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A Modern Approach to Group Risk Pricing and Credibility

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Abstract

Purpose
The purpose of this paper is to:
1. Increase understanding of the core principles of credibility and its significance to group risk pricing
2. Increase understanding of the assumptions and limitation of existing credibility models used in group risk pricing
3. Introduce an alternative approach to group risk pricing and credibility to discuss, refine and possibly implement in the medium to long term

Scope
The scope of this paper is limited to:
- Group risk lump sum products
- The analysis of trend adjusted claims experience
- The determination of risk premium net of the catastrophe risk premium

Key Points
1. Credibility models are a requirement of competition and should be tested in competition
2. Credibility models are rating mechanisms; in competition, inaccurate credibility models can lead to anti-selection and portfolio level underpricing
3. Different credibility models can lead to very different estimates of risk premium. The choice of credibility model can have a significant impact on an insurer's portfolio composition and profitability
4. The determination of a theoretically sound credibility weighted risk premium involves:
   - Sufficient consideration for and responsiveness to the effectiveness of the insurer's rating classes and the accuracy of its base rates and loading factors
   - Sufficient consideration for and responsiveness to the variability and reliability of the group risk plan's historical claims experience
   - The use of a theoretically accurate method of calculating the credibility adjusted risk premium on the basis of the two abovementioned inputs
5. On theoretical grounds, both the Bühlmann-Straub model and the Limited Fluctuation model have significant limitations in addressing each of the three abovementioned steps
6. The proposed credibility model theoretically accounts for all three steps
7. The results of a simulation of a hypothetical group risk portfolio support the theoretical arguments posed above

Model	Proposed Model	Model 1	Model 2	Model 3
Prem Won/28,000	**$948,063**	**$295,025**	**$0**	**$308,917**
Expected Future Claims/28,000	**$922,489**	**$632,815**	**$0**	**$603,694**
Premium/Expected Future Claims	**103%**	**47%**	**NA**	**51%**
Proportion of Business Won	61%	19%	0%	20%

Table 1: The figures in the table are based on a simulation of 28,000 hypothetical group risk plans over a period of 5 years. The simulations are used to produce claims data for each plan. The plan and claims information is inserted into four competing credibility models to produce four quotations for each hypothetical plan. The winning model is determined on the basis of cheapest premium.
Models 1 and 2 are variations of the Bühlmann-Straub model, Model 3 is a Limited Fluctuation model.

Key words: Credibility, Bayesian, Bühlmann, Bühlmann-Straub, Limited Fluctuation, Classical, Cross Subsidisation, Anti-Selection, Stochastic, Maximum Likelihood Estimator, Bernoulli, Group Risk, Pricing, Life Insurance

1 Introductory Comments

1.1 Background

The Australian group risk market is a high growth, highly competitive and price elastic market. In this environment the need for pricing accuracy is heightened.

The pricing of group risk business is predominantly concerned with the determination of expected future claims cost as this is by far the most significant component of office premium.

The commonly accepted approach to the determination of future claims cost involves:

1. The determination of expected historical claims based on base rates, loading factors, membership and exposure information
2. The determination of actual historical claims based on actual reported claims and estimation of various reserves
3. The adjustment of actual historical claims for future trend effects and changes in plan design
4. The determination of an experience rating factor (ERF) which is applied to expected future claims

The calculation methods used in steps 1 and 2 are reasonably advanced. Portfolio level monitoring and analysis is undertaken periodically. Mortality and morbidity investigations, claims development factor recalibration and many other established actuarial methods are used to improve accuracy at steps 1 and 2.

Step 3 is a subjective process requiring the expert judgement, knowledge and experience of the pricing actuary.

Of the abovementioned four steps, disproportionately little emphasis is placed on step 4. The calculation for step 4 has traditionally involved a formula based approach. The credibility formulae used have been adopted from other actuarial fields without specific customisation for group risk. The assumptions and derivations of these models are not well understood by group risk pricing practitioners, although they are aware that these models are not very reliable and sometimes produce counterintuitive and unreasonable results.

In this paper, the credibility model is viewed as being the weakest link and offering the greatest potential for improved accuracy in group risk pricing.

1.2 Purpose

The purpose of this paper is to:

1. Increase understanding of the core principles of credibility and its significance to group risk pricing
2. Increase understanding of the assumptions and limitation of existing credibility models used in group risk pricing
3. Introduce an alternative approach to group risk pricing and credibility to discuss, refine and possibly implement in the medium to long term

1.3 Grounds of Conclusions

The focus of this paper is pricing accuracy rather than simplicity. The discussion of the limitations of various models and approaches is accuracy related.

The conclusions drawn from this paper are predominantly based on theoretical grounds. The theoretical arguments and conclusions are further supported by the results of a simulation used to compare alternative credibility models.

1.4 The Scope of This Paper

1.4.1 Product Scope

This paper is only concerned with credibility models for group risk pricing. The specific model proposed in sections 4 to 6 applies only to lump sum products. Group Salary Continuance (GSC), Incurred But Not Reported (IBNR) and Claims In the Course of Payment (CICP) credibility models are outside the scope of this paper.

1.4.2 Trends and Other Time Effects

Credibility models assume that the past is indicative of the future. They usually make no allowance for trends and fully project forward the identified statistical significance of historical experience. As a result, the pricing actuary is dealt the responsibility of adjusting the credibility model's historical inputs such that they are indicative of known trends and plan changes expected to continue.

In this paper, all references to the past, present and future claims experience of plans will be on the assumption that the claims experience and plan exposure have been adjusted for trends and plan changes.

1.4.3 Risk Premium

All discussions of premium relate to risk premium only unless otherwise stated. This risk premium does not include the catastrophe risk premium which should be collected in addition, to cover the risk of more than one life claiming due to the same event.

2 An Overview of Key Concepts

2.1 Cross Subsidisation and Anti-Selection

The Australian group risk environment has the following characteristics:

- Price elastic market – Most consumers are price sensitive and have a strong incentive to choose the insurer quoting one of the lowest premiums
- Competitive market – There is strong pricing competition amongst insurers and the profit margins built into office premiums are generally small
- Heterogeneity of quotes – Competing insurers often quote significantly different premiums for the same tender
- Existence of cross subsidies – Every quotation has some degree of over or under pricing because insurers do not have perfect rates or models, pricing actuaries do not have perfect judgement and many pricing decisions are made on subjective grounds

Given this environment, some level of anti-selection is being experienced by every Australian group risk insurer. This statement is justified by the following:

- Every insurer underprices some plans and overprices others
- Tenders are more likely to be awarded to the cheaper insurer
- Insurers are more likely to win plans that they underprice and less likely to win plans they overprice

Assuming that the Australian group risk insurance market is a viable and profitable business, the cost of the anti-selection experienced by all insurers must be recouped by charging higher average risk rates than the 'fair' average risk rate. This additional margin will be referred to as the anti-selection premium.

At this point it is important to note the following:

1. Insurers with greater cross subsidies between plans will experience greater anti-selection and will require a higher anti-selection premium to remain profitable
2. If one of the group risk insurers starts quoting nil cross subsidy premiums for all tenders, this insurer will no longer experience anti-selection and hence can lower their risk rates whilst continuing to remain profitable
3. The resulting impact of 'step 2' on competitors (who do not reduce their level of inter-plan cross subsidies) will be:

    - Higher anti-selection (since they now only win plans which they overpriced in the infrequent event where the customer chooses them over the cheaper nil cross subsidy insurer)
    - Losses due to the experience of greater anti-selection
    - The realisation of mortality and morbidity losses arising from experience investigations and a subsequent increase in average risk rates
    - Ongoing reduction in portfolio size as premiums continue to increase and losses continue to accumulate

Reality is never quite so black and white and the results of anti-selection are unlikely to be as spectacular as the theory suggests. Insurers attempt to distinguish themselves by better service, technology et cetera. Some clients value the relationship with their incumbent insurer or have greater focus on insurer flexibility. With these considerations in mind, it is tempting to disregard the impacts of anti-selection, but this would be inappropriate.

The truth is that the group risk environment meets all of the requirements for anti-selection to occur. Further, given the magnitude of inter-plan cross subsidies, the level of anti-selection is likely to be significant. Insurer differentiation, customer loyalty and other mitigating factors will dampen the magnitude of anti-selection and allow premium rates and portfolio sizes to reach equilibrium but they are not pronounced enough to make anti-selection insignificant.

2.2 Credibility and Anti-Selection

The task of a credibility model is to adjust the insurer's initial estimate of a plan's average claims cost (based on base rates and loading factors) by accounting for the additional insight provided by the historical claims experience of the plan.

In essence, the historical claims experience can be viewed as a risk classification no different to age, sex and occupation. Here, the credibility model is the mechanism for determining the appropriate premium loading (or discount) for the given historical claims experience. Thus, an insurer is exposed to anti-selection through cross subsidies in their credibility model in the same way they would be exposed to anti-selection if they had cross subsidies in their age/sex rates or occupation loading factors.

The sole purpose of a credibility model is to minimise cross subsidies between plans with different historical claims experiences. Without competition or with nil price elasticity, a credibility model is redundant and the average claims rate could be used for all plans without any adverse impacts on insurer profitability.

2.3 Key Definitions

2.3.1 Actual

The Actual (or Actual claims) is the observed historical claims cost of a plan over the period of investigation. In practice, this is in part estimated by the use of the IBNR and Reported But Not Admitted (RBNA) reserves.

2.3.2 Underlying

The Underlying (or Underlying claims) is the expected value or true mean of Actual claims. The Underlying of a plan is unknown.

2.3.3 Expected

The Expected (or Expected claims) is the insurer's best estimate of the Underlying without having knowledge of the Actual claims of the plan. The Expected is calculated using base rates and loading factors which are derived though portfolio and population level experience studies.

2.3.4 Underlying Rating Factor (URF)

The URF is the ratio of the Underlying to Expected. On average, plans with a URF greater than one will experience claims greater than their Expected and vice versa. The URF of a plan is unknown since the Underlying is unknown.

To aid with the interpretation of the abovementioned definitions, it is helpful to draw a parallel with a coin toss. Suppose a group risk plan's claims results are represented by a coin toss where heads represents 1 claim occurring and tails represents nil claims.

Based on the insurer's long history of experience with similar coins, the insurer estimates the probability of a heads outcome to be 50%. On this basis, the insurer determines that the average number of claims is 0.5. This estimate represents the Expected number of claims.

Unknown to the insurer is the fact that this particular coin is biased and has a 55% probability of a heads outcome. Given this fact, the true average number of claims is 0.55. This figure represents the Underlying number of claims.

Finally, the coin is flipped and an outcome is observed. This outcome represents the Actual number of claims.

Note: The Actual, Underlying and Expected can describe the number of claims, amount of claims, frequency of claims or any other claims measure. In the remainder of this paper, the Actual, Underlying and Expected will refer to the amount of claims unless otherwise stated.

2.4 The Framework for Credibility Modelling

A credibility model's aim is to marry the Expected and Actual claims of a plan in order to attain a statistically sound estimate of the expected value of the plan's Underlying. To achieve this aim, a logical relationship must be established between:

- The Expected and Underlying claims of a plan
- The Underlying and Actual claims of a plan

Finally, a statistical method must be used to quantify these two relationships into a single numerical estimate of the expected value of the Underlying claims.

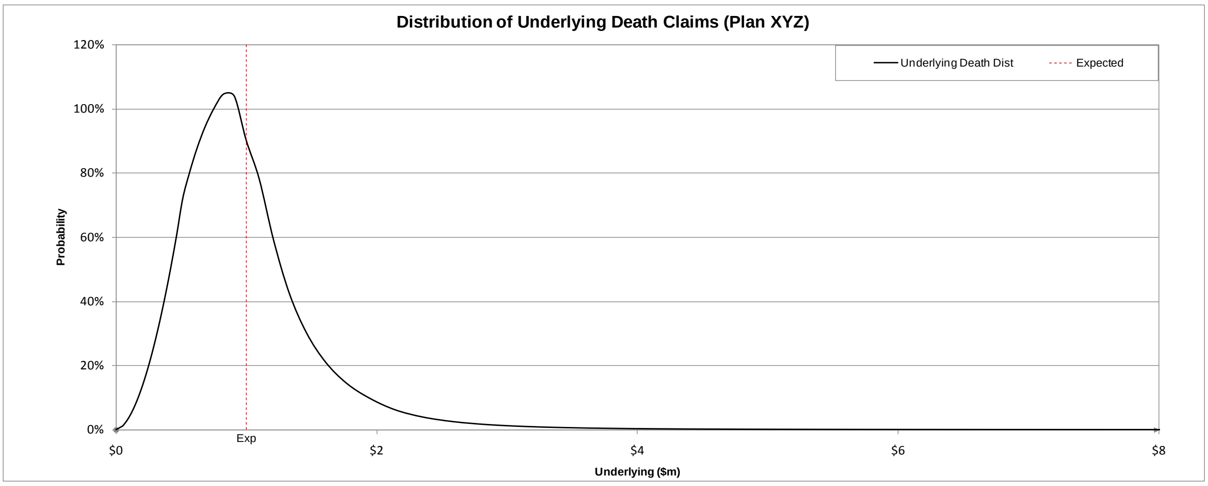
2.4.1 The Relationship Between the Expected and Underlying Claims

The Underlying of a plan is unknown to the insurer. Thus, from the insurer's perspective, the Underlying is a random variable and the insurer is concerned with determining the distribution of this random variable.

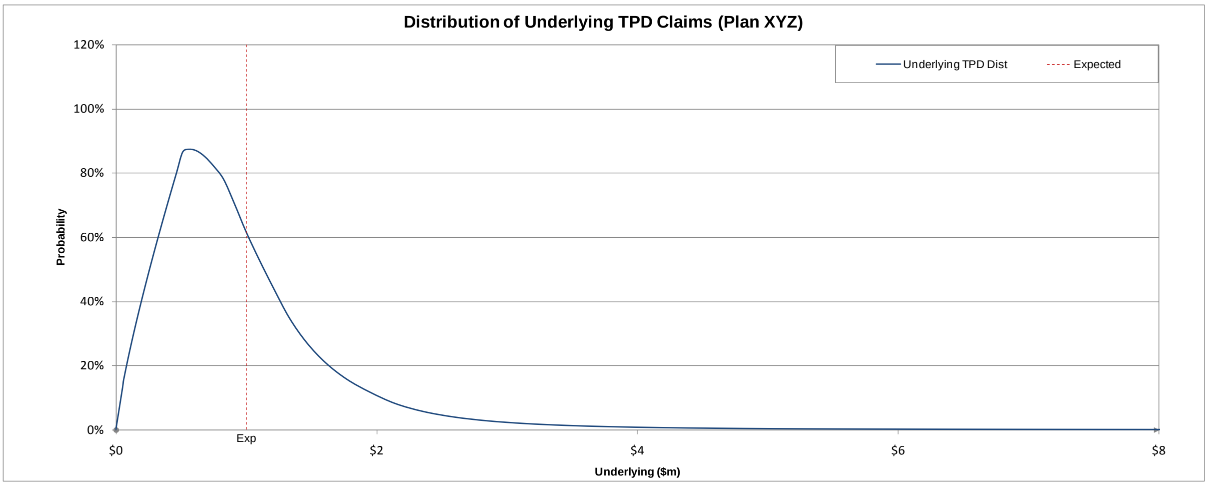
Prior to knowledge of the Actual claims, the insurer's best estimate of the mean of the Underlying distribution is the Expected.

Graphs 1 and 2 show an example of the Underlying death claims distribution and the Underlying TPD claims distribution of plan "XYZ".

Graph 1:



Graph 2:



The variance of the Underlying distribution represents the level of potential discrepancy between the true average claims cost of a plan (Underlying) and the insurer's best estimate (prior to the knowledge of Actual claims) of the average claims cost of the plan (Expected). This variance gauges the effectiveness of the insurer's rating classes (age, sex and occupation). If an additional and effective rating class is introduced, the variance of the Underlying distribution will reduce.

The shape of the Underling distribution may be influenced by a number of factors. The insurer can empirically measure the shape of the distribution of Underlying claims (see section 6) but is limited to only considering the factors that have the most significant impact on the distribution's shape. These factors are likely to be:

- Product (death cover or TPD cover)
- Availability of data such as exposure, occupation, age and sex (full data or partial data)
- Plan type (corporate, industry fund or master trust)
- Expected claims

From graphs 1 and 2, it can be observed that the variance of the Underlying TPD claims is larger than that of the Underlying death claims. This occurs because attributes such as occupational health and safety culture, job satisfaction, awareness of insurance cover et cetera vary significantly between plans and have a greater impact on TPD claims. Further, Underlying death claims are relatively insensitive to most unrated plan characteristics.

Missing or poor quality data also impacts on the accuracy of the Expected. As a result, a wider range of Underlying claims is possible. A wider flatter Underling distribution shape is appropriate for plans with incomplete data.

Corporate group risk plans are likely to have a higher variance of Underlying (after adjusting for exposure) compared to industry funds and master trusts. This is because of the plan level influence of the employer on the probabilities of claim of the plan's members. The employer can influence the probabilities of claim within the membership through various environmental factors as well as hiring policy. This employer impact will be diversified away in large master trusts. Industry funds with a specific industry exposure may achieve some diversification at the individual employer level but will remain exposed to the impact of their industry on the individual members' probabilities of claim.

The Expected claims will impact the shape of the Underlying distribution because the variance of the Underlying distribution will increase as the Expected claims increases.

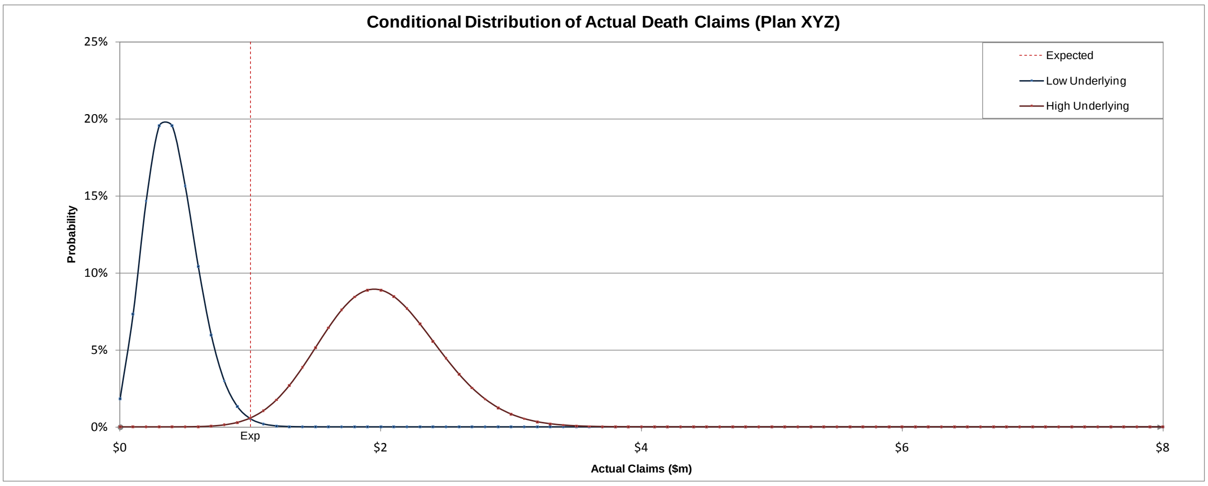
The distribution of Underlying claims is relevant to credibility because it represents how effective the Expected is at determining the Underlying. For example, if the Underlying claims distribution has a low variance, then the Underlying is unlikely to be too different from the Expected. All else being equal, greater confidence should be placed in the Expected and hence a lower level of credibility attributed to the Actual.

