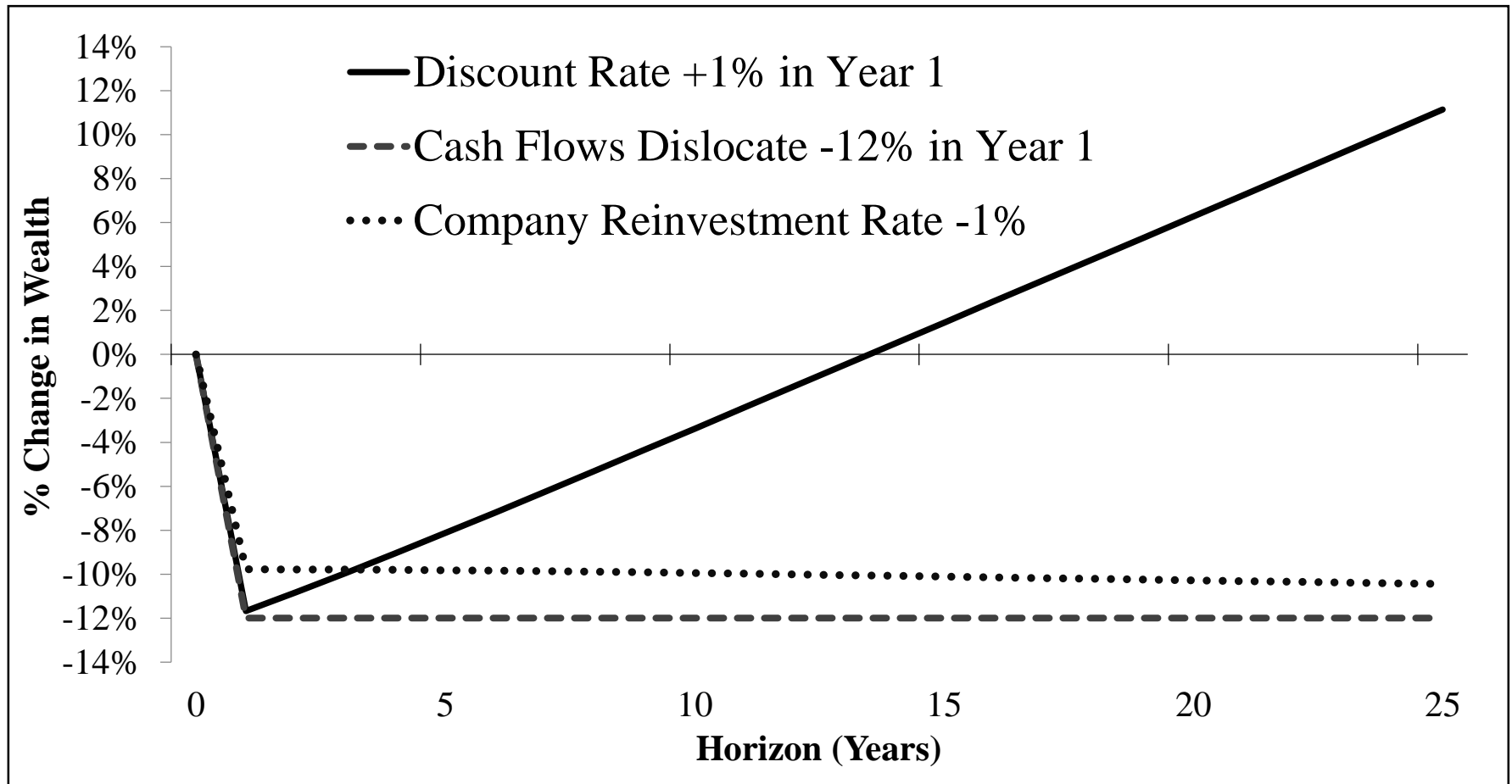


## Impact of +1% increase in discount rates



*(Duration determines initial price decline, and breakeven point)*

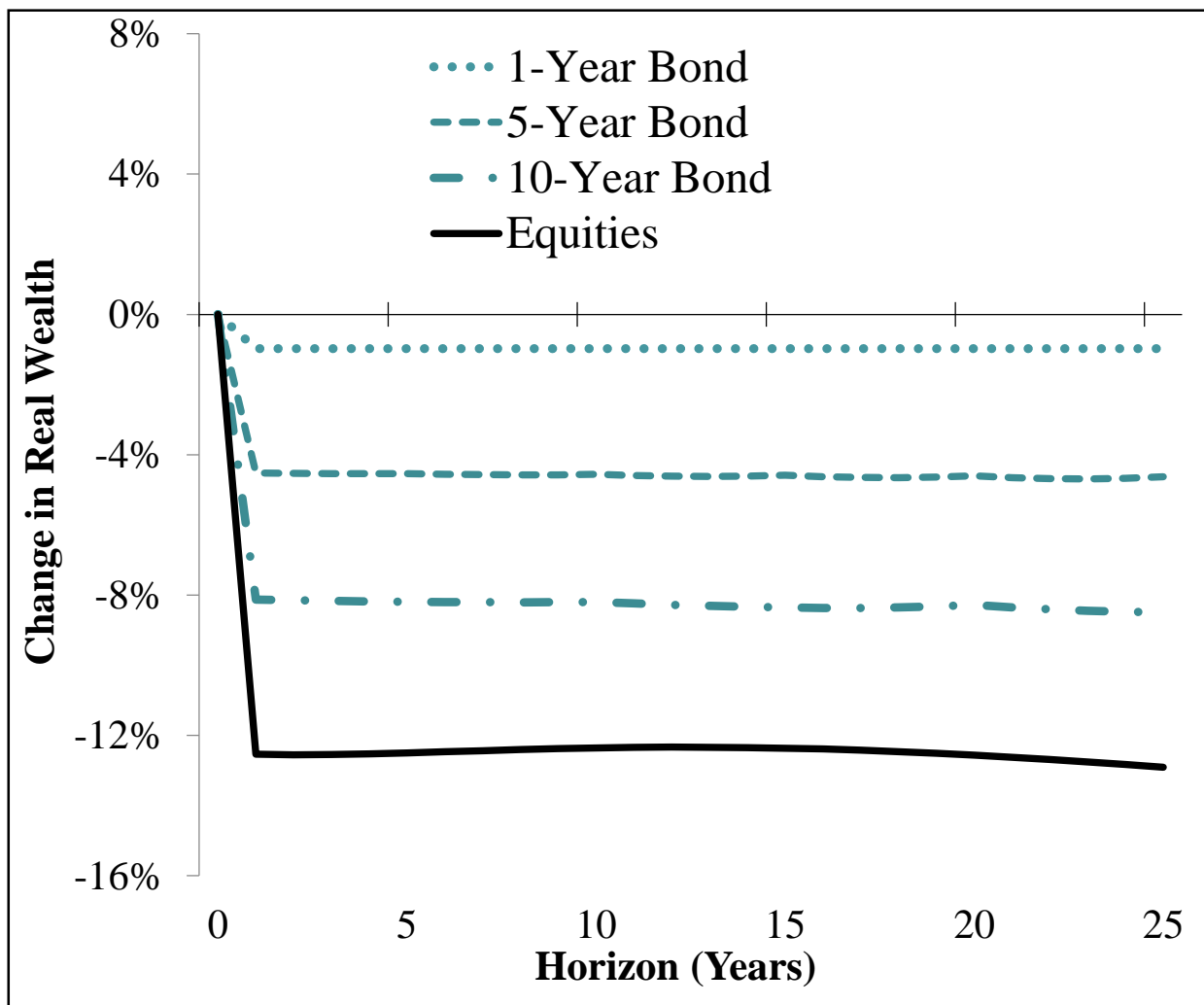
# Wealth effects of various innovations for equities



## Inflation – Real effects are the issue

- **Discount rates:** Inflation changes affect nominal rates. But do they affect real rates? Or do rates fully adjust, and neutralise the impact?
- **Cash flows:** How do real cash flows respond to inflation changes?
  - *Nominal bonds* – real value of promised cash flow decreases. This is a cash flow effect under the framework.
  - *Equities* – depends on how cash flows respond
  - *Inflation-linked bonds* – cash flow is guaranteed in real terms (but they are still exposed to discount rate and reinvestment rate effects)

