INSTITUTE OF ACTUARIES OF AUSTRALIA

COURSE 1 INVESTMENTS **OCTOBER 2011 EXAMINATIONS**

Answer all 5 questions.

QUESTION 1 **(25 Marks)**

You have been retained by the Board of an Australian defined contribution superannuation fund to provide investment advice. The Board has relayed to you the contents of a recent presentation to them in which the presenter claimed that, as the standard deviation of the returns of directly held property was generally lower than that of listed shares, there were good risk management reasons for participating in this asset class. The Board has asked you for your opinion.

In addition the Board has contemplated investing in a particular property company. The company buys plots of land, builds residential developments on them and (generally) sells the completed homes to purchasers. The company retains and rents a small number of houses in its premium developments to create a pool of residential properties to supply recurrent income to assist in the funding of working capital.

(a) Briefly summarise the various types of directly held property the superannuation fund might contemplate. All things being equal, which of these types would generally be the most suitable for the fund to consider? (5 Marks)

(b) What general characteristics of directly held property make it an attractive asset class for this fund? (6 Marks)

(c) Comment on the risk management proposition claimed by the presenter of the above-mentioned presentation. Within your commentary indicate whether you agree with it, the extent to which standard deviation adequately summarises the risks associated with directly held property and other risk areas that need to be considered. (6 Marks)

(d) Briefly assess the residential property development company as a potential holding within the fund's direct property holdings. What issues would you need to account for in undertaking a discounted cashflow valuation of the company? (8 Marks)

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QUESTION 2 **(18 Marks)**

(a) What do you understand by the terms "relative return investor" and "absolute return investor"? Give an example of a risk-return measure for an active investor of each type. (4 Marks)

(b) What are the arguments for and against performance-based fees in funds management? What elements are important in the design of performance-based fees? (7 Marks)

(c) A fund manager offers an Australian equities-themed hedge fund based on fundamental analysis. The fund can take both bought and sold positions in stocks in the agreed universes, and cash, so long as the fund's effective exposure is 100% in shares and cash. The fund can also increase funds invested by borrowing up to 25% of money raised from its investors from time to time. The fund charges both a management fee of 1% of net funds under management and a performance fee.

(i) Why would the fund's manager seek to borrow?
(ii) Why would an investor allow an active manager to short-sell?
(iii) Leaving aside generally equity market risk and the other information an investor would require in order to assess this fund, what "special" risk/s would an investor need to bear in mind with the fund?

(7 Marks)