2.4.2 The Relationship Between the Underlying and Actual Claims

The Actual claims of a plan has a given mean (the Underlying) but is subject to randomness and will fluctuate around its mean from one period to another. The insurer can model the distribution of Actual given the Underlying (conditional Actual distribution). The fact that the Underlying of the plan is unknown can be overcome by modelling a separate conditional Actual distribution for the full range of values that the Underlying can take.

Graph 3 depicts an example of plan XYZ's conditional Actual distribution for a high and a low value of the plan's Underlying.

Graph 3:



When modelling the conditional distribution of Actuals, it is important to consider and account for the plan features which will significantly impact on the shape of this distribution. These are:

- The distribution of sums insured within the plan
- The distribution of probabilities of claim within the plan
- The life years of exposure

An additional consideration of importance is the impact of estimating the observed Actual claims through IBNR and RBNA. These reserves add an additional level of error and reduce the accuracy of the observed Actual as a predictor of the Underlying.

The conditional distribution of Actual claims is relevant to credibility because it represents how effective the Actual is at determining the Underlying. For example, if the conditional distributions of Actual claims all have a low variance, then the Underlying is unlikely to be very different from the Actual. All else being equal, greater confidence should be placed in the Actual and hence a higher level of credibility attributed to it.

2.4.3 The Credibility Formula

Upon choosing the appropriate distribution of Underlying and the set of distributions of the conditional Actual claims, the insurer's task is to determine its best estimate of the expected value of the Underlying claims. This involves the determination of a revised Underlying distribution in light of the Actual claims experience (conditional Underlying distribution).

The conditional Underlying distribution can be precisely derived from the input distributions using Bayesian conditional probability. This method is optimal because it does not require any assumptions and produces exact results.

2.4.4 Concluding Comments on the Framework

The determination of credibility involves a balance between the insurer's confidence in the accuracy of its Expected claims in predicting the Underlying versus the accuracy of the Actual claims in predicting the Underlying. The distribution of Underlying captures the former while the conditional distributions of the Actual capture the latter. The credibility formula is simply the means of weighing up the two opposing predictors.

In light of this, it is imperative that the two opposing predictors be reflective of the plan being priced rather than some generic distribution independent of the plan's characteristics. Methods for achieving this critical requirement are discussed in sections 4 to 6.

2.5 Concluding Comments

Poor credibility modelling results in the cross subsidisation of plans with high Underlying claims (relative to Expected) by plans with low Underlying claims (relative to Expected). This cross subsidisation currently exists in the Australian group risk market and results in some degree of anti-selection. As a result, Australian group risk insurers have over time built margins into their rates which offset the impact of the existing level of cross subsidisation.

If one group risk insurer adapts a more accurate, low cross subsidy credibility model, they can justify lowering their base rates by the anti-selection premium built into their rates. Alternatively, the insurer can leave their rates unchanged effectively increasing their profit margin. The impact on the competitors (if they also do not improve their credibility models) will be the experience of greater anti-selection and the need for a greater anti-selection margin built into their base rates.

The accurate, low cross subsidy modelling of credibility requires three steps:

1. The accurate determination of the distribution of Underlying claims
2. The accurate determination of the conditional Actual claims distributions
3. The use of a theoretically sound and accurate credibility formula

The three abovementioned steps will be discussed in relation to various credibility models in the following sections.

3 An Overview of Existing Credibility Models

There are currently three well established credibility model types:

- Bayesian Credibility Models
- Bühlmann Credibility Models
- Limited Fluctuation Models (LFM)

These credibility models are used in a wide range of applications and are not specifically built for group risk pricing. Australian group risk insurers have adopted the Bühlmann and LFM for the purpose of pricing and at present, variations of these two models are predominantly used.

In this section, greater focus will be placed on the Bühlmann and LFM. The assumptions, practical limitations and the key steps of the derivations of these models are discussed specifically in the group risk pricing context.

3.1 Bayesian Credibility Models

There are many variations of Bayesian credibility models. What these models have in common is the use of Bayesian conditional probability.

The use of Bayesian conditional probability requires at least two input distributions. The methods used to determine the input distributions are the main distinguishing features between various Bayesian credibility models.

The Bayesian approach offers a theoretically sound and realistic framework for addressing the credibility problem. The Bayesian conditional probability formula, the heart of the Bayesian approach, is theoretically optimal. It requires no assumptions and produces exact results.

The obstacle for Bayesian credibility models has been to achieve an equally sound and effective means of determining the necessary input distributions. These inputs are difficult to determine and due to the historically pressing need for closed form numerical models, assumptions regarding these distributions were commonly made based on mathematical convenience rather than on the basis of compelling evidence.

At present, due to the advancement of computer technology, the calculation intensive Bayesian credibility models no longer carry their historical restrictions. The computer age has allowed the estimations of input distributions to be based on empirical data rather than mathematically convenient approximations.

The Bayesian approach is discussed in more detail in appendix 13.1.

3.2 The Bühlmann Credibility Models

Variations of the Bühlmann-Straub model (BSM) are commonly used in group risk pricing in Australia. The significant drivers of this model's appeal are the fact that it is non-parametric, has a closed form and is simple to apply. The BSM requires periodic calibration; however, this requirement is neglected in practice.

The Simple Bühlmann Model (SBM) is a special case of the BSM where each year's exposed to risk is the same. The final results and steps in the derivation of both models are comparable. As such, in the interests of simplicity and clarity, this paper will only discuss and show the main steps in the derivation of the SBM (Bühlmann and Gisler, 2005).

3.2.1 <u>Derivation of the Simple Bühlmann Model (SBM)</u>

In this section, the terms Underlying frequency, Expected frequency and Actual frequency will be used. These terms are equivalent to the Underling claims, Expected claims and Actual claims expressed as a frequency (per life year of exposure).

In the derivation of the SBM, a random variable Θ, known as the structural parameter, is used to represent the risk profile of a plan. Θ can be thought of abstractly, as a variable representing the unique risk features of a group risk plan. It is a more general means of representing the risk profile of a plan than the Underlying. Under this method, the Underlying is a function of Θ and is a random variable because Θ is a random variable.

The SBM uses the Bayesian framework for credibility described in section 2.4. The aim of the SBM is to estimate the Underlying frequency of a plan given the Actual frequency.

In mathematical terms, the aim is to estimate:

$$U_j = \mu(\Theta) = E[A_{jT} | \Theta]$$

Where:

- U_j is the Underlying claims frequency for plan j
- A_{jT} is the random variable representing the Actual claims frequency for plan j over the period of investigation

Note: Most credibility models estimate the historical Underlying claims over the period of investigation. The task of determining how the historical Underlying will relate to the future Underlying of a plan is left to the pricing actuary.

The SBM aims to estimate $U_j = \mu(\Theta)$ using the estimator $\widehat{\mu(\Theta)}$.

To do this, the SBM assumes $\widehat{\mu(\Theta)}$ takes a form which can be expressed as a linear combination of the observed Actual claim frequencies i.e.

$$\widehat{\mu(\Theta)} = a + \sum_{i=1}^{n_j} b_i{}^{o} A_{j\,(t-i+1)}$$

Where:
- n_j is the number of years of experience for plan j
- a is an unknown constant
- b_i is an unknown constant for $i = 1, 2, 3, ..., n_j$
- ${}^oA_{j\,(t-i+1)}$ is the observed Actual claim frequency for plan j in year $(t - i + 1)$

Note: The assumption that $\widehat{\mu(\Theta)}$ is a linear function of Actual frequencies restricts the estimator's accuracy; however, as a result of this assumption, the SBM's input requirements are reduced to only the means and variances of the Bayesian credibility model's input distributions.

The SBM optimises the estimator $\widehat{\mu(\Theta)}$ by varying the values of a and b_i such that the expected square error of the estimator is minimised. That is:

$$\min_{a,b_i \in \mathbb{R}} E[(\mu(\Theta) - \widehat{\mu(\Theta)})^2]$$

Assuming A_{jt} is identically distributed across t, it can be shown that $b_1 = b_2 = \cdots = b_n = b$ is optimal. Thus, the above becomes:

$$\min_{a,b \in \mathbb{R}} E[(\mu(\Theta) - a - bA_{jT})^2]$$

Where:

$$A_{jT} = \frac{1}{n_j}\sum_{i=1}^{n_j} A_{j\,(t-i+1)}$$

To find the values of a and b which minimise the expected square error, the above expectation is differentiated with respect to a and b and the results are set to equal zero. This yields the following two equations:

$$E[\mu(\Theta) - a - bA_{jT}] = 0$$

$$Cov[\bar{A}_{jT}, \mu(\Theta)] - bVar[A_{jT}] = 0$$

This simplifies to:

$$a = (1 - b)E_j$$

$$b = \frac{1}{1 + \frac{E[Var(A_{jT}|\Theta)]}{Var(E[A_{jT}|\Theta])}}$$

Where:
- E_j is the Expected frequency of plan j and is equal to $E[\mu(\Theta)] = E\big[E[A_{jT}|\Theta]\big]$

Finally, the equation for determining the credibility factor b is further broken down to produce the familiar formula:

$$b = \frac{n_j N_j}{n_j N_j + \frac{E[\sigma^2(\Theta)]}{Var[\mu(\Theta)]}}$$

Where:

- N_j is the number of lives in plan j
- $\sigma^2(u) = N_j \times Var[A_{j\,(t+1)}|\Theta = u]$ one can think of this as the conditional variance of the frequency of Actual claims standardised to a per member variance

And hence, the SBM's final credibility weighted estimator is:

$$\widehat{\mu(\Theta)} = (1 - b)E_j + b\,{}^{o}A_{jT}$$

3.2.2 Estimation of the Credibility Factor Components

A number of methods are available for estimating $E[\sigma^2(\Theta)]$ and $Var[\mu(\Theta)]$. An example of an estimation method based on a portfolio of p plans is (Bühlmann and Gisler, 2005):

$$E[\widehat{\sigma^2(\Theta)}] = \frac{1}{p}\sum_{j=1}^{p}\frac{1}{n_j - 1}\sum_{i=1}^{n_j} N_j\,({}^{o}A_{j\,(t-i+1)} - {}^{o}A_{jT})^2$$

And:

$$Var[\widehat{\mu(\Theta)}] = \frac{1}{p-1}\sum_{j=1}^{p}({}^{o}A_{jT} - {}^{o}A)^2 - \frac{E[\widehat{\sigma^2(\Theta)}]}{n_j N_j}$$

Where:

$${}^{o}A = \frac{\sum_{j=1}^{p} n_j N_j\,{}^{o}A_{jT}}{\sum_{j=1}^{p} n_j N_j}$$

Note: It is possible for the estimator $Var[\widehat{\mu(\Theta)}]$ to be negative in which case it should be set to zero.

3.2.3 A Discussion of the SBM Credibility Formula

Bühlmann's optimised linear estimation approach leads to the credibility factor formula:

$$b = \frac{1}{1 + \frac{E[Var(A_{jT}|\Theta)]}{Var(E[A_{jT}|\Theta])}}$$

It also results in a reduced input parameter requirement. That is, rather than requiring inputs of the Underlying distribution and the set of conditional Actual distributions (the inputs of the Bayesian approach), the SBM only requires the expected values and variances of these distributions.

The SBM's linear approximation to the Bayesian approach potentially introduces a significant level of inaccuracy; however, historically this was a practical necessity due to the calculation intensiveness of the Bayesian approach.

3.2.4 A Discussion of the SBM's Input Parameters

From the SBM formula below:

$$b = \frac{1}{1 + \frac{E[Var(A_{jT}|\Theta)]}{Var(E[A_{jT}|\Theta])}}$$

it is clear that the value of the credibility factor is entirely dependent on the ratio:

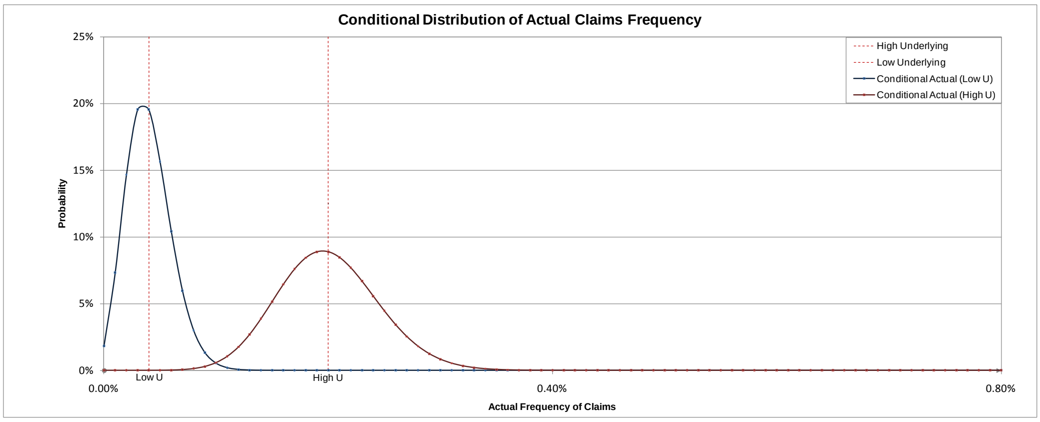
$$\frac{E[Var(A_{jT}|\Theta)]}{Var(E[A_{jT}|\Theta])}$$

To clearly understand the implications of the SBM, it is important to first understand what the numerator and denominator of this ratio represent and what influences the value of these variables.

- What does $E[Var(A_{jT}|\Theta)]$ really mean?

  Graph 3 in section 2.4.2 shows two examples of the conditional distribution of Actual claims. One curve shows the distribution for a 'low' value of the Underlying and the other for a 'high' value of the Underlying. The same graph is presented below but the horizontal axis is changed to depict the frequency of Actual claims.

Graph 4:



If one draws a parallel between Θ and the Underlying frequency, it becomes clear that the variance of the above distributions is equivalent to $Var(A_{jT}\,|\Theta = u)$ and is dependent on the value of u. Thus, $E[Var(A_{jT}\,|\Theta)]$ is just the weighted average of the $Var(A_{jT}\,|\Theta = u)$ for each possible value of u. In other words, one can think of $E[Var(A_{jT}\,|\Theta)]$ as the variance of the Actual random variable around the Underlying frequency of claims averaged over all possible values of the Underlying frequency of claims.

Note: $Var(A_{jT}\,|\Theta = u)$ increases as u increases. This means that as the Expected frequency of claims increases, the $E[Var(A_{jT}\,|\Theta)]$ will also increase.

- What influences the value of $E[Var(A_{jT}\,|\Theta)]$?

The factors which will make $E[Var(A_{jT}\,|\Theta)]$ vary from plan to plan include:

- The life years of exposure of the plan
- The Expected probabilities of claim across the plan's members
- The sum insured distribution of members (this will only impact calculations based on amount of claims not number of claims)

- What does $Var(E\left[A_{jT}\,|\Theta\right])$ really mean?

This is the variance of the mean of the Actual claim frequency. In other words, it is the variance of the plan's Underlying claim frequency.

- What influences the value of $Var(E\left[A_{jT}\,|\Theta\right])$?

As discussed in section 2.4.1, the variance of the Underling is entirely dependent on how closely the Expected predicts the Underlying. As such, the significant drivers of the variance of Underlying frequency are likely to be:

- Product
- Exposure and membership data quality
- Plan type (corporate, industry fund or master trust)
- Expected frequency of claims

- Would the ratio $\frac{E[Var\left(A_{jT}\,|\Theta\right)]}{Var\left(E\left[A_{jT}\,|\Theta\right]\right)}$ remain constant even though the numerator and denominator change from one plan to another?

  No. From the above discussion, it is clear that most of the significant drivers of $Var(E\left[A_{jT}\,|\Theta\right])$ do not have a direct impact on $E[Var\left(A_{jT}\,|\Theta\right)]$. Further, the plan characteristics that may influence both the numerator and denominator are unlikely to affect both variables by the same factor.

Considering the implications of the above discussion, it is clear that the credibility model is not complete until the ratio or its two components can be calculated for each plan. To achieve this, Bühlmann removes the impact of exposure embedded in the ratio's numerator and makes it explicate. That is:

$$b = \frac{1}{1+\frac{E[Var\left(A_{jT}\,|\Theta\right)]}{Var\left(E\left[A_{jT}\,|\Theta\right]\right)}} \qquad \text{becomes} \qquad b = \frac{n_j N_j}{n_j N_j + \frac{E[\sigma^2(\Theta)]}{Var\left(E\left[A_{jT}\,|\Theta\right]\right)}}$$

The model then specifies the methods of estimating $E[\sigma^2(\Theta)]$ and $Var(E\left[A_{jT}\,|\Theta\right])$ at a portfolio level (section 3.2.2) which implies that they are constant or have a constant ratio from one plan to another and from one product to another. Clearly, this is inconsistent with the view that exposure is just one of a number of significant factors which influence the ratio $\frac{E[Var\left(A_{jT}\,|\Theta\right)]}{Var\left(E\left[A_{jT}\,|\Theta\right]\right)}$.

3.2.5 Limitations of the BSM and SBM

1. **The treatment of $\boldsymbol{E[\sigma^2(\Theta)]}$ as a constant**

   Under the BSM, $E[\sigma^2(\Theta)]$ is treated as a constant across all plans and products. This can be seen from the equations for the estimator $E[\widehat{\sigma^2(\Theta)}]$. That is:

$$E[\widehat{\sigma^2(\Theta)}] = \frac{1}{p}\sum_{j=1}^{p}\frac{1}{n_j - 1}\sum_{i=1}^{n_j} N_j\,({}^{o}A_{j\,(t-i+1)} - {}^{o}A_{jT})^2$$

   One will note that the unbiased estimator of variance for plan j is:

$$\widehat{\sigma^2\left(\Theta_j\right)} = \frac{1}{n_j - 1}\sum_{i=1}^{n_j} N_j\,({}^{o}A_{j\,(t-i+1)} - {}^{o}A_{jT})^2$$

   Therefore,

$$E[\widehat{\sigma^2(\Theta)}] = \frac{1}{p}\sum_{j=1}^{p}\widehat{\sigma^2\left(\Theta_j\right)}$$

Thus, this estimator is simply a portfolio level average of each plan's estimator of variance.

This means that the model does not take into account some of the key factors influencing a plan's distribution of Actual frequencies around the plan's Underlying. Instead, it uses an average across all plans. By doing this, the model will either assign too much or too little credibility and hence result in a higher or lower credibility premium than is optimal. This will expose the insurer using the BSM to anti-selection provided that a competing insurer is using a credibility model which accounts for the other factors affecting credibility.

2. **The treatment of $\boldsymbol{Var}[\mu(\boldsymbol{\Theta})]$ as a constant**
   The practical application of the BSM does not account for the main drivers of the $Var(E[A_{jT}|\Theta])$. As a result of this, the model will either assign too much or too little credibility and hence result in a higher or lower credibility premium than is optimal.

3. **The linearity assumption**
   The BSM assumes that the estimator for $\mu(\Theta)$ takes the form:

$$\widehat{\mu(\Theta)} = a + \sum_{i=1}^{n_j} b_i{}^o A_{j(t-i+1)}$$

This assumption leads to the commonly recognisable credibility formula:

$$\widehat{\mu(\Theta)} = (1-b)E_j + b^o A_{jT}$$

This is an assumption which restricts the accuracy of the BSM. This is because there is no justifiable reason for the linearity assumption on the grounds of accuracy. The reasons for using the assumption are mathematical convenience and simplicity.