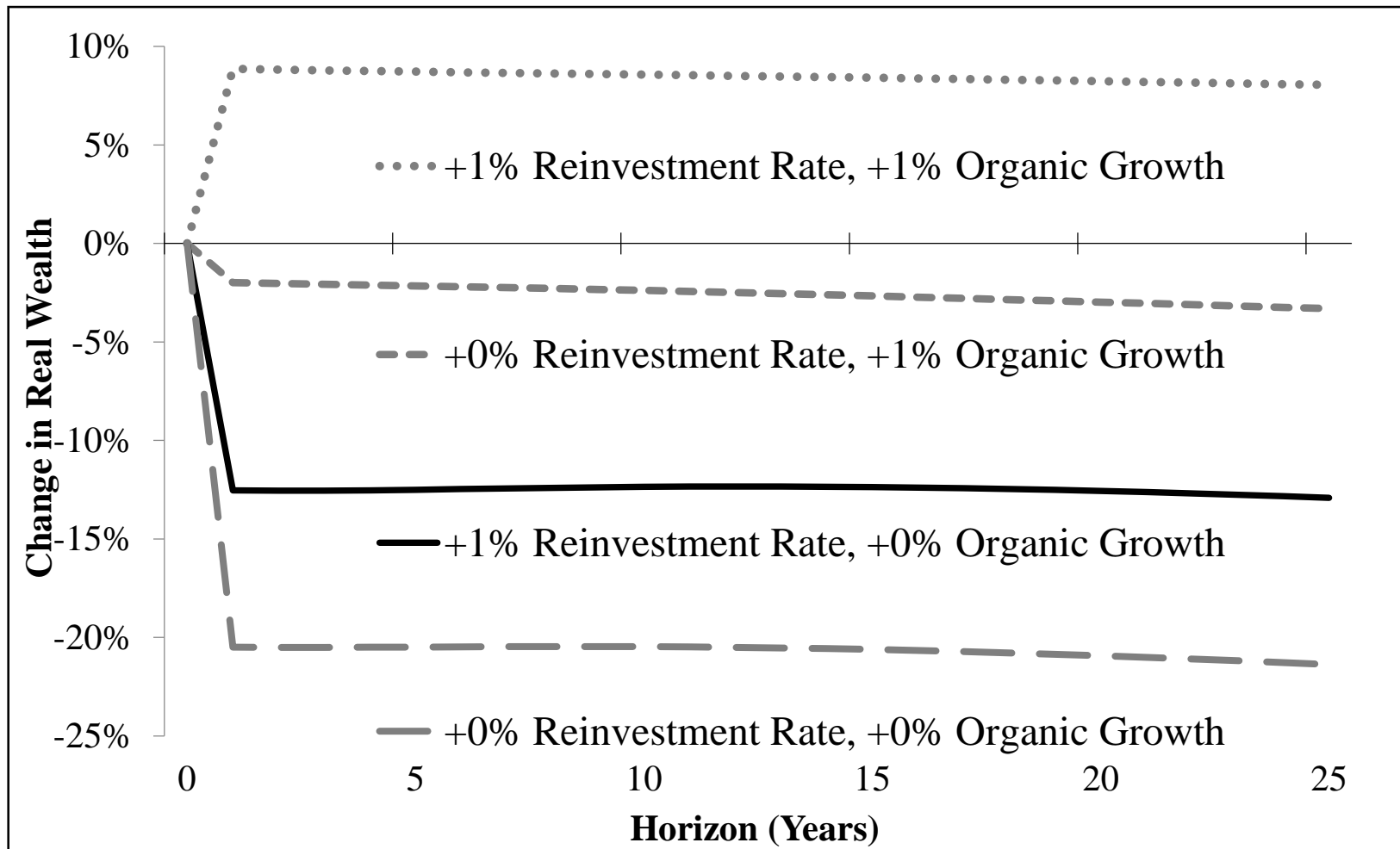
## Effect of +1% inflation innovation



*Chart assumes:*

- Inflation +1%
- Nominal rates +1%; real rates unchanged
- Equity reinvestment rates increase by 1% (+0.5% growth at 50% retention rate); but existing cash flows do not adjust