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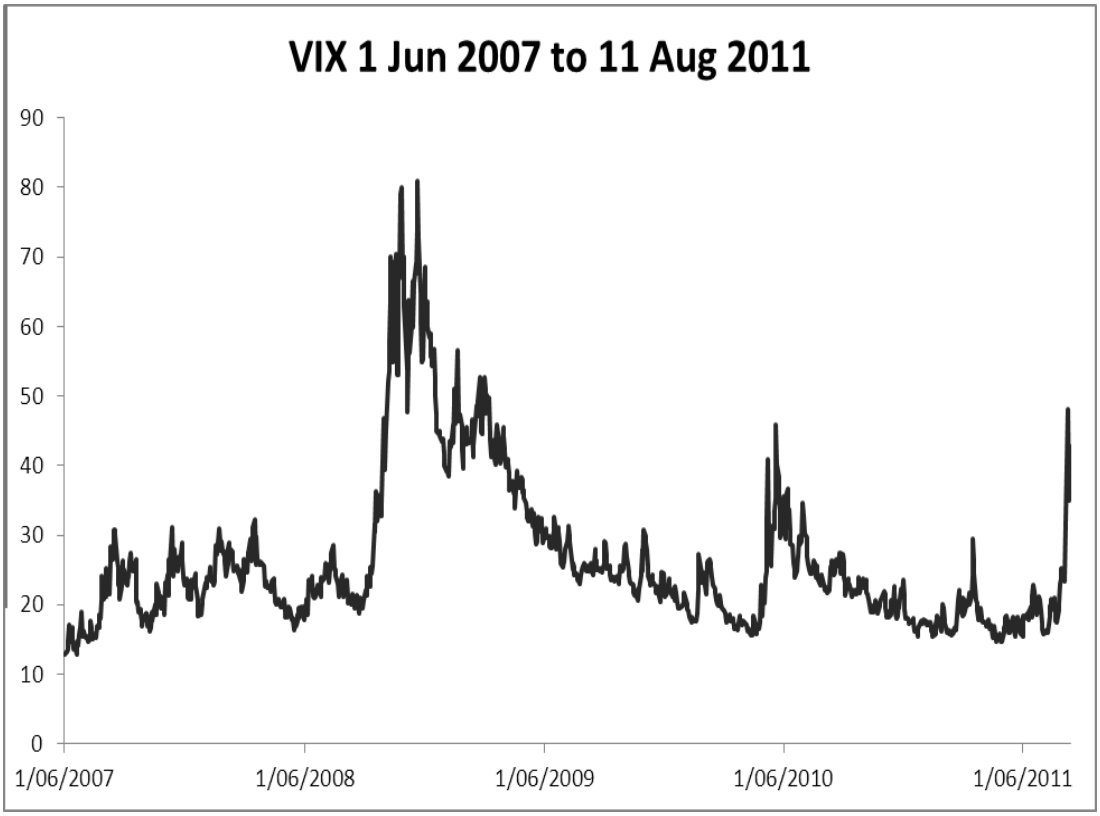
QUESTION 3 **(20 Marks)**

(a) The graph below shows the Chicago Board Options Exchange (CBOE) VIX volatility index movements for the past 4 years. The VIX is sometimes referred to as 'the fear index' and represents option market estimates of the annualised volatility (standard deviation) of the S&P500 index for the next 30 days. The VIX index is closely correlated with the actual realised sample path of S&P500 volatility.



Stochastic Volatility (SV) models such as the Heston and GARCH models are called Diffusion models and assume that SV is a stochastic process (random function) that has continuous sample paths without jumps. Another class of SV models called Jump-Diffusion models (e.g. Jorion) assumes the presence of discontinuities or 'jumps' in the SV sample path.

With reference to the VIX graph above, which of these types of model do you think would most accurately describe the stochastic volatility process of the S&P500? Give reasons for your answer. What recent financial market events may be associated with increases in volatility? (3 Marks)

(b) The following stochastic differential equation (SDE) is a continuous stochastic process commonly used to describe the return process r_t of assets traded on financial markets:

$$dr_t = \mu_t dt + \sigma_t dW_t$$

where W_t is a standard Brownian motion and μ_t and σ_t denote drift and volatility respectively.

Suppose the volatility σ_t is constant (as assumed in the Black-Scholes framework).

(i) Show that this implies the marginal distribution of the financial market return process is normally distributed.

(ii) Is it reasonable to use a model that assumes normality of the return process? Give reasons.

(iii) Would it be more reasonable to use a suitable leptokurtic ('fat-tailed') distribution to model the probability of large return movements on the S&P500 than to assume normality of returns? Why?

(5 Marks)

(c) The Single Index Model (SIM) is a model of stock returns where returns can be decomposed into stock specific returns due to firm-specific economic events and external economic events that are fully captured in market returns:

$$r_i - r_f = \alpha_i + \beta_i \left(r_m - r_f \right) + \varepsilon_i$$

where

r_i **is the return of stock *i***

r_f **is the risk free rate**

r_m **is the market return**

α_i **is the expected (constant) return of stock i if market excess return is zero,**

$r_m - r_f = 0$

β_i **is the beta of stock *i*,** ε_i **is the stock *i* specific return random variable with a mean of zero and is uncorrelated with the stock specific returns of other stocks or market returns.**