Attempting to fit a line of best fit through an irregular curve can result in large errors.

3.2.6 Concluding Comments on the Limitations of the BSM and SBM

Section 2.5 stated the three key requirements of an accurate credibility model. These are:

1. The accurate determination of the distribution of Underlying claims
2. The accurate determination of the conditional Actual claims distributions
3. The use of a theoretically sound and accurate credibility formula

The discussion in section 3.2.3 to 3.2.5 shows that the practical application of BSM:

- Does not account for the impact of any significant drivers of the distribution of Underlying claims
- Does not account for the impact of a range of factors which influence the conditional distribution of Actual claims
- Uses a linear approximation to the optimal Bayesian conditional probability formula

3.3 The Limited Fluctuation Model (LFM)

The LFM or variations of it are also commonly used in group risk pricing. This model is also referred to as the Classical Credibility Model.

The LFM firstly determines the number of expected claims required for full credibility. It then determines the partial credibility attributable to each plan based on the number or amount of observed claims relative to the full credibility requirement (Venter, 1986).

3.3.1 Derivation of the Full Credibility Requirement

The derivation of the LFM's full credibility requirement starts with the equation for total claims:

$$T_{jT} = \sum_{k=1}^{C_j} SI_{jk}$$

Where:

- T_{jT} is the Actual claims amount for plan j over the period of investigation
- C_j is the Actual number of claims for plan j over the period of investigation
- SI_{jk} is the claim size for the k^th claim of plan j

Using the Central Limit Theorem, it is assumed that T_{jT} is normally distributed with mean μ_T and variance ${\sigma_T}^2$ i.e.

$$T_{jT} \sim N[\mu_T, {\sigma_T}^2]$$

The LFM assigns full credibility to a plan's experience if the expected number of claims is sufficiently large to ensure that:

$$Prob(-\varepsilon\mu_T < T_{jT} - \mu_T < \varepsilon\mu_T) = \alpha$$

Where ε is a small factor such as 10% and α is a high probability, for example 90%

It can be shown (see appendix 13.2) that this is achieved when:

$$\mu_C = (z/\varepsilon)^2 \times \left[\frac{{\sigma_{SI}}^2}{{\mu_{SI}}^2} + \frac{{\sigma_C}^2}{\mu_C}\right]$$

Where:

- μ_C is the expected value of the number of claims i.e. $\mu_C = E[C_j]$
- ${\sigma_C}^2$ is the variance of the number of claims i.e. ${\sigma_C}^2 = Var[C_j]$
- μ_{SI} is the expected value of a claim's sum insured i.e. $\mu_{SI} = E[SI_{jk}]$
- ${\sigma_{SI}}^2$ is the variance of a claim's sum insured i.e. ${\sigma_{SI}}^2 = Var[SI_{jk}]$
- z is a constant such that the probability that a standard normal random variable is larger than z is $\frac{1-\alpha}{2}$ e.g. for $\alpha = 90\%$, $\frac{1-\alpha}{2} = 5\%$ and $z = 1.645$.

Note that the equation contains μ_C on both the left and right hand side, however, $\frac{{\sigma_C}^2}{\mu_C}$ can be treated as a constant equal to 1 by assuming that C_j is distributed as a Poisson.

Hence, under the Poisson assumption, the equation for full credibility becomes:

$$C_j{}^F = ({}^{Z}/_{\varepsilon})^2 \times \left[\frac{{\sigma_{SI}}^2}{{\mu_{SI}}^2} + 1\right]$$

Where:

- $C_j{}^F$ is the expected number of claims required for full credibility

In the fixed sum insured case, this formula simplifies to:

$$C_j{}^F = ({}^{Z}/_{\varepsilon})^2$$

3.3.2 Derivation of the Partial Credibility Formula

Under the LFM, it is assumed that the credibility weighted premium is a weighted average of the Expected and Actual claims. That is:

$$P_{jT} = (1-b)E_{jT} + b\,{}^{o}T_{jT}$$

Where:

- P_{jT} is the credibility weighted risk premium for plan j over the entire experience period
- E_{jT} is the Expected risk premium for plan j over the entire experience period
- b is the credibility factor
- ${}^{o}T_{jT}$ is the observed Actual claims amount for plan j over the period of investigation

The credibility factor b is chosen such that the following equation is satisfied:

$$Prob\left(-\varepsilon\mu_T < b(T_{jT} - \mu_T) < \varepsilon\mu_T\right) = \alpha$$

This results in the following equation for the credibility factor b (see appendix 13.2):

$$b = \sqrt{\frac{\mu_C}{C_j{}^F}}$$

In practice, the Expected number of claims is replaced by the observed Actual number of claims, thus resulting in the familiar credibility formula:

$$b = \sqrt{\frac{{}^{o}C_j}{C_j{}^F}}$$

3.3.3 Limitations of the LFM

The LFM has a number of inaccuracies. The most significant of these are:

1. **The disregard of the distribution of the Underlying claims**

   The LFM ignores the fact that $\mu_T = E[T_{jT}]$ is a random variable with a distribution around E_{jT}. By doing so, it fails to increase the level of credibility when E_{jT} is unreliable and decrease it when E_j is unlikely to be very different from μ_T.

   This also results in the requirement of the subjective inputs ε and α which are difficult to choose on the basis of any logical approach.

   The LFM could be improved by adjusting ${C_j}^F$ according to the variability of Underlying on Expected. This would, however, require some sort of basis for the determination of ${C_j}^F$ such as a theoretically more sound credibility model to calibrate against. Aside from the obvious redundancy, this exercise may not be worthwhile considering the other inaccuracies of the LFM.

2. **The normality assumption for total claims**

   Since the total claims amount is a sum of a number of claim amounts, it will be normally distributed when the total number of claims is sufficiently large (Central Limit Theorem). Under a constant sum insured scenario, a Poisson distribution for the number of claims and $C_j > 10$, the distribution of total claims approximates a normal quite closely. Below this level, the normality assumption is less reliable.

   As the variance of sums insured increases, a greater number of claims are required for the normality assumption to remain reasonable accurate.

3. **The approximation of the expected value of the number of claims**

   In the determination of partial credibility, the Actual number of claims is used as an approximation to the expected value of the number of claims. The formula for partial credibility as derived under the LFM is:

$$b = \sqrt{\frac{\mu_C}{{C_j}^F}}$$

   Since μ_C is unknown, the observed Actual number of claims, ${}^oC_j$, is used in its place. This approximation reduces the reliability of the partial credibility results particularly at the smaller values of ${}^oC_j$.

4. The partial credibility formula

The LFM's credibility factor moves in the right direction as $(T_{jT} - \mu_T)$ becomes more variable; however, it does not move by the right amount. This is a substantial inaccuracy.

The LFM's credibility formula ignores the impact the value of b has on the term $(1-b)E_{jT}$ in the following formula:

$$P_{jT} = (1-b)E_{jT} + b^o T_{jT}$$

As a result, a doubling in the standard deviation of $(T_{jT} - \mu_T)$ results in a halving of the credibility factor (all else being equal). This can be seen from the equation below:

$$Prob\left(-\varepsilon\mu_T < b(T_{jT} - \mu_T) < \varepsilon\mu_T\right) = \alpha$$

The other half of the credibility equation (ignored by the LFM) is:

$$Prob\left(-\varepsilon\mu_T < (1-b)(E_{jT} - \mu_T) < \varepsilon\mu_T\right) = \alpha'$$

When considering the impact that the value of b has on both credibility inputs, it becomes evident that a better choice for the value of b would be one which equates α with α'. That is:

$$Prob\left(-\varepsilon\mu_T < b(T_{jT} - \mu_T) < \varepsilon\mu_T\right) = Prob\left(-\varepsilon\mu_T < (1-b)(E_{jT} - \mu_T) < \varepsilon\mu_T\right)$$

From the above equation, it is clear that a doubling in the standard deviation of $(T_{jT} - \mu_T)$ should not reduce b by as much as half (all else being equal).

This limitation is evidenced in practice by the fact that under certain conditions, an increase in the number of claims leads to a reduction in the credibility weighted premium. The conditions which result in this counterintuitive result are:

- The credibility factor is less than one after the increase in the number of claims
- The plan has level sums insured
- $E_{jT} \geq 3T_{jT}$ after the increase in the number of claims

3.3.4 <u>Concluding Comments on the Limitations of the LFM</u>

Section 2.5 stated the three key requirements of an accurate credibility model. These are:

1. The accurate determination of the distribution of Underlying claims
2. The accurate determination of the conditional Actual claims distributions
3. The use of a theoretically sound and accurate credibility formula

Section 3.3.3 states that the LFM:

- Ignores the distribution of Underlying and any bearing this has on credibility
- Accounts for the distribution of Actual reasonably well but is exposed to inaccuracy when the number of observed claims is relatively small
- Uses a credibility formula that is substantially inaccurate

3.4 Some Specific Models Used in Group Risk Pricing

Three types of credibility models were discussed above. In the Australian group risk market, variations of these models are used. In this section, three commonly used variations of these models are shown.

3.4.1 Model 1

Define:

1. X as the total number of life years of exposure
2. ${}^{o}C$ as the annualised observed Actual amount of claims
3. E as the annualised Expected amount of claims
4. P as the credibility weighted risk premium

Under this model, P is determined using:

$$P = (1-b)E + b\,{}^{o}C$$

And the credibility factor b is calculated as:

$$b = \frac{(X-400)}{(X-400)+1400}$$

This model overrides the credibility formula and uses full credibility when:

- X > 9400
- ${}^{o}C/E$ > 2.35

Also, when ${}^{o}C/E$ < 0.47, the credibility factor is to be determined on a case by case basis.

This model appears to be a variation of the BSM where:

- Instead of using actual exposure, exposure minus 400 is used
- $\frac{E[\sigma^2(\Theta)]}{Var\,[\mu(\Theta)]} = 1{,}400$
- Special rules apply outside of certain variable ranges

3.4.2 Model 2

Define:

1. X as the total number of life years of exposure
2. ${}^{o}N$ as the observed Actual number of claims
3. ${}^{o}C$ as the annualised observed Actual amount of claims
4. E as the annualised Expected amount of claims
5. P as the credibility weighted risk premium

Under this model, P is determined using:

$$P = (1-b)E + b\,{}^{o}C$$

And the credibility factor b is calculated as:

$$b = \frac{k}{k + 0.3}$$

Where:

$$k = \frac{{}^{o}N}{60} + \frac{X}{60{,}000}$$

This model appears to be a variation of the BSM where:

- Instead of using actual exposure, a weighted average of number of claims and exposure is used
- If one redefines k as $k = \frac{X}{30{,}000}$, this would imply $\frac{E[\sigma^2(\Theta)]}{Var\,[\mu(\Theta)]} = 9{,}000$

3.4.3 Model 3

Define:

1. ${}^{o}N$ as the observed Actual number of claims
2. μ_{SI} as the mean sum insured
3. ${\sigma_{SI}}^2$ as the variance of sum insured
4. ${}^{o}C$ as the annualised observed Actual amount of claims
5. E as the annualised Expected amount of claims
6. P as the credibility weighted risk premium

Under this model, P is determined using:

$$P = (1 - b)E + b^{o}C$$

And the credibility factor b is calculated as:

$$b = \sqrt{\frac{{}^{o}N}{C^F}}$$

Where:

$$C^F = 271 \times \left[\frac{{\sigma_{SI}}^2}{{\mu_{SI}}^2} + 1\right]$$

This model is the LFM with $\varepsilon = 10\%$ and $\alpha = 90\%$.

4 A New Approach to Group Risk Pricing

This section of the paper and sections 5 and 6 deal with the introduction of a proposed new approach to group risk pricing and credibility. This approach aims to improve pricing accuracy by addressing the three requirements of an accurate credibility model. This is achieved by the use of two interlinked models. These are:

- A credibility model (proposed credibility model)
- A URF distribution measurement tool (URF distribution model)

Sections 4.1 to 4.3 discuss the high level approach taken to address the three requirements of an accurate credibility model.

Note: The URF is defined in section 2.3.4 as the ratio of Underlying claims to Expected claims.

4.1 The Credibility Formula

Under the proposed approach, the credibility formula is based on Bayesian conditional probability. The credibility formula forms part of the proposed credibility model.

4.2 The Conditional Actual Claims Distributions

The set of conditional Actual distributions is modelled through simulation. These calculations are conducted by the proposed credibility model.

4.3 The Underlying Claims Distribution

The determination of the Underlying claims distribution is in part modelled and in part empirically measured. The Underlying of a plan is modelled as the product of the Expected claims and the URF. This is calculated by the proposed credibility model; however, the distribution of the URF is generated by the URF distribution model.

4.4 The Interaction Between the Two Models

4.4.1 <u>The Proposed Credibility Model's Inputs and Outputs</u>

The credibility model requires an input assumption for the distribution of URF in addition to the regular pricing data such as membership details, sum insured data, exposed to risk and historical claims.

The model produces two outputs: the primary output is a risk premium produced for each plan; the model's secondary output is a set of conditional probabilities produced for each plan.

Note: The conditional probability output is independent of the URF distribution input; however, the risk premium calculation requires an appropriate URF distribution input.

4.4.2 <u>The URF Distribution Model's Inputs and Outputs</u>

The URF distribution model requires the conditional probabilities for a sufficient number of plans in order to derive a URF distribution based on the experience of these plans.

4.4.3 <u>The Management of the Two Models' Interactions in Practice</u>

For practical reasons, it is likely that upon implementation, these models will be required to produce premiums without the significant wait required for model calibration. For this reason, it will be necessary to make an initial assumption for the distribution of URF. This assumption can then be gradually phased out over two to five years at which point, the models can be fully reliant on their own output of the distribution of URF. Below is a three stage flow chart of how the models might be implemented.

Chart 1A: The management of the URF distribution calibration process in the first year of model implementation.

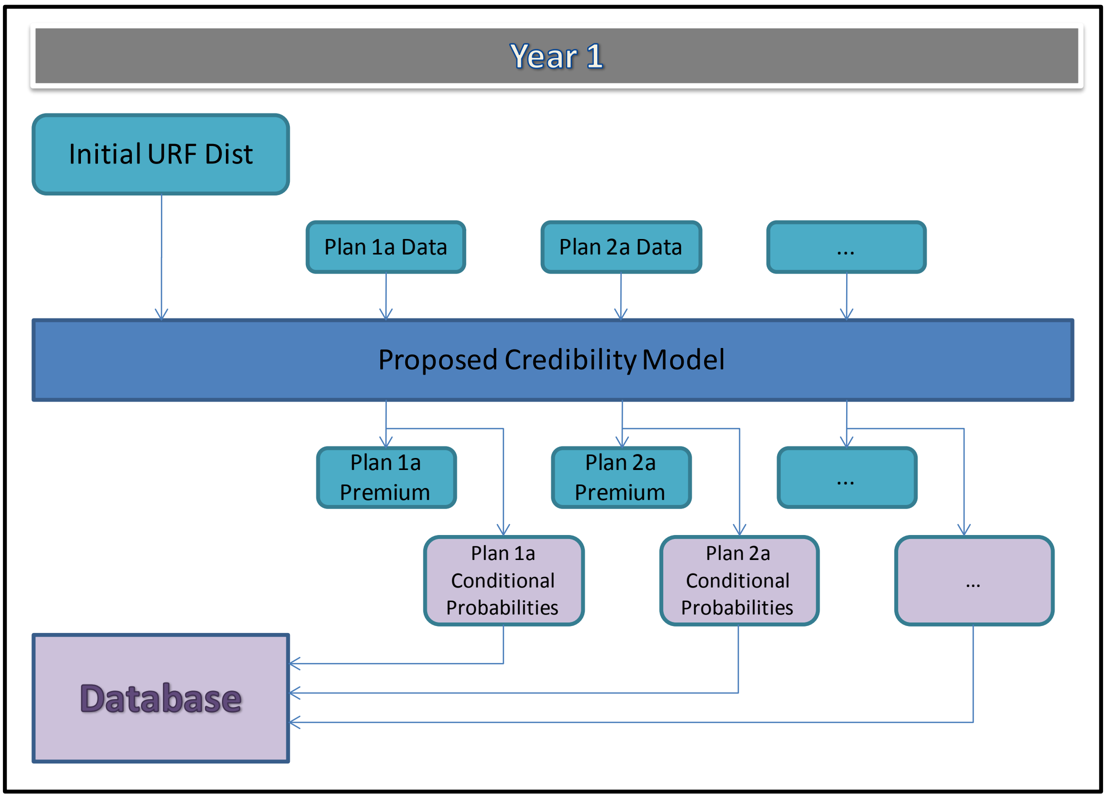


Chart 1B: The management of the URF distribution calibration process in the second year of model implementation.