# Possible equity cash flow impacts of inflation +1%



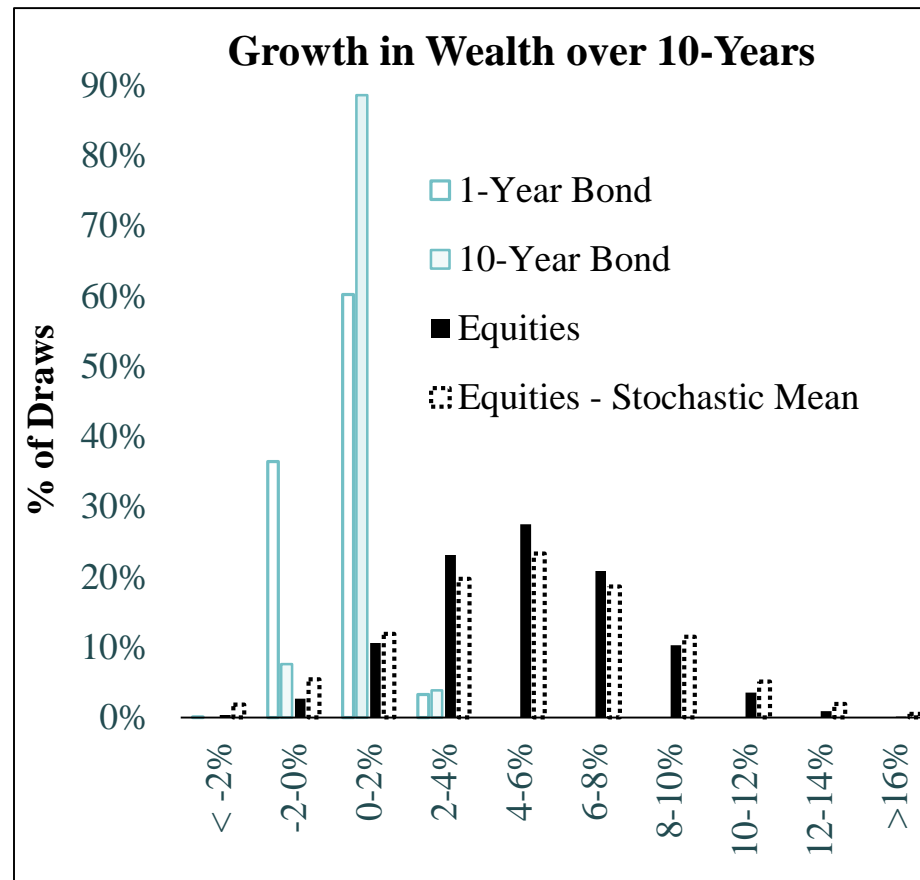
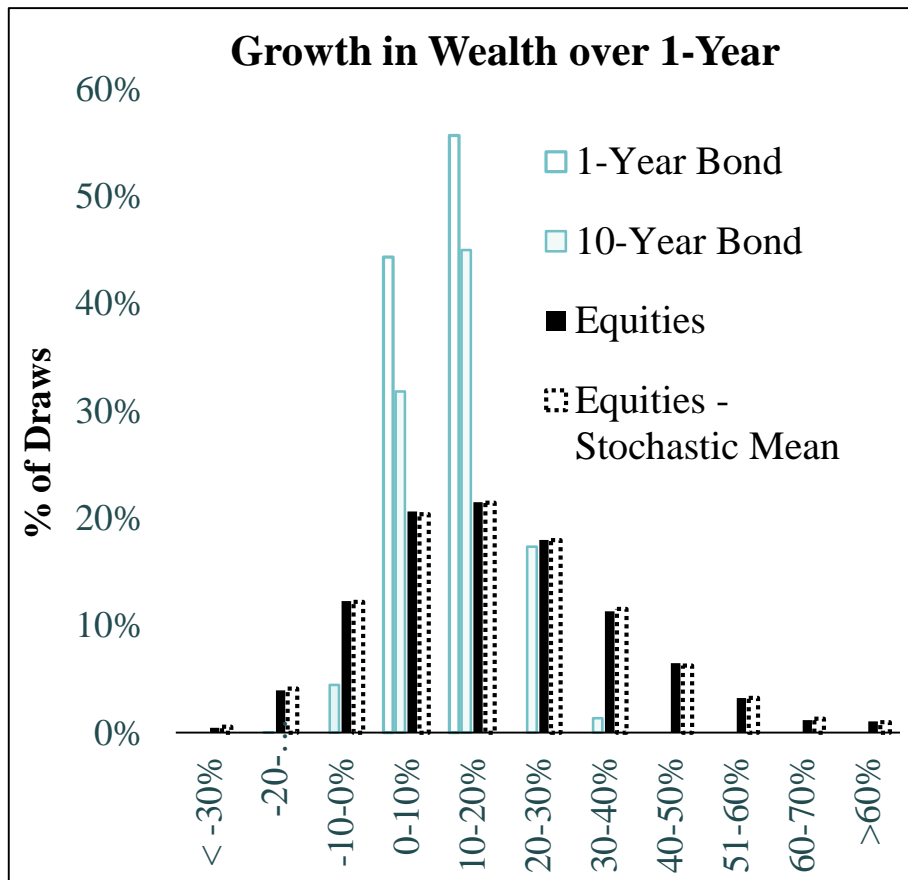
## Risk drivers: Implications for investment processes