(i) Suppose we construct a portfolio of *n* equally weighted stocks. For simplicity, assume that the volatility (standard deviation) of the stock specific returns of all stocks in the portfolio are the same $\sigma[\varepsilon_i] = \sigma\left[\varepsilon_j\right] = \sigma_{ss}$. Assuming the SIM model of stock returns, give an expression (and briefly justify it) for the <u>stock specific</u> volatility (standard deviation) of the portfolio as a function of the number *n* of stocks in the equal weighted portfolio? (2 Marks)

(ii) Assume the SIM model of stock returns is correct. What is the covariance of returns between any two stocks? Explicitly derive your answer. (2 Marks)

(d) The Sharpe-Lintner CAPM model assumes the market portfolio is highly diversified (a large number of stocks) and that the market portfolio risk due to stock specific returns is zero. The Security Market Line (SML) in the CAPM model relates expected excess stock/portfolio returns to expected market returns.

$$E\left[r_i\right] - r_f = \beta_i \left(E\left(r_m\right) - r_f\right)$$

(i) What is the additional assumption we require to formulate the SML from the Single index Model? (1 Mark)

(ii) The ex-ante Treynor ratio *T* is a measure of the non-diversifiable risk adjusted expected excess return of a stock or portfolio: $T = \dfrac{E\left[r_i\right] - r_f}{\beta_i}$.

Suppose we estimate the Treynor ratio of a stock and find that it is higher than the CAPM market portfolio. What implications does this have for the CAPM model and the SIM model? (2 Marks)

(e) The beta β_i **of a stock/portfolio can be formulated as** $\beta_i = \dfrac{\sigma_i \sigma_m}{\sigma_m^2} \rho_{i,m}$, **where** σ_i **is the standard deviation of stock/portfolio returns** σ_m **is the standard deviation of market returns** $\rho_{i,m}$ **is correlation of stock and market returns. Suppose we hold a well-diversified portfolio with a beta = 1. What is the volatility (standard deviation) of the portfolio? (2 Marks)**

(f) The Markowitz mean-variance model is the foundation of modern portfolio theory. The Markowitz mean-variance utility function is formulated as follows (in vector notation):

$$U\left(\lambda, h\right) = \mathrm{h}^{\mathrm{T}}\mathrm{r} - \lambda \mathrm{h}^{\mathrm{T}}\mathrm{V}\mathrm{h}.$$

Give a proof that portfolios generated by Markowitz mean-variance optimisation lie on the efficient frontier.

(3 Marks)

QUESTION 4 **(20 Marks)**

You are an investment analyst working for a firm of Asset Consultants.

You have been asked to critically examine the fee and active management structure of a $5 billion Australian Equity Fund. This fund is currently managed externally with a single active manager. The Management Expense Ratio (MER) of this manager is 0.85% of the equity fund asset value managed and the manager has a 10 year record with an average information ratio of 0.25 before fees against the ASX200 benchmark. The ex-ante information ratio of this manager is expected to continue at 0.25. The current target active risk of the equity fund is 3%.

A new structure for the equity fund is proposed where the ASX200 index market return of the equity fund is generated by an external wholesale index fund manager with an MER of 0.15% and active returns are to be generated by 2 specialist 'pure alpha' active managers proportionally overlaid over the index fund. These managers implement 'pure alpha' long-short portfolios where they are short the ASX200 benchmark and are long an active equity portfolio with no net investment.

Active 'pure alpha' manager 'A' is an active investor who cycles between the Value and Growth investment style. The ex-ante information ratio of manager 'A' is expected to be 0.50. The management expense ratio (MER) of manager 'A' is 0.90% of the proportion $0 \le \delta \le 1$ of the value of the underlying equity fund managed.

Active 'pure alpha' manager 'B' is an active investor who cycles between the Large and Small capitalisation investment style. The ex-ante information ratio of manager 'B' is expected to be 0.40. The MER of manager 'B' is 0.70% of the proportion $1 - \delta$ of the value of the underlying equity fund managed.

The active investment styles of the two 'pure alpha' managers are not independent and their active returns have a correlation of 0.3. The combined active return strategy should introduce an active risk of 3% to the underlying equity portfolio.