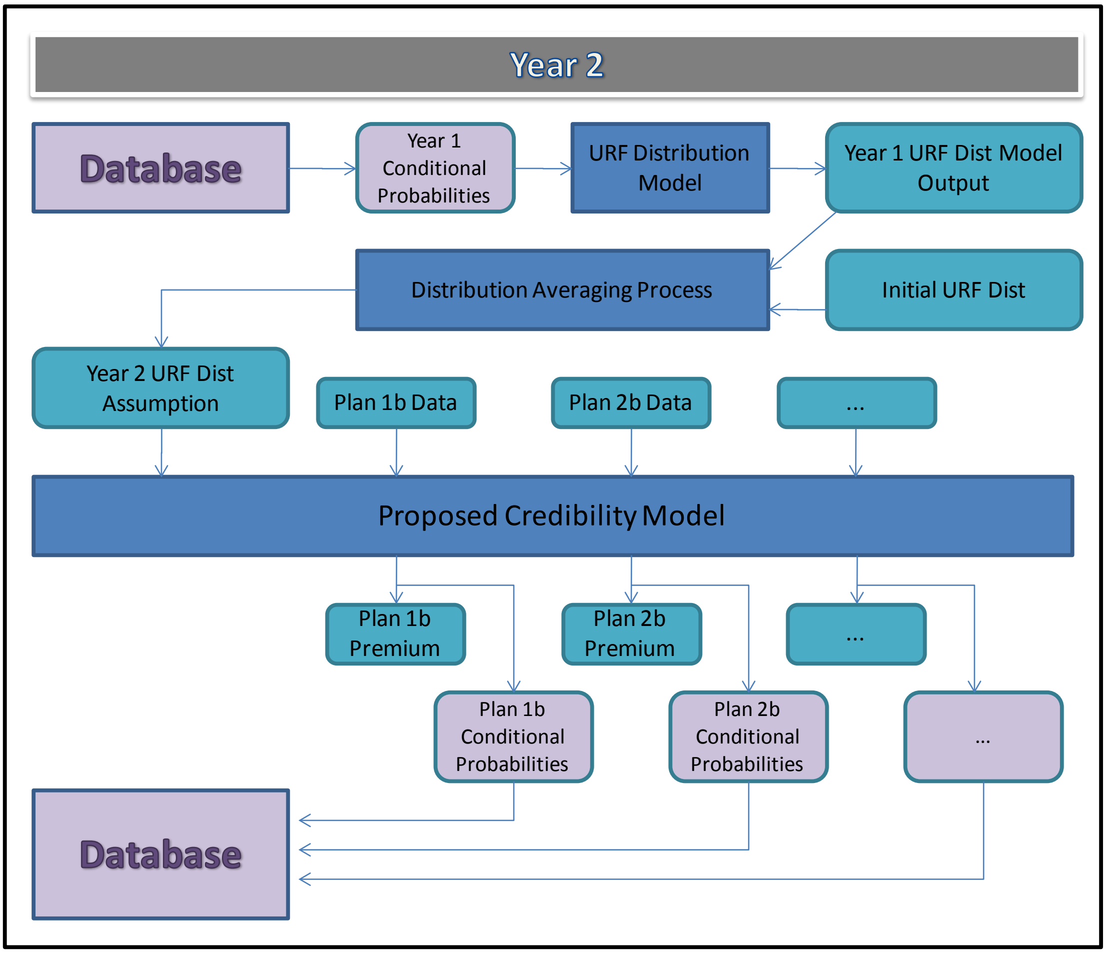
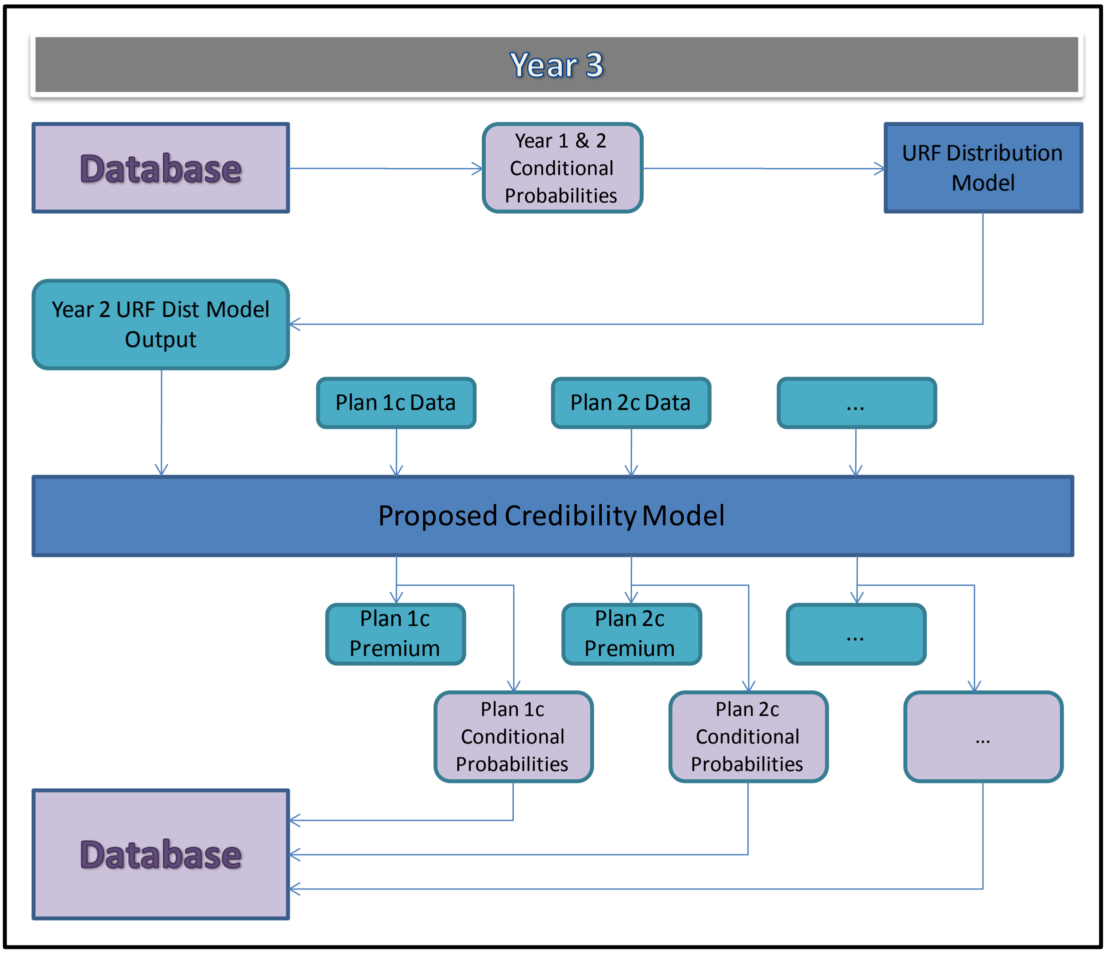


Chart 1C: The management of the URF distribution calibration process in the third year of model implementation.



4.5 The Value Added by the Two Models

4.5.1 The Credibility Model

The most significant advantage of the proposed new credibility model is its plan level pricing accuracy. It achieves this by making use of all the available data and accounting for the information though a logical, scientific means.

Existing models have greater inter-plan cross subsidisation and are less responsive to the individual characteristics of the plans they price. Section 10 shows the potential impact of these limitations in a competitive environment.

4.5.2 The URF Distribution Model

The distribution of URF provides a great deal of insight into the accuracy of the insurer's base rates, loading factors and plan data as well as the effectiveness of the insurer's rating classes. Having a model which can produce a distribution of URF across various groupings of plans can be very beneficial and can aid the development of a greater understanding of different market segments.

4.6 The Simulation Based Approach and its 'Fringe Benefits'

The proposed credibility model relies on a simulation based approach. In addition to its use in credibility modelling, the simulation approach lays a foundation for:

- The identification of new rating classes
- The measurement of the effectiveness of existing rating classes, rates and loadings
- Determining the cost of various profit share arrangements
- The plan level allocation of risk based capital
- The measurement of the impact of various reinsurance arrangements
- The reporting of various probability based statistics based on the portfolio level expected distribution of Actual claims
- Determining the cost of rate guarantees of various durations
- The measurement of the impact of the economic cycle on the URF distribution of various market segments

Note: Some of these 'fringe benefits' can be realised with relatively little additional modelling. The more complex tasks will require significant additional data collection, research and modelling.

5 The Proposed Credibility Model for Group Risk

5.1 Assumptions

5.1.1 Assumption 1: Modelling of Individual Member Claim Amounts

Individual member claim amounts are modelled using the following formula:

$$A_{jtk} = SI_{jtk}\, I_{jtk}$$

Where:

- A_{jtk} is the Actual claims amount of k^{th} member of plan j in year t
- SI_{jtk} is the sum insured of k^{th} member of plan j in year t
- I_{jtk} is a Bernoulli random variable. That is:

$$I_{jtk} \sim \text{Bernoulli}\,(P_{jtk}^{U})$$

Where:

- P_{jtk}^{U} is the Underlying probability of plan j's k^{th} member making a claim in year t

The Bernoulli distribution is appropriate for the number of claims in any one year by any one member as only two outcomes are possible, a claim either occurs or not. Note that the claim does not have to be reported, just incurred.

Using a Bernoulli distribution with annual probability rates implies that members who claim, on average are not replaced for 6 months. This seems realistic.

5.1.2 Assumption 2: Modelling of Underlying Probabilities of Claim at Member Level

The Underlying probability of claim P_{jtk}^{U} is modelled as:

$$P_{jtk}^{U} = URF_j \times P_{jtk}^{E}$$

Where:

- URF_j is the Underlying Rating Factor for plan j
- P_{jtk}^{E} is the probability of plan j's k^{th} member making a claim in year t based on the insurer's risk rates and loading factors

An alternative and more complete model is:

$$P_{jtk}^{U} = URF_j \times P_{jtk}^{E} + URT_j + \varepsilon_{jkt}$$

Where:

- URT_j is an Underlying Rating Term specific to group j
- ε_{jkt} is an error term accounting for individual variations between members and over time

In the proposed credibility model, the simple model for P_{jtk}^{U} is used. This is appropriate because it reduces the complexity of the proposed credibility model.

The simulation results in section 9 suggest that the credibility model's results are not significantly impacted by the use of the simple model for P_{jtk}^{U}.

5.1.3 Assumption 3: Modelling of the Actual Claims Amount

It is assumed that the Actual claims cost of plan j is the sum of the individual claims costs of its members. That is:

$$A_{jt} = \sum_{k=1}^{N_j} A_{jtk} = \sum_{k=1}^{N_j} SI_{jtk} \, I_{jtk}$$

And

$$A_{jT} = \sum_{t=1}^{\mathrm{T}} A_{jt}$$

Where:

- A_{jt} is the Actual claims amount of plan j, in year t
- A_{jT} is the Actual claims amount of plan j, over the period of investigation
- N_j is the number of members in plan j in year t. For simplicity, membership has been assumed to be static over time
- I_{jtk} are conditionally independent, that is:

I_{jta} and I_{jtb} are, conditional on a given URF_j, independent for $a \neq b$

The conditional independence assumption is appropriate because:

- Unlike a retail portfolio, a single group risk plan does not cover the same life twice
- The proposed model only covers the component of risk premium net of catastrophe risk premium. Thus, multiple claims arising from a single event should be treated as a single claim under this model with additional premiums charged for the catastrophe component.

5.2 The Proposed Credibility Model

The proposed credibility model produces credibility adjusted risk premiums instead of credibility factors. Credibility factors and experience rating factors (ERF) can be inferred from the model's premium output.

5.2.1 The Credibility Formula

This model's objective is to find the minimum cross subsidy risk premium for plan *j* given the Actual experience of this plan. On this basis, the next one year's premium, $Prem_{jT+1}$, is calculated as:

$$Prem_{jT+1} = \mathrm{E}[A_{jT+1} | A_{jT} = {}^{o}A_{jT}]$$

Where:

- A_{jT+1} is the Actual claims of plan j, over the next year
- ${}^{o}A_{jT}$ is the observed Actual claims of plan j, over the period of investigation

This can be broken down (see appendix 13.3) into:

$$Prem_{jT+1} = \sum_{n=1}^{N} \mathrm{E}[A_{jT+1} | URF_j = URF_j^n] \times Prob(URF_j = URF_j^n | A_{jT} = {}^{o}A_{jT})$$

Where:

- N is the number of discrete URF points used to approximate the continuous URF random variable
- URF_j^n is the n th value in the discrete set of URF values used to approximate the continuous range of URF values

This step is based on assumptions 1, 2 and 3 which imply that the only information that ${}^{o}A_{jT}$ can provide that is useful for determining $\mathrm{E}[A_{jT+1}]$ is the value of URF_j.

After some manipulation (see appendix 13.3) the above equation can be expressed as:

$$Prem_{jT+1} = \sum_{n=1}^{N} \{\mathrm{E}[A_{jT+1} | URF_j = URF_j^n] \times \frac{Prob(A_{jT} = {}^{o}A_{jT} | URF_j = URF_j^n) \times Prob(URF_j = URF_j^n)}{\sum_{n=1}^{N} Prob(A_{jT} = {}^{o}A_{jT} | URF_j = URF_j^n) \times Prob(URF_j = URF_j^n)}\}$$

Now, the equation is in a form where all of the components can be determined and hence the value of $Prem_{jT+1}$ can be calculated.

Note that there are only three components to the above equation. These are:

- $\mathrm{E}[A_{jT+1} | URF_j = URF_j^n]$ for $n = 1, 2, 3, \ldots, N$
- $Prob(URF_j = URF_j^n)$ for $n = 1, 2, 3, \ldots, N$
- $Prob(A_{jT} = {}^{o}A_{jT} | URF_j = URF_j^n)$ for $n = 1, 2, 3, \ldots, N$

5.2.2 Calculation of the Components of the Credibility Formula

The determination of E$[\boldsymbol{A_{jT+1}}|\boldsymbol{URF_j} = \boldsymbol{URF_j^n}]$

Based on assumption 3, E$[A_{jT+1}|URF_j = URF_j^n]$ can be broken down as follows:

$$\mathrm{E}\left[A_{jT+1}\middle|URF_j = URF_j^n\right] = \sum_{k=1}^{N_j} \mathrm{E}[A_{j(T+1)k}|URF_j = URF_j^n]$$

$$= \sum_{k=1}^{N_j} \mathrm{E}[A_{j(T+1)k}|URF_j = 1] \times URF_j^n$$

And using assumption 1 and 2, it can be shown that:

$$\mathrm{E}\left[A_{jT+1}\middle|URF_j = URF_j^n\right] = \sum_{k=1}^{N_j} SI_{j(T+1)k} \times URF_j^n \times P_{j(T+1)k}^E$$

Now, $SI_{j(T+1)k}$ and $P_{j(T+1)k}^E$ are both known; thus, E$\left[A_{jT+1}\middle|URF_j = URF_j^n\right]$ can be calculated for each of the N values of URF_j^n.

The determination of $\boldsymbol{Prob}(\boldsymbol{URF_j} = \boldsymbol{URF_j^n})$

The $Prob(URF_j = URF_j^n)$ is calculated by the URF distribution model (see section 6).

The determination of $\boldsymbol{Prob}(\boldsymbol{A_{jT}} = {}^o\boldsymbol{A_{jT}}|\boldsymbol{URF_j} = \boldsymbol{URF_j^n})$

The $Prob(A_{jT} = {}^oA_{jT}|URF_j = URF_j^n)$ is calculated using a simulation modelled on the basis of assumptions 1, 2 and 3. Appendix 13.4 covers the details of how this simulation is conducted.

6 The URF Distribution Model

In the previous section, the credibility formula was discussed. The credibility formula's derivation (see appendix 13.3), shows that:

$$Prob\left(A_{jT} = {}^{o}A_{jT}\right) = \sum_{n=1}^{N} Prob\left(A_{jT} = {}^{o}A_{jT} \,|URF_j = URF_j^n\right) \times Prob\left(URF_j = URF_j^n\right)$$

This is the probability that the Actual claims for plan j over the period of investigation are as observed.

Assuming that A_{jT} and A_{iT} are independent for $j{\neq}i$, it can be shown that the probability of a grouping of J plans all experiencing an Actual claims exactly as observed, is:

$$Prob\left(A_{1T} = {}^{o}A_{1T} \cap A_{2T} = {}^{o}A_{2T} \cap \ldots \cap A_{JT} = {}^{o}A_{JT}\right) = \prod_{j=1}^{J} Prob\left(A_{jT} = {}^{o}A_{jT}\right)$$

Note that the conditional probabilities for each plan i.e. $Prob\left(A_{jT} = {}^{o}A_{jT} \,|URF_j = URF_j^n\right)$ are calculated as a part of the pricing process. Also, as discussed in section 4.4.1, the conditional probabilities are independent of the URF distribution. Thus, provided that the N conditional probabilities generated in the final pricing of each plan are stored in a database, $Prob\left(A_{1T} = {}^{o}A_{1T} \cap A_{2T} = {}^{o}A_{2T} \cap \ldots \cap A_{JT} = {}^{o}A_{JT}\right)$ can be calculated for any given distribution of URF.

Using the Maximum Likelihood Estimator (MLE) approach, the N values of $Prob\left(URF_j = URF_j^n\right)$ are chosen such that the likelihood of observing the experienced claims of the portfolio is maximised.

7 Limitations of the Proposed Approach

7.1 The Assumption of Inter-Plan Independence

The URF distribution model's assumption that A_{jT} and A_{iT} are independent for $i \neq j$ is not realistic. This assumption may be stronger than required and the determination of a URF distribution may be possible under a more realistic assumption. More work is needed in this area.

7.2 The URF Distribution Model's Results

The output produced by the URF distribution model is sufficiently accurate for the purposes of credibility modelling; however, without further refinement, its effectiveness in rating class identification is limited. Testing shows that the estimated URF distribution's mean and variance are quite accurate. The estimated URF distribution also captures some of the clear irregularities of the URF distribution's shape. These irregularities can be useful in identifying rating classes. The main limitation of the estimated URF distribution is its tendency to overfit to data and thus, highlight URF distribution irregularities that are not actually present. There are a number of optimisation techniques and other approaches to explore which may address this limitation. Further work is needed in this area.

The estimation of a URF distribution requires the conditional probability input from a number of plans. At this stage, it is unclear how the number of plans and their size would affect the reliability of the estimated URF distribution. A better understanding of this is required before the proposed approach can be implemented in practice.

7.3 The Magnitude of Pricing Accuracy Improvement

The pricing inaccuracy of existing credibility models is largely driven by the shape of the URF distribution and how much it changes between:

- Death and TPD cover
- Full plan data and incomplete plan data
- Corporate plans, master trusts and industry funds

At present, the URF distributions and the size of the impact of the above factors is unknown. In the absence of this information, it is difficult to know exactly how inaccurate the existing credibility models are.

The pricing accuracy improvement offered by the proposed approach is dependent on the level of inaccuracy of existing models. Thus, it is difficult to determine if the extra complexity introduced by the proposed approach is outweighed by the improvement in accuracy. Before the proposed approach can be implemented, there is a need for some initial research into the shape of the URF distribution and its sensitivity to product, data quality et cetera.

8 The Credibility Model Assessment Method

This section introduces a method which is later used to assess the accuracy of various credibility models. In Section 9, the credibility model assessment method is used to assess the accuracy of the proposed model and the three commonly used credibility models introduced in section 3.4. This analysis is done in a non competitive environment. In section 10, the method is used to assess the accuracy of all four models in a competitive environment.

8.1 An Overview of the Credibility Model Assessment Method

The main steps in the assessment method are:

1. The creation of a hypothetical group risk portfolio of plans. This includes exposure and membership information as well as Underlying probability of claim data at member level.
2. The simulation of Actual claims for the portfolio of plans based on each member's Underlying probability of claim.
3. The calculation of risk premiums for the portfolio of plans using the credibility models being tested.
4. The calculation of various accuracy based measures of the credibility models' performance. The accuracy of various credibility models is determined by comparing each credibility model's risk premium to the corresponding Underlying claims.

The assessment method is slightly different for the proposed model due to its calibration requirement.

Chart 2A shows how the assessment method is applied to the proposed credibility model.