- **Investors of all horizons need to worry about cash flow effects**
  - Cash flow innovations amount to a permanent change in wealth.
  - Short-horizon investors might focus on when cash flow innovations will change market *expectations*, and hence impact on prices.
  - Timing is unimportant to long-horizon investors. For them, it is about what cash flows will be *delivered* eventually.
- **Discount rate effects vary with investor horizon**
  - Short-horizon investors need to worry about ‘repricing’ effects
  - Long-horizon investors should care about impact on reinvestment rates
  - Asset duration *vs.* investor horizon matters
- **Long-term investors should also be more concerned with:**
  - Reinvestment rates under agency arrangements
  - Initial expected returns

## PART B: Creating distributions of wealth

- **Various methods available** (should embed covariance structure)
  - Simulations from historical data
  - Statistical models, e.g. VAR, regime switching
  - Structural models imposing relations between variables (e.g. Wilkie)
  - Value-based models, e.g. plowback models for equities
  - Scenario analysis
- **Framework of Part A implemented using basic model:**
  - Structural model with two state variables:
    - a) Inflation* – drives discount rate and reinvestment rate syndrome
    - b) Equity cash flows from existing operations* – random walk
  - Statistical models based on US equity, bond and inflation data
  - Calibrated to generate plausible expected returns and volatility

# Distributions over 1-year and 10-year horizons





## PART C: Portfolio construction

- A distribution of wealth outcomes for candidate portfolios can be generated using wealth projections for each asset
  - Covariance should be embedded within the joint distribution
  - Modelling may assume rebalancing or other pre-specified conditional strategies, if desired.
- A ‘score’ is given to each point on the resulting wealth distribution using an objective (i.e. utility) function
- Optimal portfolio is the one that maximises expected utility

# Objective functions

**Reference-dependent - general form** (drawing on Tarré, 2017):

$$U_{PT} = I_{\frac{W}{W^*} \geq 1} \gamma \left( \left( \frac{W}{W^*} \right)^\alpha - 1 \right) - I_{\frac{W}{W^*} < 1} \lambda \left( \left( \frac{W}{W^*} \right)^\beta - 1 \right)$$

$U_{PT}$  = prospect theory utility

$W / W^*$  = wealth / target wealth

$I$  = indicator function (1, 0)

$\alpha$  = curvature parameter on gains, i.e. wealth > target ( $\alpha = 0.62$ )

$\beta$  = curvature parameter on losses, i.e. wealth < target ( $\beta = 0.88$ )

$\gamma$  = weighting parameter on gains, i.e. wealth > target ( $\gamma = 1$ )

$\lambda$  = weighting parameter on losses, i.e. wealth < target ( $\lambda = 2.25$ )