(a)

(i) What is the after fees information ratio of the current external active manager? (2 Marks)

(ii) Suppose now that active manager 'A' manages 50% ($\delta = 0.5$) of the value of the underlying equity fund and active manager 'B' manages the other 50%. Assuming that both managers also have equal active risk, what is the resultant information ratio of the combined active strategies before MER fees? (3 Marks)

(iii) What is the resultant information ratio of the combined active strategies after all MER fees? (2 Marks)

(b) The optimal allocation of active risk between active managers in an investment portfolio to maximise expected active return (information ratio) is known as 'risk budgeting'. Outline briefly in words how you would formally calculate the optimal proportion of active risk allocated to active manager 'A' and active manager 'B'. (2 Marks)

(c) The formula for a t-statistic to test a sample mean against the null hypothesis (the probability that the population mean is zero) is as follows: $\frac{\bar{S}}{\bar{\sigma}}\sqrt{n}$, where *t* is the *t*-statistic, $\bar{S}$ is the sample mean, $\bar{\sigma}$ is the sample standard deviation and *n* is the sample size. How would we use this statistic to assess if active manager ex-post returns are due to skill rather than luck ? (2 Marks)

(d) You calculate the annualised monthly performance of an equity portfolio where retail investors can buy and sell investment units that are priced daily using the Net Asset Value (NAV) of portfolio. Assume that the unit price was U_0 at the beginning of the month and U_{31} at the end of the month. Give reasons for the choice of portfolio performance calculation methodology you use and formulate in detail how you would calculate the annualised monthly portfolio return to a retail investor assuming that the buy price is the same as sell price on any day. (4 Marks)

(e) You have been asked to analyse the performance of an investment portfolio that is invested in two sectors: 50% cash and 50% domestic equities. It trades only at the end of each month and has no deposits or redemptions. It has a benchmark of 100% domestic equities. Cash returned 0.5% for the month, the portfolio's equity sector increased in value by 2.0% for the month. The benchmark return was -1.0% for the month.

(i) Describe and quantify the performance effect of sector allocation for the month. (2 Marks)

(ii) Describe and quantify the performance effect of stock selection for the month. (1 Mark)

(iii) Describe how your calculation process would change if there was a large cash deposit made to the fund during through the month. (2 Marks)

QUESTION 5 **(17 Marks)**

You are a financial consultant to a developing republic which was formerly a member of the Soviet Union. This country is reviewing its retirement incomes policy. Previously, a prescribed level of state pension, equal to 50% of income at retirement, was available to all workers on retirement at age 65. The government is concerned with the cost of maintaining this policy, and is interested in adopting a market based policy for retirement incomes.

You are asked to consider the effectiveness of offering tax concessions to workers for contributions to retirement accounts, to be established under regulation. The tax concessions operate in conjunction with reductions in the state pension. The proposal is similar to the retirement income policy in Australia. The proposal is as follows:

- **The state pension will be subject to an income test, as well as an assets test;**
- **Workers' income that is diverted to retirement accounts is taxed concessionally;**
- **Investment income on retirement accounts is taxed concessionally;**
- **Restrictions on the investment of retirement accounts will be imposed, to ensure diversification;**
- **Only on retirement after age 65 are the funds in retirement accounts accessible to workers. The income drawn from these accounts is subject to minimum and maximum limits, similar to the allocated pension rules operating in Australia;**
- **The income drawn from retirement accounts is not taxed, but is included in the income test.**

The government has engaged you to provide advice on the review. Draft your advice on the following issues:

(a) Discuss briefly how and why the proposed policy could promote economic growth. **(2 Marks)**

(b) What type of asset model would be suitable for assessing the impact of the policy? **(3 Marks)**

(c) How would you assess the financial objectives of the policy, in terms of the trade off between short term tax concessions and long term savings in state pensions? **(3 Marks)**

(d) What controls are available to the government in implementing the policy? **(5 Marks)**

(e) What risks are inherent in the policy? **(4 Marks)**

END OF PAPER

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