Chart 2A:

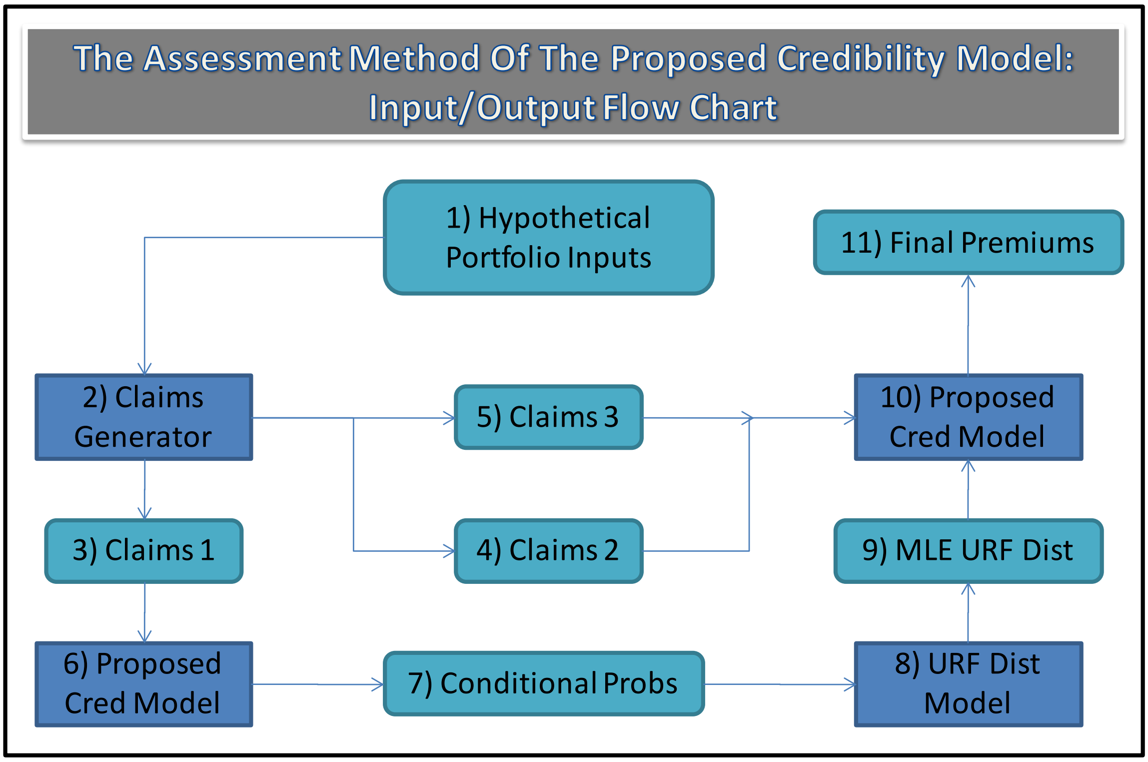
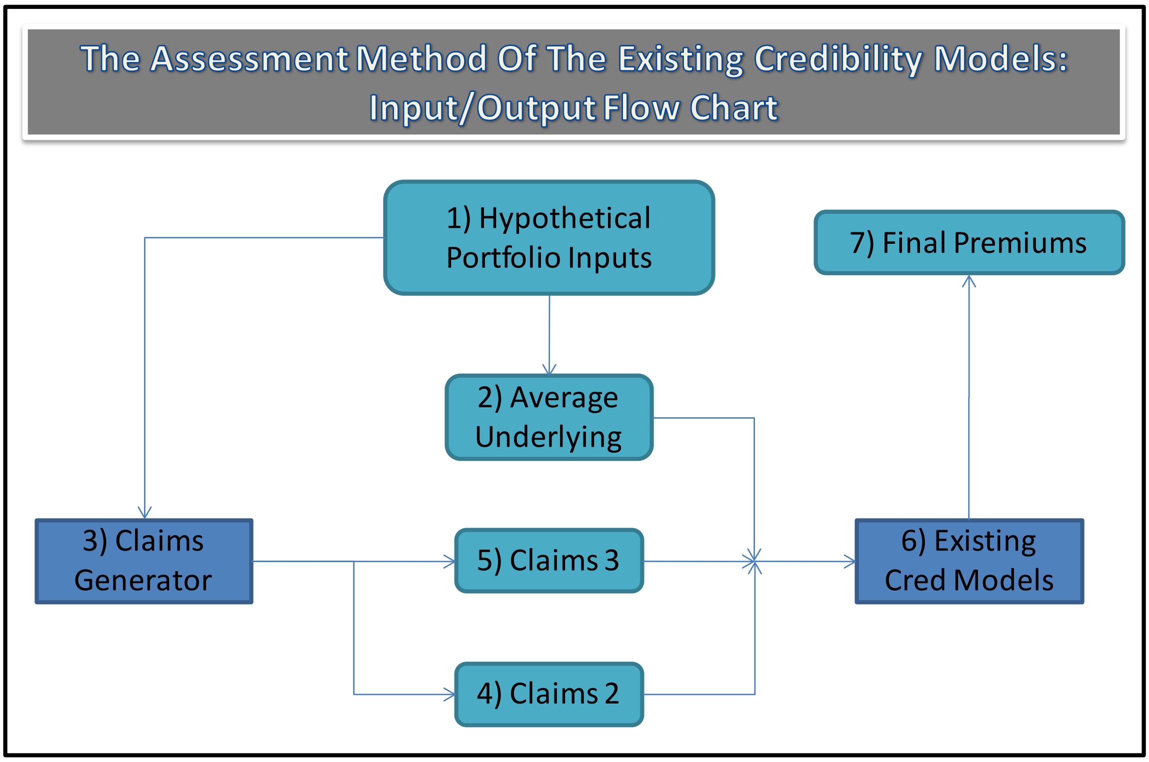


Chart 2B shows how the assessment method is applied to each of the existing credibility models.

Chart 2B:



In the above flow charts, the blue rectangles represent processes or functions which are performed. The round edged boxes represent inputs and outputs of these processes.

The assessment method begins with the creation of the hypothetical portfolio inputs (see section 8.2). The hypothetical portfolio inputs are inserted into a tool which generates claims on the basis of each member's sum insured and probability of claim. The Actual claims are generated using a Bernoulli model simulation approach which is consistent with assumptions 1 and 3 of the proposed credibility model.

Three conditionally independent sets of Actual claims are generated for each plan. The first set of claims, 'Claims 1', is used by the proposed credibility model for the purpose of calculating the N conditional probabilities for each plan in the hypothetical portfolio.

Each plan's set of conditional probabilities are inserted into the URF distribution model. An estimate of the hypothetical portfolio's URF distribution is produced on the basis of the Actual claims in 'Claims 1'. The estimated URF distribution is inserted into the proposed credibility model.

At this point, the proposed credibility model is calibrated allowing it to be used for pricing.

The existing credibility models require the Expected claims as an input. The average Underlying claims across the hypothetical portfolio of plans is used for this purpose.

The second and third set of Actual claims, 'Claims 2' and 'Claims 3', are inserted into the proposed credibility model and the three existing models. Credibility adjusted risk premiums are produced by each model for each plan in the hypothetical portfolio. The 'Claims 2' results are used for testing plan level premium accuracy while the 'Claims 3' results are used for testing model accuracy by claims event.

Note: At no point are the true probabilities of claim for each member in each plan used to calibrate assumptions or calculate premiums of the proposed model. These probabilities are only used in the generation of the claims experience and later in the assessment of the accuracy of the pricing models.

8.2 The Hypothetical Portfolio

The hypothetical portfolio used is a portfolio consisting of 140 plans. For the purposes of this example, the plans have been chosen to be identical in their:

- Membership
- Exposure
- Sum insured distribution

The only difference between the 140 plans is each plan's member level Underlying probability of claim.

The plans are chosen to be identical in their exterior to make the model comparisons clearer and highlight certain features of the proposed credibility model. This does not influence the accuracy of the proposed model.

Each plan has specified probabilities of claim at the individual member level. The chosen formula for determining the probability of claim of any member is more general than the formula used in assumption 2 of the proposed model. In other words, this model is intentionally made to be inconsistent with assumption 2 of the proposed model. Under the hypothetical portfolio, the individual member's Underlying probability of claim is modelled as:

$$P_{jk}^{U} = Maximum[URF_j \times P_{jk}^{E} + URT_j + \varepsilon_{jk}, 0]$$

Where 20 possible values of URF_j are chosen and 7 possible values of URT_j such that each plan represents a unique combination of URF and URT values. The full set of values for the URF and URT combinations are produced in appendix 13.5 along with the average value of P_{jk}^{U} across all members and the Underlying claims mean of each plan.

The values of URT range from -0.002 to +0.002. This is equivalent to a per mil loading of ±$2 across each and every member of the plan.

ε_{jk} is randomly generated for each member of each plan from a Normal[0,0.001] distribution. This is equivalent to a $1 per mil standard deviation.

The hypothetical plans consist of 3000 members each. They have 10% of these members insured for $1,000,000 death only cover and the remaining 90% insured for $100,000 death only cover.

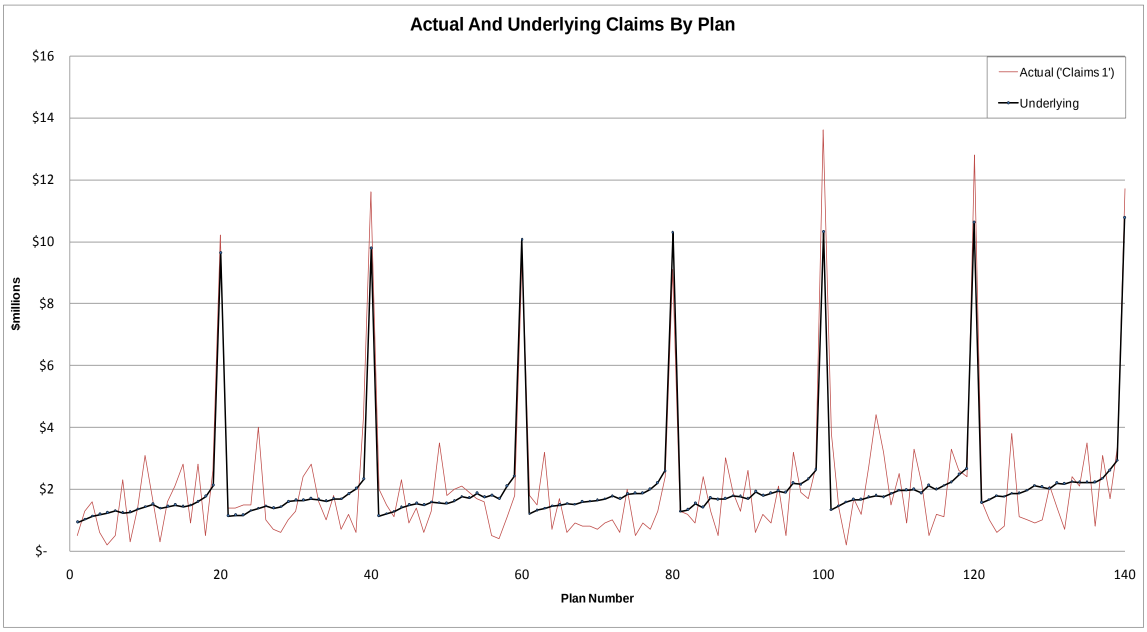
8.3 The Generated Claims

On the basis of the hypothetical portfolio, claims are generated for a period of five years. Here, for simplicity, the members are assumed to not age or exit the plan over this period.

8.3.1 'Claims 1'

'Claims 1' contains 140 Actual claims events each corresponding to one of the 140 plans in the hypothetical portfolio. Graph 5 shows each plan's Actual claim experience over the five years and their Underlying claims cost for the same period.

Graph 5:



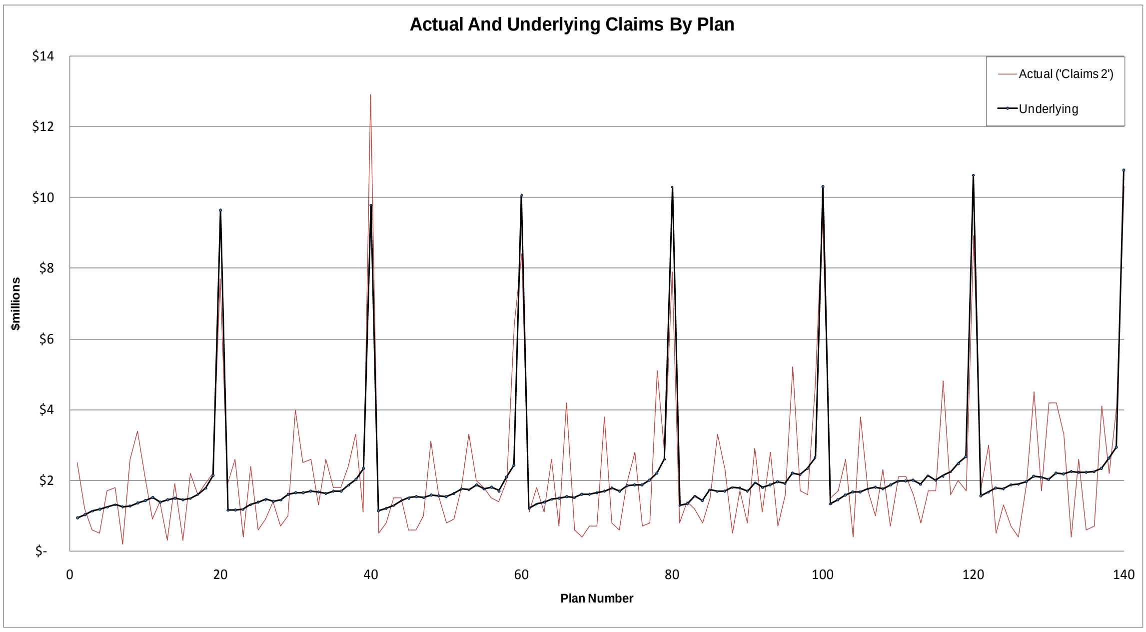
Note: In graph 5, there are peaks in the Underling and Actual claims occurring every 20 plans. These are due to the hypothetical portfolio's assumptions (see appendix 13.5)

8.3.2 'Claims 2'

'Claims 2' also consists of 140 claims events each corresponding to one of the 140 plans in the Hypothetical portfolio. 'Claims 2' is used as the Actual claims experience of the 140 plans for the purposes of calculating credibility adjusted risk premiums.

Graph 6 shows each plan's Actual claim experience over the five year period of investigation along with its Underlying claims cost for the same period.

Graph 6:



8.3.3 <u>'Claims 3'</u>

'Claims 3' consists of 28,000 claims events, with 200 claims events corresponding to each of the 140 plans in the hypothetical portfolio. 'Claims 3' contains 200 conditionally independent equivalents of 'Claims 2'.

When 'Claims 3' is used for credibility model testing, the hypothetical portfolio can be thought of as consisting of 28,000 plans with 200 duplicates of each of the original 140 plans.

9 Model Results Without Competition

In this section, premiums are calculated using the proposed credibility model and the three existing credibility models previously introduced in section 3.4. All of the results obtained are on a like-for-like basis to make the results between all of the models comparable. In comparing the results of the proposed credibility model and the existing models, the following should be noted:

- The same base risk rates are used
- The same hypothetical portfolio, simulated claims results and exposure are used
- 'Claims 1' is used to calibrate the URF distribution of the proposed model
- The Expected claims (used as an input to existing models) is the average of the Underlying claims of the 140 plans in the hypothetical portfolio

9.1 Results of the Proposed Model

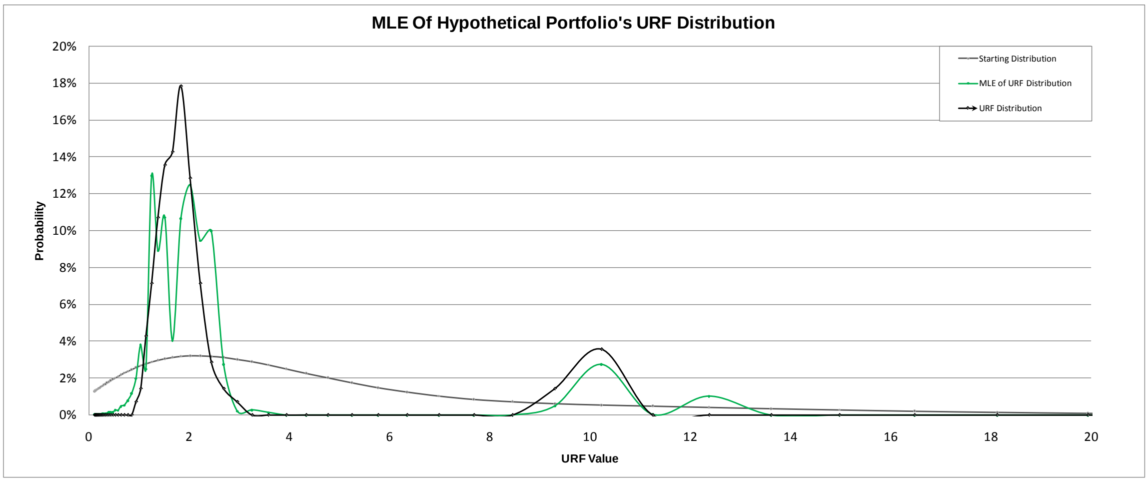
9.1.1 The URF Distribution

On the basis of the Actual claims experience of the 140 plans, N conditional probabilities are generated for each plan. These conditional probabilities are then used to generate a MLE for the URF distribution.

Note: The results presented in this paper use 63 URF points. That is N=63.

Graph 7 shows the hypothetical portfolio's URF distribution and the MLE of the URF distribution.

Graph 7:



The MLE distribution is found through a recursive function. The grey curve in the above graph represents the starting point used to find the MLE. It is chosen to be the average of the corresponding conditional probability of each plan.

This method appears to be reasonably effective for extracting the URF distribution embedded in the Actual claims data. The mean URF implied by the MLE is 2.09 which is only 1.2%

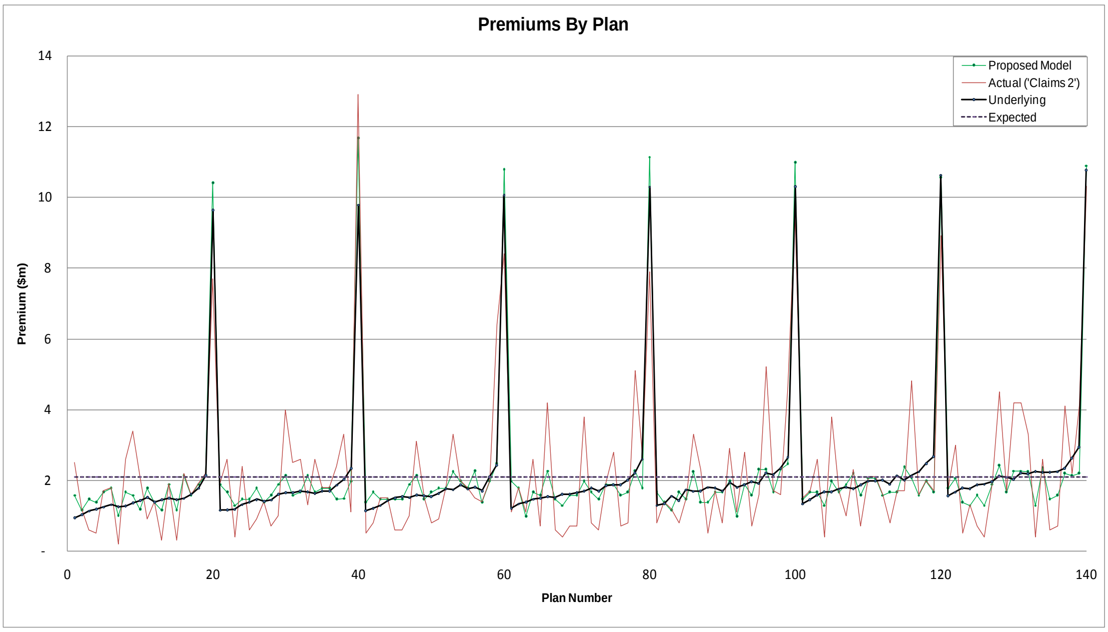
lower than that of the true URF distribution. The Actual claims data that this MLE is based on is 3% lower than the Underlying claims.

The standard deviation of the URF implied by the MLE is 1.90 which is 2.8% higher than the standard deviation of the hypothetical portfolio's URF distribution.

9.1.2 Premiums by Plan

Graph 8 shows the Actual claims ('Claims 2') and Underlying claims cost along with the proposed model's premium results:

Graph 8:



The horizontal purple line represents the insurer's Expected claims based on the portfolio's average Underlying.

In the above graph, the following is noted:

- The premiums track the Underlying claims quite well and have a significantly lower variation around the Underlying than the Actual claims
- The premiums are not a weighted average between Actual and Expected claims with a fixed weight for all plans

At the portfolio level, the total premiums are 1.8% higher than Underlying claims.

The average absolute deviation between premiums and Underlying claims is 14% of average Underlying claims. This is approximately one third of the average absolute deviation between Actual claims and Underlying claims.