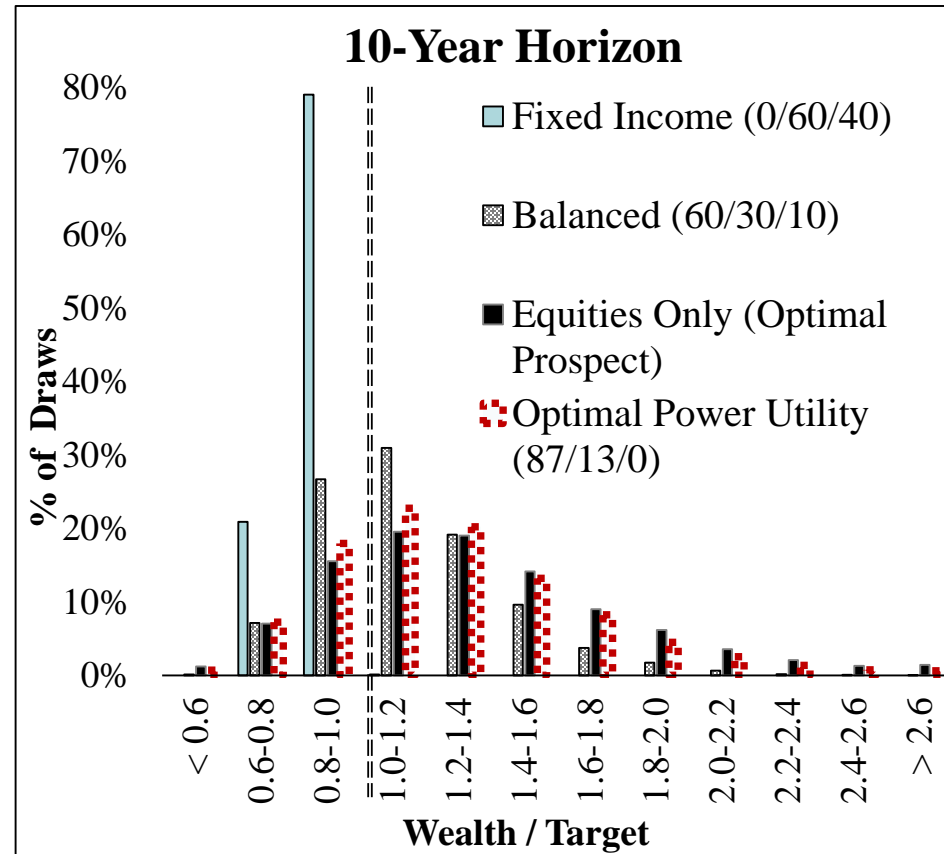
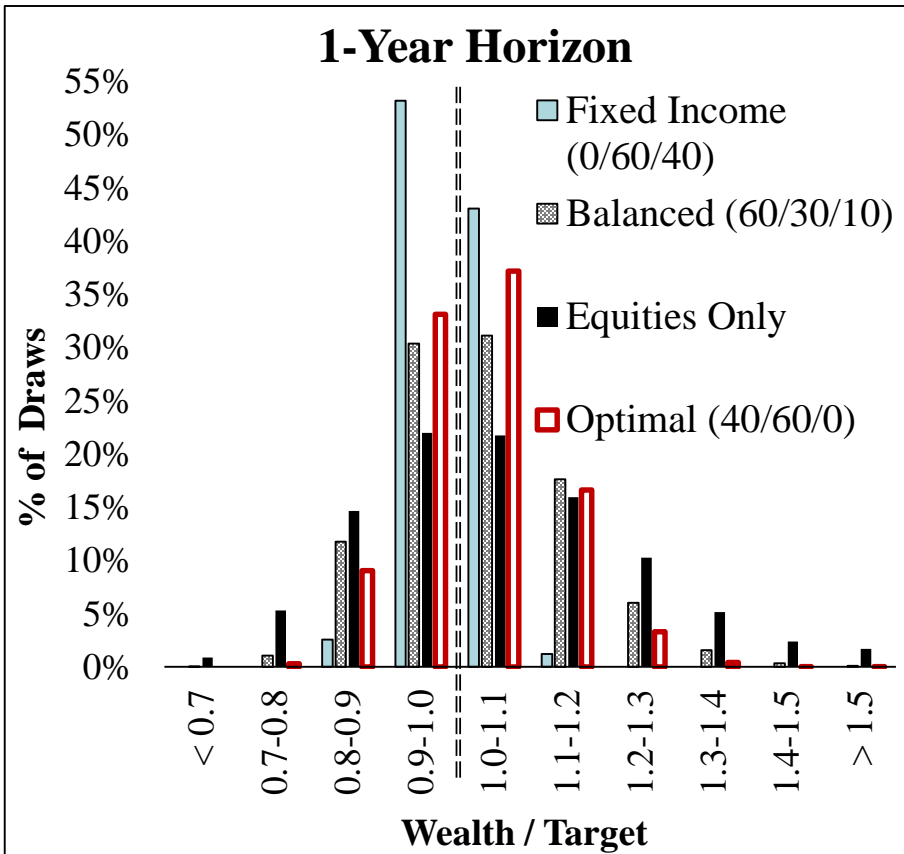
## Power utility