The standard deviation of the error between premiums and Underlying claims is 35% of the standard deviation of the error between Actual claims and Underlying claims.

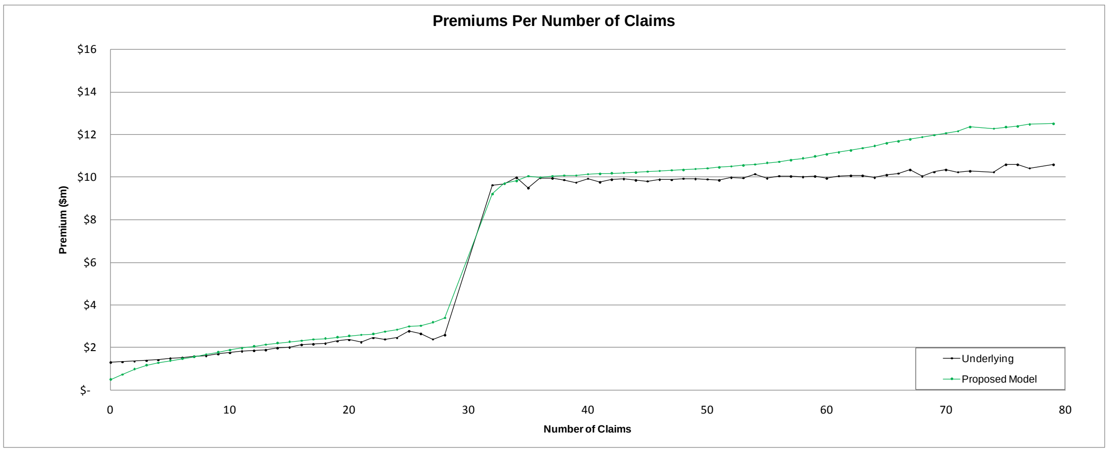
9.1.3 <u>Premiums by Claims Event</u>

The results produced in this section are based on 'Claims 3' data.

In the previous section, plan level premiums are compared to the plan level Underlying claims. While this may seem logical, it is worth noting that the insurer's calculated premiums are only based on externally visible features of a plan such as exposure, membership and observed Actual claims data. Thus, the same premium will be charged for any of the 140 plans in the hypothetical portfolio provided that they exhibit the same claims experience. On this basis, a credibility model cannot eliminate all inter-plan cross subsidisation and is instead limited to minimising cross subsidisation between different Actual claims events.

In graph 9, the average Underlying for all plans which experienced the same number of claims is compared with the corresponding premium. The discrepancy between the average Underlying and the corresponding premium represents the level of overpricing or underpricing for the given claims event.

Graph 9:



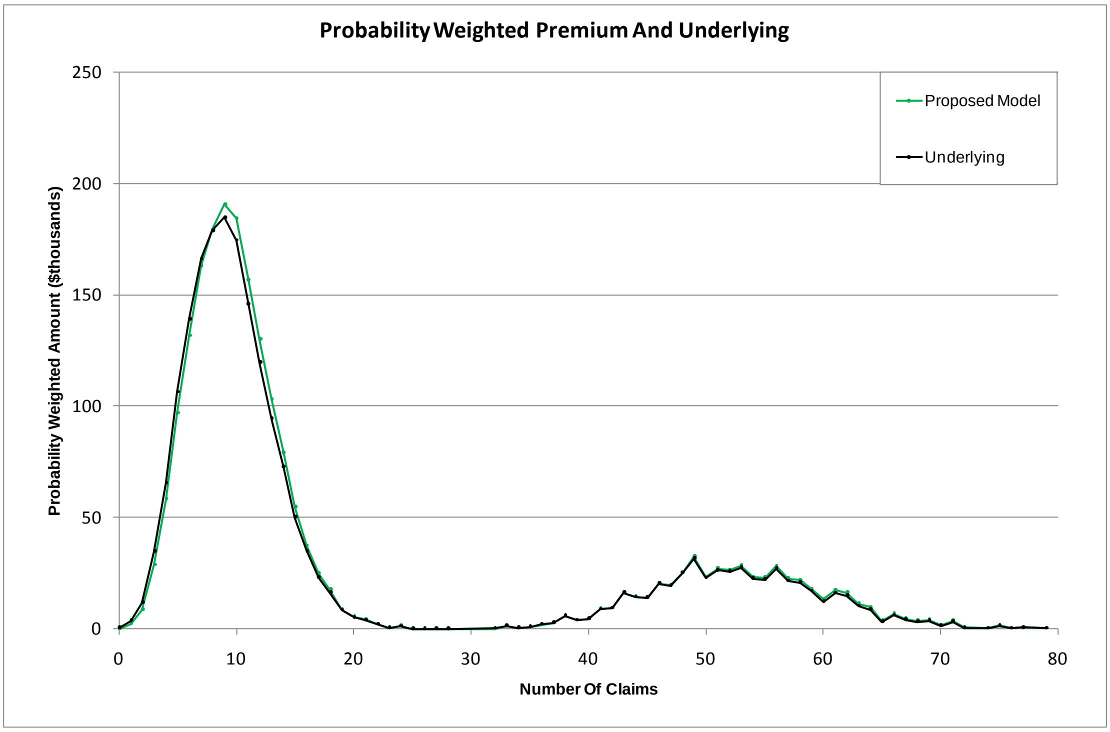
Note: The premium in graph 9 is based on a level sum insured of $190,000. This corresponds to the average sum insured of the hypothetical portfolio.

Graph 9 shows a significant increase in the average Underlying occurring between the 25 and 35 number of claims interval. This dramatic increase is caused by the shape of the URF distribution (graph 7). The lower level average Underlying corresponds to the larger URF distribution peak occurring between the URF values of one and three. The higher level average Underlying corresponds to the smaller URF distribution peak occurring between the URF values of eight and eleven.

The proposed model tracks the average Underlying very closely. This occurs because the proposed model accounts for the shape of the URF distribution. A growing deviation between the proposed model's premium and the average Underlying occurs at 50 claims and above. This is caused by the inaccurate estimation of the third small peak in the URF distribution. This is of little significance because the occurrence of more than 50 claims is uncommon.

In graph 10, the premium and average Underlying from graph 9 are weighted by the probability of occurrence of the corresponding number of claims. Thus, the relative importance of the premium error can be gauged because the large but infrequent errors can be compared with the smaller but frequent pricing errors.

Graph 10:



Graph 10 indicates that the premiums are a close match to the Underlying claims mean. The proposed credibility model is designed to produce premiums that exactly match the Underlying claims mean. The little deviation that does exist between the premium for a given number of claims and the average Underlying for the same number of claims is due to:

- The fact that assumption 2 in the credibility model is intentionally different to the assumption used in the hypothetical scenario.
- The estimate of the distribution of URF is not perfect due to the random variation in claims over the 5 year claims experience period.

Note: The proposed credibility model will always track the average Underlying and thus, retain its low cross subsidy property regardless of the shape of the URF distribution.

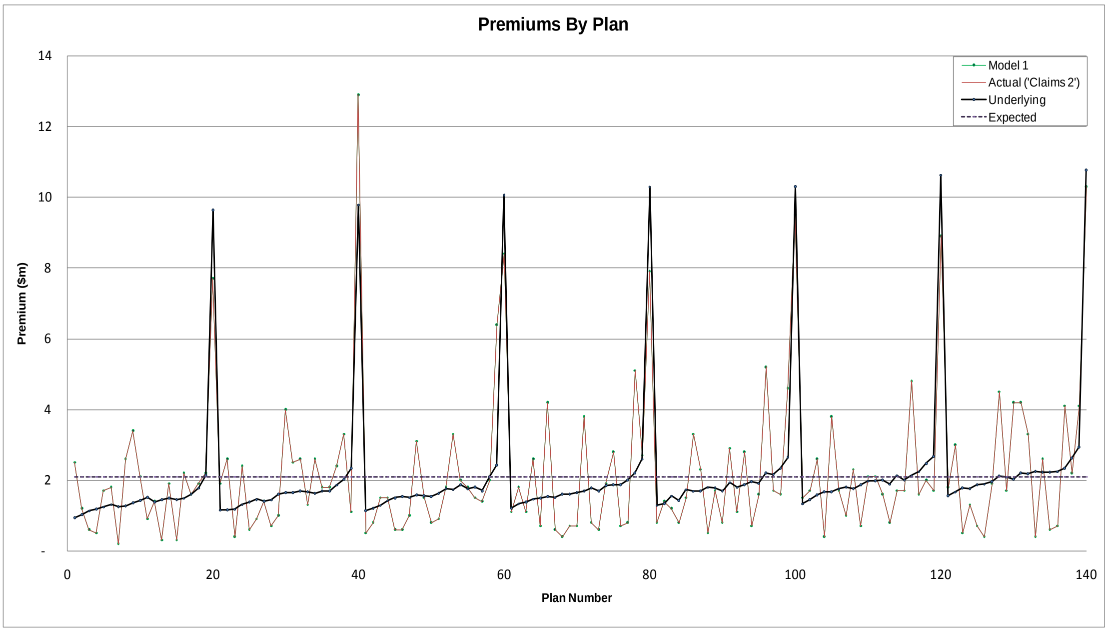
9.2 Results of Model 1

Model 1 assigns full credibility at exposure greater than 9400 life years resulting in 100% credibility for all plans. Thus, model 1's premium results are identical to Actual claims.

9.2.1 Premiums by Plan

Graph 11 shows the Actual claims and Underlying claims cost along with the calculated premiums:

Graph 11:



In the above graph, the following can be noted:

- The premiums track the Underlying claims quite poorly and have the same variation around the Underlying as the Actual claims
- Premiums are the same as the Actual claims

At the portfolio level, the total premiums are 2.8% higher than Underlying claims.

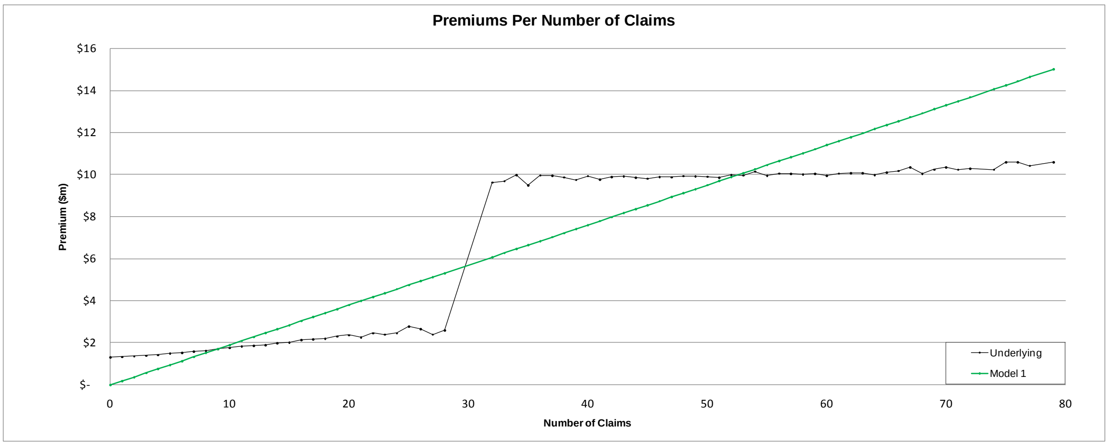
The average absolute deviation between premiums and Underlying claims is 42% of average Underlying claims. This is 100% of the average absolute deviation between Actual claims and Underlying claims.

The standard deviation of the error between premiums and Underlying claims is 100% of the standard deviation of the error between Actual claims and Underlying claims.

9.2.2 Premiums by Claims Event

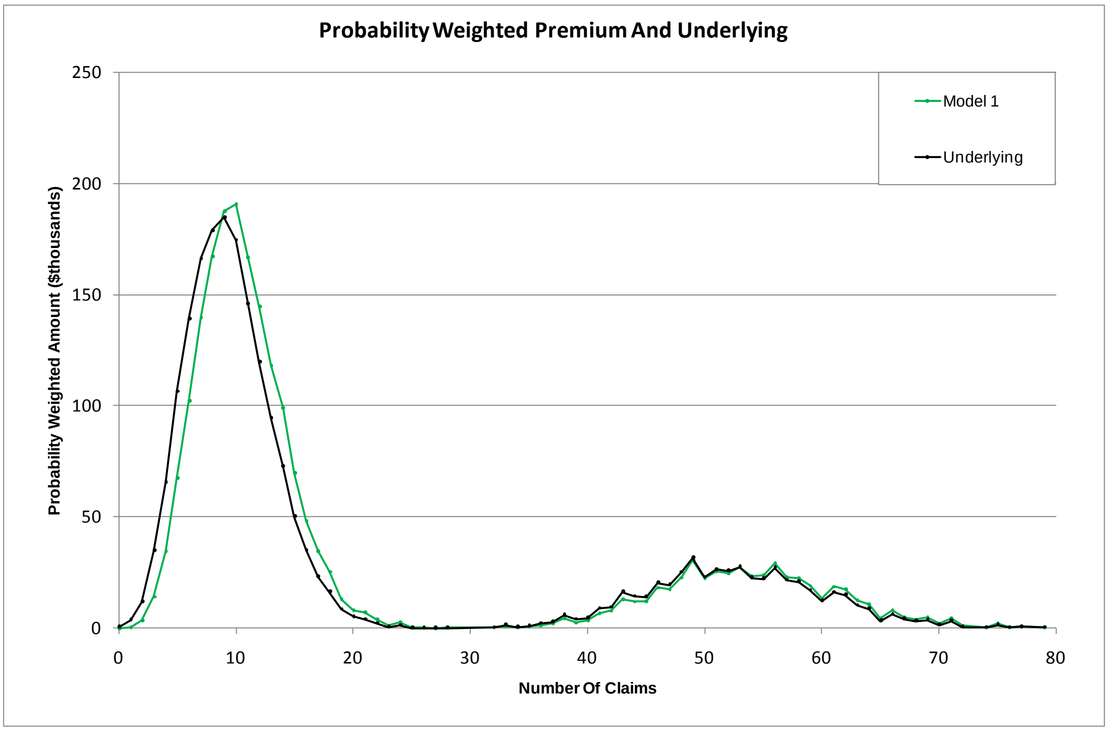
As per section 9.1.3, below are the corresponding graphs for Model 1.

Graph 12:



Graph 12 shows a linear relationship between premium and number of claims. Although the exposure limit of 9400 life years results in full credibility, this linear relationship would still occur at exposures below the 9400 limit. This is a feature of the BSM because the credibility factor is not dependent on the number of claims.

Graph 13:

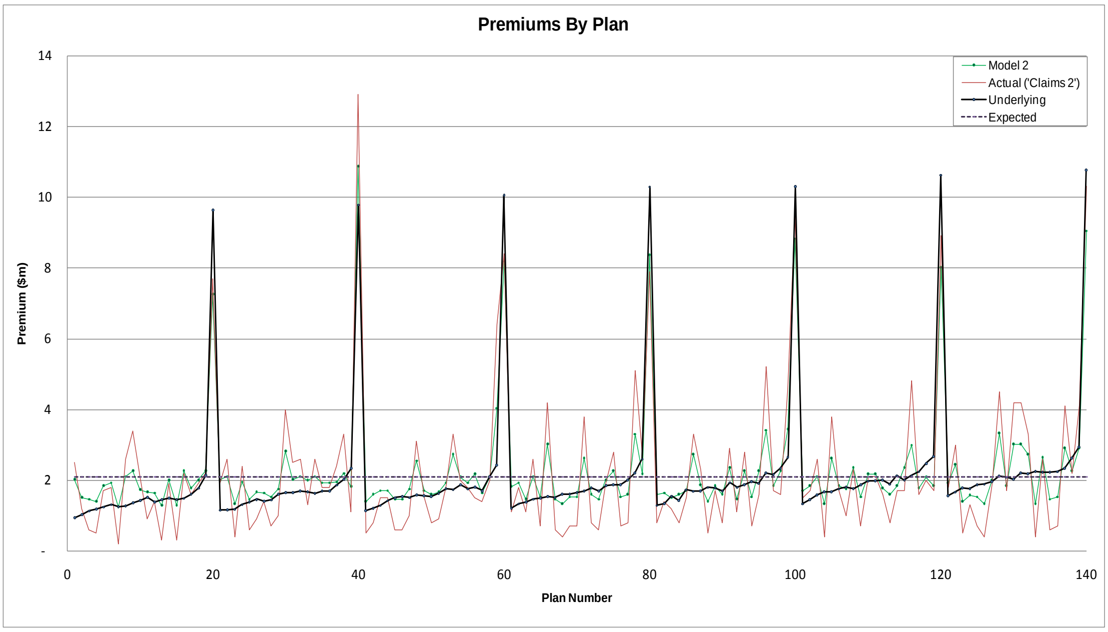


As can be observed, Model 1's premiums are quite different to the average Underlying claims.

9.3 Results of Model 2

9.3.1 Premiums by Plan

Graph 14:



In graph 14, the following can be noted:

- The premiums track the Underlying claims poorly but have a lower variation around the Underlying than the Actual claims
- Premiums are a weighted average between Actual and Expected with the weight varying depending on the number of claims.

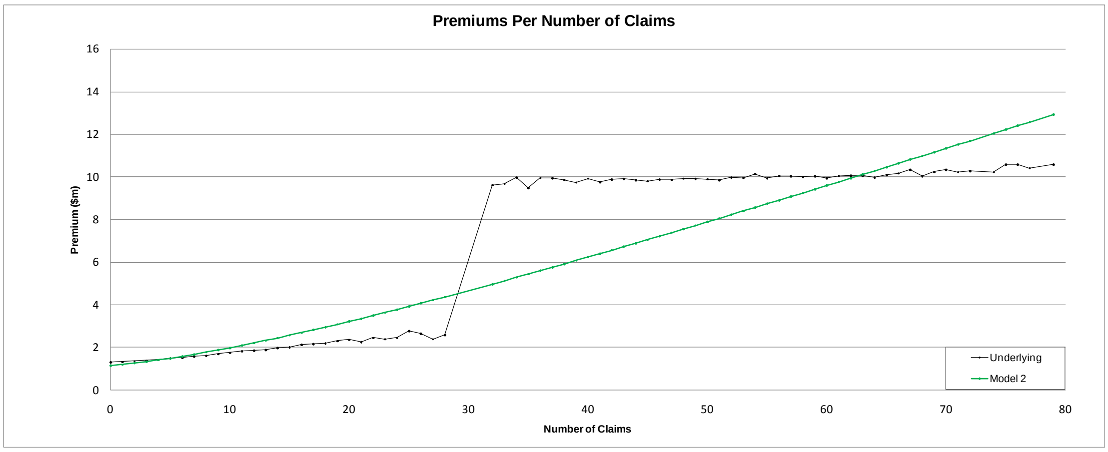
At the portfolio level, the total premiums are 6.8% higher than Underlying claims.

The average absolute deviation between premiums and Underlying claims is 23% of average Underlying claims. This is approximately 54% of the average absolute deviation between Actual claims and Underlying claims.

The standard deviation of the error between premiums and Underlying claims is 56% of the standard deviation of the error between Actual claims and Underlying claims.