$$U_{PU} = \frac{W^{(1-CRRA)}}{1 - CRRA}$$

$U_{PU}$  = power utility

CRRA = coefficient of relative risk aversion (= 5.1)

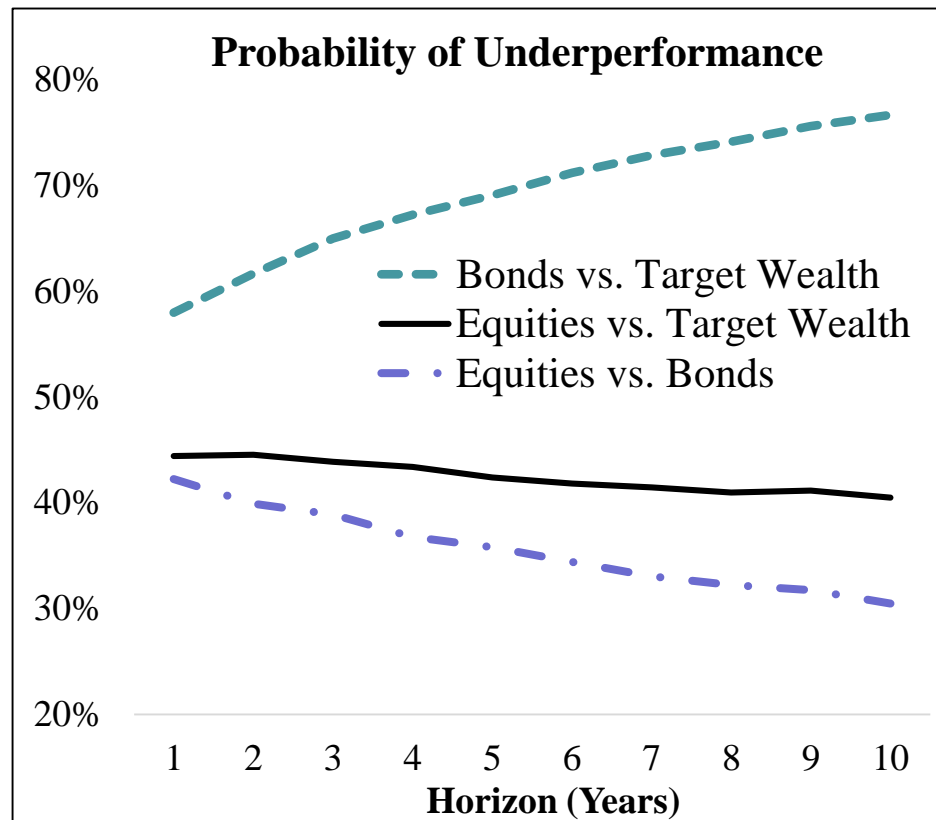
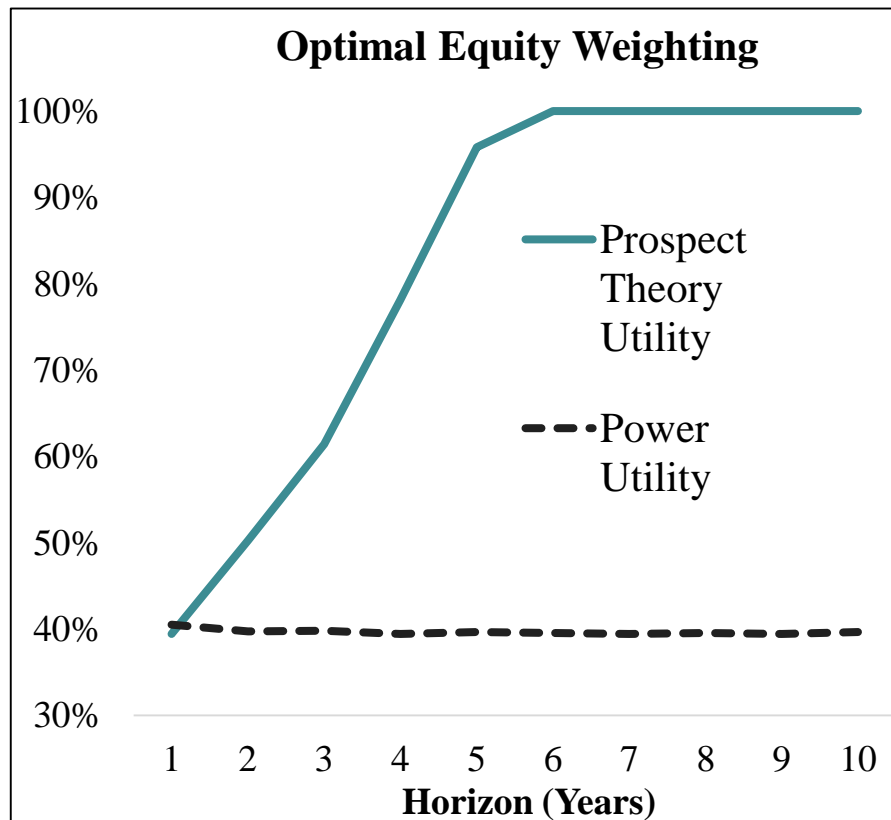
# Asset distributions through to portfolios ...