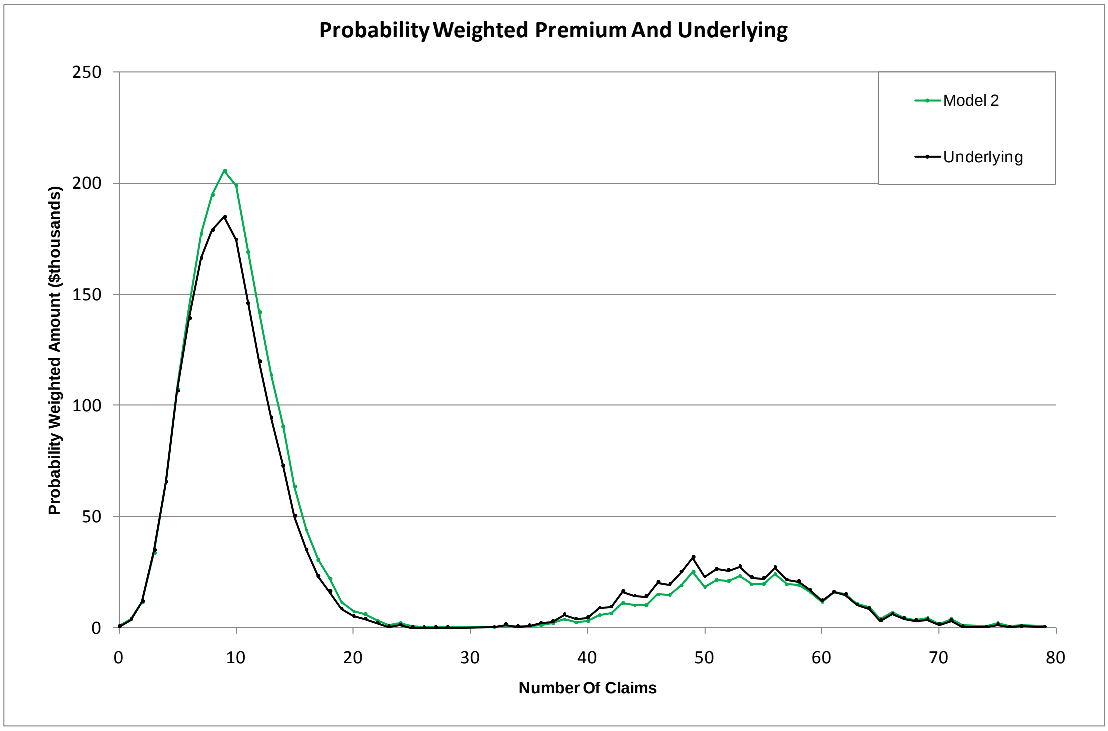
9.3.2 Premiums by Claims Event

Graph 15:



Graph 15 shows the non-linear relationship between premium and the number of claims. Model 2 does not take the shape of the URF distribution into consideration.

Graph 16:

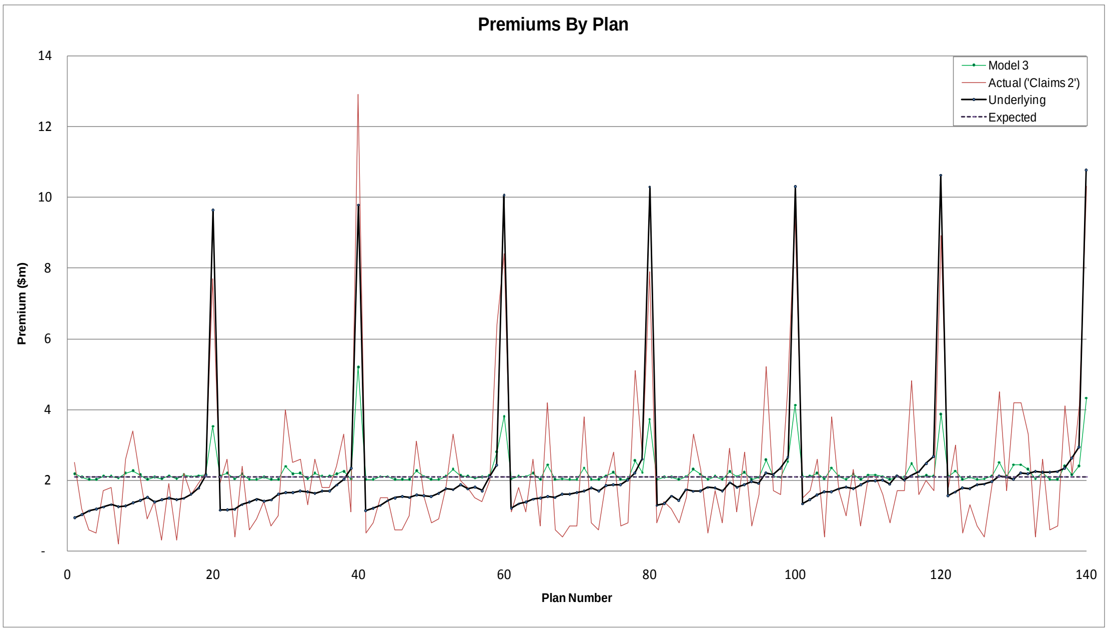


As can be observed, the premiums are quite different to the Underlying claims. There is also a clear overpricing at the lower number of claims and significant underpricing at the higher claims levels.

9.4 Results of Model 3

9.4.1 Premiums by Plan

Graph 17:



In graph 17, the following can be noted:

- The premiums track the Underlying claims very poorly
- The premiums substantially deviate from the insurer's Expected only for the few plans where the Actual claims are very large
- Premiums are a weighted average between the Actual and Expected claims with the weight varying depending on the number of claims

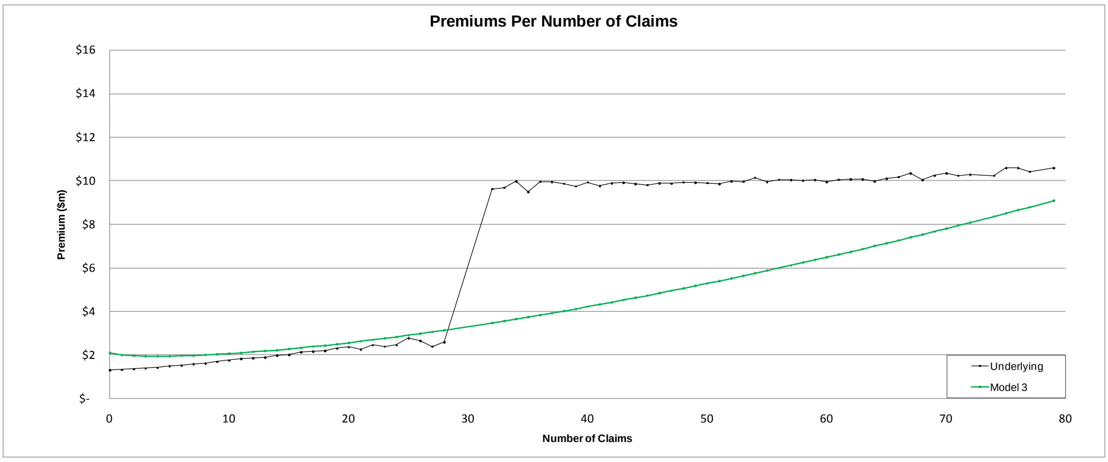
At the portfolio level, the total premiums are 3.9% higher than Underlying claims.

The average absolute deviation between premiums and Underlying claims is 35% of average Underlying claims. This is approximately 82% of the average absolute deviation between Actual claims and Underlying claims.

The standard deviation of the error between premiums and Underlying claims is 125% of the standard deviation of the error between Actual claims and Underlying claims.

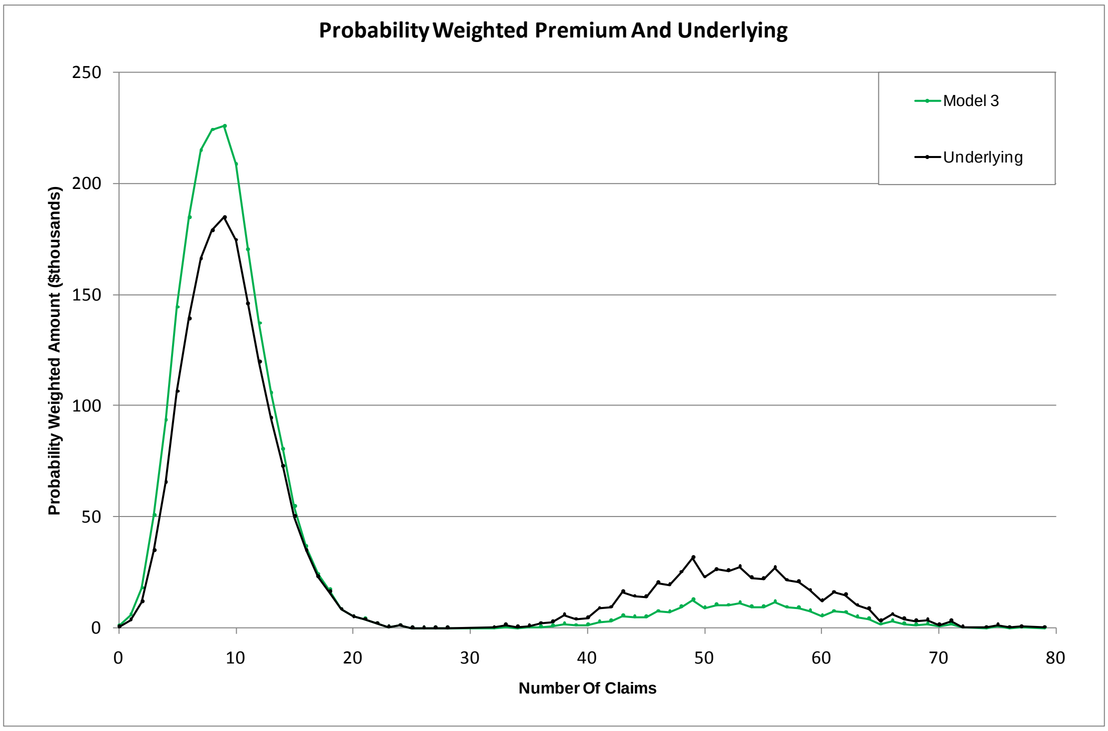
9.4.2 Premiums by Claims Event

Graph 18:



Graph 18 shows the non-linear property of the LFM. Between zero and four claims, the premium charged reduces as the number of claims increases. Above four claims, the premium starts increasing with the number of claims but does not surpass the premium charged for zero claims until 12 or more claims are observed. This is discussed in section 3.3.3.

Graph 19:



As can be observed, the premiums are quite different to the Underlying claims. There is also an even more pronounced overpricing at the lower number of claims with underpricing at the higher claims levels when compared to Model 2. This occurs because the credibility attributed to each plan is too low.

10 Existing Models and Proposed Model in Competition

For the below analysis, the 'Claims 3' data is used. On the basis of the premiums generated for each claims event, the four models competed for the 28,000 plans with each plan being won by the cheapest model. Below are each model's proportion of business won (weighted by premium) and the ratio of the total premium collected divided by the total Underlying claims mean of the business won.

10.1 No Competition

Table 2:

Model	Proposed Model	Model 1	Model 2	Model 3
Prem Won/28,000	$ 2,206,604	$ 2,168,336	$ 2,276,031	$ 2,235,983
Underlying Claims/28,000	$ 2,158,999	$ 2,158,999	$ 2,158,999	$ 2,158,999
Premium/Underlying	**102%**	**100%**	**105%**	**104%**
Proportion of Business Won	**100%**	**100%**	**100%**	**100%**

From table 2, it can be observed that all four models have a premium to Underlying ratio roughly equal to 100%. This shows the fact that all four models have portfolio level accuracy in a non competitive environment.

10.2 In Competition

10.2.1 Proposed Model versus Model 1

Table 3A:

Model	Proposed Model	Model 1
Prem Won/28,000	$ 1,283,893	$ 536,618
Underlying Claims/28,000	$ 1,237,745	$ 920,595
Premium/Underlying	**104%**	**58%**
Proportion of Business Won	**71%**	**29%**

10.2.2 Proposed Model versus Model 2

Table 3B:

Model	Proposed Model	Model 2
Prem Won/28,000	$ 1,202,463	$ 874,505
Underlying Claims/28,000	$ 1,185,066	$ 973,933
Premium/Underlying	**101%**	**90%**
Proportion of Business Won	**58%**	**42%**

10.2.3 Proposed Model versus Model 3

Table 3C:

Model	Proposed Model	Model 3
Prem Won/28,000	$ 1,466,750	$ 410,501
Underlying Claims/28,000	$ 1,456,666	$ 702,333
Premium/Underlying	**101%**	**58%**
Proportion of Business Won	**78%**	**22%**

10.2.4 All Four Models in Competition

Table 4:

Model	Proposed Model	Model 1	Model 2	Model 3
Prem Won/28,000	$ 948,063	$ 295,025	$ 0	$ 308,917
Underlying Claims/28,000	$ 922,489	$ 632,815	$ 0	$ 603,694
Premium/Underlying	**103%**	**47%**	**NA**	**51%**
Proportion of Business Won	**61%**	**19%**	**0%**	**20%**

From the ratio of premiums to Underlying in tables 3A to 4, the following can be observed:

- The existing models are exposed to anti-selection and portfolio level pricing inaccuracy in competition
- The proposed model maintains a premium to Underlying ratio close to 100% as a result of its low cross subsidisation property
- As a result of greater competition in the four model competition scenario, greater losses are incurred by the existing models

Note: Model 2 is to some extent a hybrid of Model 1 and Model 3. As a result of this, it produces premiums which usually lie between Model 1 and Model 3. Thus, although it is more accurate under the hypothetical portfolio, it wins little business. Further, when Model 2 is also competing with the proposed model, it loses the little business it does win over Model 1 and Model 3.

10.3 The Premium Differences

When the proposed model is cheapest, it is on average 32% cheaper than Model 1, 19% cheaper than Model 2 and 22% cheaper than Model 3.

The results of section 10.2 are based on full price elasticity, as the cheapest model always wins. Considering the size of the premium differences relative to the price elasticity of the group risk customer, full price elasticity is not too unrealistic.

11 Interpretation and Limitations of the Results

The results of the analysis in sections 9 and 10 support some important conclusions of this paper's theoretical arguments. These are:

- Existing models are unable to respond to the URF distribution
- Existing models are very inaccurate under certain URF distribution shapes
- The proposed model is able to respond to the URF distribution
- The proposed model is sufficiently accurate regardless of the shape of the URF distribution
- Premium cross subsidisation can lead to significant portfolio level underpricing in competition

The results have some limitations which are important to note. These are:

- The hypothetical portfolio's URF distribution is unlikely to be indicative of the URF distribution of the Australian group risk market. Without the use of real life market data, the results can only be used to highlight principles and potential magnitudes of error rather than as a quantitative analysis of credibility model impacts at the portfolio level.
- The hypothetical portfolio consists of 140 group risk plans with identical membership and exposure data. Theoretically the results of the proposed model maintain their low cross subsidy property regardless of plan size and membership details; however, the existing models' results will be impacted by changes to exposure.

12 Summary of Key Points

1. Credibility models are a requirement of competition and should be tested in competition
2. Credibility models are rating mechanisms; in competition, inaccurate credibility models can lead to anti-selection and portfolio level underpricing
3. Different credibility models can lead to very different estimates of risk premium. The choice of credibility model can have a significant impact on an insurer's portfolio composition and profitability
4. There are 3 key requirements of a credibility model:

   - Sufficient consideration for and responsiveness to the distribution of the Underlying claims mean around the Expected claims
   - Sufficient consideration for and responsiveness to the distribution of the Actual claims around the Underlying claims mean
   - A theoretically accurate means of calculating the risk premium on the basis of the plan inputs and the abovementioned distributions

5. Both the Bühlmann-Straub model and the Limited Fluctuation model have significant limitations in addressing each of the three abovementioned requirements
6. The proposed credibility model accounts for all three of the abovementioned requirements
7. The simulation based approach of the proposed credibility model lays a foundation for accomplishing a number of other value adding tasks
8. The concept of the URF distribution is important and its measurement is key to monitoring the effectiveness of rating classes and the accuracy of base rates and loading factors
9. The URF distribution can be estimated reasonably well using the maximum likelihood estimator approach; however, some additional refinement and understanding of this approach is desirable
10. Group risk lump sum claims for a given Underlying can be modelled as the weighted sum of Bernoulli random variables, where the weights are the sums insured
11. Sufficient data and all the inputs are available for the use of the optimal Bayesian approach to credibility
12. The simulation based credibility model assessment method is an insightful and informative tool; however, for realistic quantitative analysis of a credibility model's impact at the portfolio level, the hypothetical portfolio needs to be representative of the Australian group risk market

13 Appendix

13.1 Single Factor Bayesian Approach

13.1.1 The Two-Urn Model

The Two-urn model (Bühlmann and Gisler, 2005) represents a general concept which applies to both the single factor Bayesian and the Bühlmann-Straub credibility models.

Consider two urns. Both urns contain an infinite number of balls which are numbered. The distribution of numbers in urn 1 is fixed, but the distribution of numbers in urn 2 depends on the number drawn from urn 1.

The number on the j^{th} randomly selected ball from urn 1 represents the structural parameter for plan j. Subsequent n drawings from urn 2 represent the n observed claim statistics for plan j.

As a simplified example, consider the normal-uniform case. Here, urn 1 has a Uniform(0,1) distribution. The j^{th} number drawn from urn 1 is the random variable U_j (structural parameter). Suppose, it is observed that $U_j = u_j$. Then, the distribution of numbers in urn 2 becomes Bin(u_j, N_j) were N_j is the number of members in plan j. Claims statistic observations X_{j1}, X_{j2}, ... X_{jn} will be drawn from urn 2.

The insurer will never know the true value of u_j, it would only see the observed claims statistics X_{j1}, X_{j2}, ... X_{jn} and attempt to determine the mean claims statistic for plan j (u_j x N_j) from these observations.

The key element that the Two-urn model captures is that each plan has a different claims distribution but the plans have something in common as each plan's structural parameter is an independent observation from the same structural distribution.

13.1.2 The Bayesian Approach

In line with the Two-urn construct above, the following are defined:

1. U_j is a random variable representing the number drawn from urn 1 on the j^{th} trial. Prior to observations made from urn 2, U_j has a probability density function of $U(u)$. U_j is often set to represent the true claims frequency of plan j (Underlying claims frequency).

2. A_{jt} is a random variable representing the number drawn from the j^{th} urn 2 on the t^{th} trial. A_{jt}'s distribution is dependent on the value of U_j and has a conditional probability density function of $A_j({}^{o}A_{jt} | U_j = u)$. A_{jt} usually represents the observed Actual claims frequency for plan j in year t.

3. Following the observations made from urn 2, U_j has a conditional probability density function of $U'\left(u \middle| A_{jt} = {}^{o}A_{jt}, A_{jt-1} = {}^{o}A_{jt-1}, A_{jt-2} = {}^{o}A_{jt-2}, \ldots\right) = U'(u | A_{jHist} = {}^{o}A_{jHist})$.

Under the Bayesian approach, the objective is to optimally estimate plan j's expected value of Actual claims in the next period i.e. $E[A_{jt+1} | A_{jHist} = {}^{o}A_{jHist}]$.