# Portfolio statistics

Portfolio	1-Year Horizon				10-Year Horizon			
	Fixed Income	Balanced (60/30/10)	Equity Only	40/60 Portfolio, Calibrated Utility Parameters	Fixed Income	Balanced (60/30/10)	Optimal: Prospect Theory Equity Only	Optimal: Power Utility
<b>Wealth vs Target</b>								
Mean	-0.7%	2.9%	5.0%	2.3%	-16.0%	14.0%	33.6%	27.3%
Standard Deviation	4.9%	11.8%	18.2%	9.5%	5.0%	27.6%	45.0%	39.3%
<i>Percentiles</i>								
100%	13%	64%	106%	51%	3%	189%	320%	277%
95%	7%	24%	38%	19%	-8%	66%	118%	101%
90%	6%	18%	29%	15%	-10%	51%	93%	79%
75%	3%	10%	16%	8%	-13%	29%	57%	48%
50%	-1%	2%	3%	2%	-16%	10%	27%	21%
25%	-4%	-5%	-8%	-4%	-19%	-6%	1%	-1%
10%	-7%	-11%	-17%	-10%	-22%	-17%	-17%	-17%
5%	-9%	-15%	-21%	-12%	-24%	-23%	-26%	-25%
0%	-18%	-31%	-42%	-28%	-35%	-47%	-60%	-55%
<b>Shortfall Measures</b>								
Probability Shortfall	56%	43%	43%	42%	100%	34%	24%	26%
Expected Shortfall	-4%	-8%	-11%	-6%	-16%	-13%	-16%	-15%

# What if mean-reversion is removed?



## Takeaway messages

- Horizon matters for the risks of most concern to an investor, and hence investment process design
- Objective functions are influential
  - Samuelson was right ... within a narrow frame. However ...
  - Adding a wealth target makes quite an impact
  - Prospect theory functions are well worth considering when a target is involved (like retirement) ... plus the units are more intuitive
- Mean reversion matters as the horizon extends beyond one period
- Mean-variance optimisation should be replaced by utility function analysis for many applications. MDUF.v1 is a start!



***Questions?***

***Discussion?***