Assuming claims frequencies are independent and identically distributed over time conditional on the value of U_j, the following is true:

$$E[A_{jT+1} | A_{jHist} = {}^{o}A_{jHist}] = \int_0^{\infty} E[A_{jT+1} | U_j = u] \times U'(u | A_{jHist} = {}^{o}A_{jHist})du$$

This can be shown to equate to:

$$\int_0^{\infty} E[A_{jT+1} | U_j = u] \times \frac{Prob_j(A_{jHist} = {}^{o}A_{jHist} | U_j = u) \times U(u)}{\int_0^{\infty} Prob_j(A_{jHist} = {}^{o}A_{jHist} | U_j = u) \times U(u)du} du$$

Where:

- $Prob_j(A_{jHist} = {}^{o}A_{jHist} | U_j = u) = A_j({}^{o}A_{jt} | U_j = u) \times A_j({}^{o}A_{j(t-1)} | U_j = u) \times A_j({}^{o}A_{j(t-2)} | U_j = u) \times \ldots$
- ${}^{o}A_{jt}$, ${}^{o}A_{j(t-1)}$, ${}^{o}A_{j(t-2)}$, ... are the observed values of A_{jt}, $A_{j(t-1)}$, $A_{j(t-2)}$, ...

Although the Bayesian approach is theoretically optimal, it requires inputs of $U(u)$, $A_j({}^{o}A_{jt} | U_j = u)$ and $E[A_{jT+1} | U_j = u]$. These inputs can be difficult to determine and assumptions regarding these are often made based on mathematical convenience rather than on the basis of compelling evidence. These are commonly the grounds for criticisms of the Bayesian approach.

13.2 Derivation of the LFM

13.2.1 <u>Derivation of the Mean and Variance of Total Claims</u>

The derivation of the LFM's full credibility requirement starts with the equation for total claims:

$$T_{jT} = \sum_{k=1}^{C_j} SI_{jk}$$

From this, one can determine μ_T and ${\sigma_T}^2$ as the follows:

$$\mu_T = E\left[\sum_{k=1}^{C_j} SI_{jk}\right] = E\left[E[\sum_{k=1}^{C_j} SI_{jk} | C_j]\right] = \mu_C \times \mu_{SI}$$

And:

$${\sigma_T}^2 = Var\left[\sum_{k=1}^{C_j} SI_{jk}\right] = E[Var(\sum_{k=1}^{C_j} SI_{jk} | C_j)] + Var(E[\sum_{k=1}^{C_j} SI_{jk} | C_j])$$

$$= E\left[C_j \times Var\left(SI_{jk}\right)\right] + Var\left(C_j \times E\left[SI_{jk}\right]\right)$$

$$= E\left[C_j\right] \times Var\left(SI_{jk}\right) + Var\left(C_j\right) \times E^2\left[SI_{jk}\right]$$

That is:

$$\sigma_T{}^2 = \mu_C \sigma_{SI}{}^2 + \sigma_C{}^2 \mu_{SI}{}^2$$

13.2.2 Derivation of the Full Credibility Requirement

Assuming T_{jT} is distributed as a normal i.e. $T_{jT} \sim N[\mu_T, \sigma_T{}^2]$, the following is true:

$$\frac{T_{jT} - \mu_T}{\sigma_T} \sim N[0,1]$$

Hence,

$$Prob\left(-\varepsilon\mu_T < T_{jT} - \mu_T < \varepsilon\mu_T\right) = \alpha$$

Can be expressed as:

$$Prob\left(\frac{-\varepsilon\mu_T}{\sigma_T} < \frac{T_{jT} - \mu_T}{\sigma_T} < \frac{\varepsilon\mu_T}{\sigma_T}\right) = \alpha$$

i.e.

$$Prob\left(\frac{T_{jT} - \mu_T}{\sigma_T} > \frac{\varepsilon\mu_T}{\sigma_T}\right) = \frac{1-\alpha}{2}$$

Now, if z is defined as a constant such that the probability that a standard normal variable is larger than z is $\frac{1-\alpha}{2}$, then $\frac{\varepsilon\mu_T}{\sigma_T} = z$ satisfies:

$$Prob\left(-\varepsilon\mu_T < T_{jT} - \mu_T < \varepsilon\mu_T\right) = \alpha$$

Now, substituting in the formulae for μ_T and σ_T, the following is attained:

$$\frac{\varepsilon \times \mu_C \times \mu_{SI}}{(\mu_C \sigma_{SI}{}^2 + \sigma_C{}^2 \mu_{SI}{}^2)^{0.5}} = z$$

i.e.

$$\mu_C = ({}^{z}/_{\varepsilon}) \times \frac{(\mu_C \sigma_{SI}{}^2 + \sigma_C{}^2 \mu_{SI}{}^2)^{0.5}}{\mu_{SI}}$$

By squaring both sides and dividing by μ_C, this simplifies to:

$$C_j{}^F = (z/\varepsilon)^2 \times \left[\frac{\sigma_{SI}{}^2}{\mu_{SI}{}^2} + \frac{\sigma_C{}^2}{\mu_C}\right]$$

13.2.3 <u>Derivation of the Partial Credibility Formula</u>

The LFM starts with the assumption that:

$$P_j = (1-b)E_j + b^o T_{jT}$$

It then chooses the value for b such that the following equation is satisfied:

$$Prob\left(-\varepsilon\mu_T < b(T_{jT} - \mu_T) < \varepsilon\mu_T\right) = \alpha$$

Now, this is the same as:

$$Prob\left(-\varepsilon'\mu_T < T_{jT} - \mu_T < \varepsilon'\mu_T\right) = \alpha$$

Where $\varepsilon' = \varepsilon/b$.

Now, from the full credibility formula in the previous section, it is known that this equation will be satisfied when:

$$\mu_C = (z/\varepsilon')^2 \times \left[\frac{\sigma_{SI}{}^2}{\mu_{SI}{}^2} + \frac{\sigma_C{}^2}{\mu_C}\right]$$

$$= (zb/\varepsilon)^2 \times \left[\frac{\sigma_{SI}{}^2}{\mu_{SI}{}^2} + \frac{\sigma_C{}^2}{\mu_C}\right]$$

$$= b^2 \times (z/\varepsilon)^2 \times \left[\frac{\sigma_{SI}{}^2}{\mu_{SI}{}^2} + \frac{\sigma_C{}^2}{\mu_C}\right]$$

$$= b^2 \times C_j{}^F$$

i.e.

$$b = \sqrt{\frac{\mu_C}{C_j{}^F}}$$

13.3 Derivation of the Proposed Model

$$Prem_{jT+1} = \mathrm{E}[A_{jT+1} | A_{jT} = {}^{o}A_{jT}]$$

i.e.

$$Prem_{jT+1} = \int_{0}^{\infty} a \times Prob(A_{jT+1} = a | A_{jT} = {}^{o}A_{jT}) da$$

Now,

$$Prob(A_{jT+1} = a | A_{jT} = {}^{o}A_{jT}) = \frac{Prob(A_{jT+1} = a \ \cap \ A_{jT} = {}^{o}A_{jT})}{Prob(A_{jT} = {}^{o}A_{jT})}$$

$$= \frac{\sum_{n=1}^{N} Prob(A_{jT+1} = a \cap A_{jT} = {}^{o}A_{jT} \cap URF_j = URF_j^n)}{Prob(A_{jT} = {}^{o}A_{jT})}$$

Now, since A_{jT+1} given $URF_j = URF_j^n$ is independent of A_{jT}, the above can be expressed as:

$$= \frac{\sum_{n=1}^{N} \{Prob\left(A_{jT+1} = a \cap URF_j = URF_j^n\right) \times Prob\left(A_{jT} = {}^{o}A_{jT} \cap URF_j = URF_j^n\right) / Prob(URF_j = URF_j^n)\}}{Prob(A_{jT} = {}^{o}A_{jT})}$$

$$= \sum_{n=1}^{N} \{Prob\left(A_{jT+1} = a | URF_j = URF_j^n\right) \times Prob\left(URF_j = URF_j^n | A_{jT} = {}^{o}A_{jT}\right)\}$$

Therefore:

$$Prem_{jT+1} = \int_{0}^{\infty} a \times \sum_{n=1}^{N} \{Prob\left(A_{jT+1} = a | URF_j = URF_j^n\right) \times Prob\left(URF_j = URF_j^n | A_{jT} = {}^{o}A_{jT}\right)\} da$$

$$= \sum_{n=1}^{N} \{\int_{0}^{\infty} a \times Prob\left(A_{jT+1} = a | URF_j = URF_j^n\right) \times Prob\left(URF_j = URF_j^n | A_{jT} = {}^{o}A_{jT}\right) da\}$$

Thus:

$$Prem_{jT+1} = \sum_{n=1}^{N} \mathrm{E}[A_{jT+1} | URF_j = URF_j^n] \times Prob\left(URF_j = URF_j^n | A_{jT} = {}^{o}A_{jT}\right)$$

Now,

$$Prob(URF_j = URF_j^n | A_{jT} = {}^oA_{jT}) = \frac{Prob(URF_j = URF_j^n \cap A_{jT} = {}^oA_{jT})}{Prob(A_{jT} = {}^oA_{jT})}$$

$$= \frac{Prob(URF_j = URF_j^n \cap A_{jT} = {}^oA_{jT})}{Prob(A_{jT} = {}^oA_{jT})} \times \frac{Prob(URF_j = URF_j^n)}{Prob(URF_j = URF_j^n)}$$

$$= \frac{Prob(A_{jT} = {}^oA_{jT} | URF_j = URF_j^n) \times Prob(URF_j = URF_j^n)}{Prob(A_{jT} = {}^oA_{jT})}$$

$$= \frac{Prob(A_{jT} = {}^oA_{jT} | URF_j = URF_j^n) \times Prob(URF_j = URF_j^n)}{\sum_{n=1}^{N} Prob(A_{jT} = {}^oA_{jT} | URF_j = URF_j^n) \times Prob(URF_j = URF_j^n)}$$

Thus:

$$Prem_{jT+1} = \sum_{n=1}^{N} \{\mathrm{E}[A_{jT+1} | URF_j = URF_j^n] \times \frac{Prob(A_{jT} = {}^oA_{jT} | URF_j = URF_j^n) \times Prob(URF_j = URF_j^n)}{\sum_{n=1}^{N} Prob(A_{jT} = {}^oA_{jT} | URF_j = URF_j^n) \times Prob(URF_j = URF_j^n)}\}$$

13.4 The Determination of the Conditional Actual Claims Distribution

The $Prob(A_{jT} = {}^oA_{jT} | URF_j = URF_j^n)$ can be calculated using a simulation modelled on the basis of assumptions 1, 2 and 3. This can be done using the following steps:

a. An $\epsilon \times$ N matrix is calculated for $P_{jtk}^U = URF_j^n \times P_{jtk}^E$

   Where:
   - N is the number of discrete URF points used to approximate the continuous URF random variable
   - ϵ is the total life years of exposure of group j and:

$$\epsilon = \sum_{t=1}^{\mathrm{T}} \mathrm{N}_j$$

   For simplicity, the k^{th} member in plan j at time t is denoted by ε where ε can takes values of 1, 2, ..., ϵ. Each value of ε has a one to one correspondence with k^{th} member in group j at time t. Thus, P_{jtk}^U , P_{jtk}^E , ... are represented by $P_{j\varepsilon}^U$, $P_{j\varepsilon}^E$, ...

b. An S $\times$ ϵ $\times$ N matrix of independent Uniform[0,1] random variables $U_{s\varepsilon n}$ is generated where S is the number of simulations and is required to be large. The results produced in section 9.1 use S=60,000.

c. For each value of S, the $\epsilon \times N$ matrix in 'step a' is compared with the $\epsilon \times N$ matrix in 'step b' and a new $S \times \epsilon \times N$ matrix, the Actual claims matrix, is produced. The Actual claims matrix contains the elements $A_{s\varepsilon n}$.

   $A_{s\varepsilon n}$ has a value of $SI_{j\varepsilon}$ if $U_{s\varepsilon n} < P_{j\varepsilon}^{U}$ and a value of 0 otherwise. That is, $A_{s\varepsilon n}$ represents the Actual claim cost for the ε^{th} exposure on the S^{th} simulation using $URF_j = URF_j^n$.

d. The $S \times \epsilon \times N$ Actual claims matrix in 'step c' can be summed across ϵ to produce a new $S \times N$ matrix where each of its values represents the total claim amount for plan *j* produced in a single simulation given $URF_j = URF_j^n$. That is:

$$A_{sn} = \sum_{\varepsilon=1}^{\epsilon} A_{s\varepsilon n}$$

   Using the S observations of the total claim amounts for each value of URF_j, one can construct the complete distribution of A_{jT} given $URF_j = URF_j^n$ and also, calculate $Prob\left(A_{jT} = {}^{o}A_{jT} \,|\, URF_j = URF_j^n\right)$.

   Note: In practice $Prob\left(A_{jT} = {}^{o}A_{jT} \,|\, URF_j = URF_j^n\right)$ is estimated using:

$$Prob\left(\lambda I \leq A_{jT} < \lambda(I+1) \,|\, URF_j = URF_j^n\right)$$

   Where:

   - λ is a relatively small dollar value claim range, for example, $\lambda = \frac{{}^{o}A_{jT}}{100.5}$
   - I is an integer increasing in increments of 1 starting from 0 and stopping at some arbitrary high value

13.5 The Hypothetical Portfolio's Assumptions

Table 5: The hypothetical portfolio assumptions

Plan Number j	URF	URT	Average Prob of Claim	Underlying claims cost p.a.
1	0.1	-0.0002	0.000341	$ 186,812
2	0.3	-0.0002	0.000358	$ 205,323
3	0.5	-0.0002	0.000398	$ 225,120
4	0.6	-0.0002	0.000406	$ 237,580
5	0.7	-0.0002	0.000424	$ 247,612
6	0.75	-0.0002	0.000424	$ 260,911
7	0.8	-0.0002	0.000430	$ 249,982
8	0.85	-0.0002	0.000445	$ 253,553
9	0.9	-0.0002	0.000455	$ 271,024
10	0.95	-0.0002	0.000489	$ 286,622
11	1	-0.0002	0.000493	$ 303,320
12	1.05	-0.0002	0.000493	$ 276,745
13	1.1	-0.0002	0.000488	$ 289,420
14	1.15	-0.0002	0.000501	$ 297,790
15	1.2	-0.0002	0.000509	$ 289,228
16	1.25	-0.0002	0.000520	$ 295,924
17	1.3	-0.0002	0.000559	$ 321,150
18	1.6	-0.0002	0.000622	$ 354,271
19	2	-0.0002	0.000736	$ 427,143
20	10	-0.0002	0.003301	$ 1,929,107
21	0.1	-0.0001	0.000371	$ 229,235
22	0.3	-0.0001	0.000396	$ 232,613
23	0.5	-0.0001	0.000415	$ 233,653
24	0.6	-0.0001	0.000453	$ 261,327
25	0.7	-0.0001	0.000491	$ 275,907
26	0.75	-0.0001	0.000486	$ 291,118
27	0.8	-0.0001	0.000496	$ 279,399
28	0.85	-0.0001	0.000503	$ 287,124
29	0.9	-0.0001	0.000536	$ 321,097
30	0.95	-0.0001	0.000549	$ 328,867
31	1	-0.0001	0.000554	$ 330,097
32	1.05	-0.0001	0.000576	$ 338,657
33	1.1	-0.0001	0.000585	$ 333,431
34	1.15	-0.0001	0.000563	$ 324,204
35	1.2	-0.0001	0.000571	$ 338,986
36	1.25	-0.0001	0.000613	$ 337,466
37	1.3	-0.0001	0.000610	$ 371,637
38	1.6	-0.0001	0.000693	$ 404,857
39	2	-0.0001	0.000792	$ 466,194

40	10	-0.0001	0.003377	$	1,957,958
41	0.1	-0.00005	0.000397	$	228,004
42	0.3	-0.00005	0.000420	$	240,823
43	0.5	-0.00005	0.000449	$	256,368
44	0.6	-0.00005	0.000466	$	283,223
45	0.7	-0.00005	0.000498	$	299,949
46	0.75	-0.00005	0.000517	$	307,948
47	0.8	-0.00005	0.000531	$	300,501
48	0.85	-0.00005	0.000537	$	317,240
49	0.9	-0.00005	0.000555	$	312,691
50	0.95	-0.00005	0.000549	$	308,807
51	1	-0.00005	0.000568	$	324,641
52	1.05	-0.00005	0.000613	$	352,744
53	1.1	-0.00005	0.000592	$	345,937
54	1.15	-0.00005	0.000612	$	374,591
55	1.2	-0.00005	0.000620	$	349,665
56	1.25	-0.00005	0.000629	$	361,713
57	1.3	-0.00005	0.000617	$	340,029
58	1.6	-0.00005	0.000728	$	419,424
59	2	-0.00005	0.000825	$	483,775
60	10	-0.00005	0.003422	$	2,012,273
61	0.1	0	0.000415	$	241,480
62	0.3	0	0.000458	$	265,340
63	0.5	0	0.000492	$	276,603
64	0.6	0	0.000515	$	291,859
65	0.7	0	0.000528	$	299,182
66	0.75	0	0.000554	$	307,563
67	0.8	0	0.000542	$	301,393
68	0.85	0	0.000554	$	319,541
69	0.9	0	0.000569	$	321,248
70	0.95	0	0.000575	$	329,745
71	1	0	0.000603	$	337,630
72	1.05	0	0.000611	$	356,576
73	1.1	0	0.000608	$	338,731
74	1.15	0	0.000641	$	369,838
75	1.2	0	0.000669	$	374,269
76	1.25	0	0.000659	$	373,715
77	1.3	0	0.000678	$	400,391
78	1.6	0	0.000753	$	439,722
79	2	0	0.000880	$	518,721
80	10	0	0.003500	$	2,060,100
81	0.1	0.00005	0.000447	$	256,754
82	0.3	0.00005	0.000473	$	267,310
83	0.5	0.00005	0.000532	$	309,080
84	0.6	0.00005	0.000526	$	284,145

85	0.7	0.00005	0.000555	$	345,897
86	0.75	0.00005	0.000578	$	336,298
87	0.8	0.00005	0.000600	$	338,186
88	0.85	0.00005	0.000607	$	358,308
89	0.9	0.00005	0.000627	$	355,853
90	0.95	0.00005	0.000628	$	339,856
91	1	0.00005	0.000629	$	384,586
92	1.05	0.00005	0.000644	$	358,473
93	1.1	0.00005	0.000651	$	374,132
94	1.15	0.00005	0.000681	$	390,702
95	1.2	0.00005	0.000666	$	380,403
96	1.25	0.00005	0.000714	$	441,305
97	1.3	0.00005	0.000728	$	432,805
98	1.6	0.00005	0.000803	$	466,605
99	2	0.00005	0.000890	$	528,473
100	10	0.00005	0.003546	$	2,063,860
101	0.1	0.0001	0.000463	$	265,740
102	0.3	0.0001	0.000491	$	290,832
103	0.5	0.0001	0.000548	$	316,748
104	0.6	0.0001	0.000571	$	334,088
105	0.7	0.0001	0.000587	$	335,119
106	0.75	0.0001	0.000613	$	349,309
107	0.8	0.0001	0.000616	$	360,141
108	0.85	0.0001	0.000611	$	351,552
109	0.9	0.0001	0.000639	$	372,803
110	0.95	0.0001	0.000674	$	393,551
111	1	0.0001	0.000664	$	396,063
112	1.05	0.0001	0.000684	$	401,543
113	1.1	0.0001	0.000679	$	376,807
114	1.15	0.0001	0.000716	$	425,109
115	1.2	0.0001	0.000707	$	401,460
116	1.25	0.0001	0.000740	$	424,190
117	1.3	0.0001	0.000768	$	447,693
118	1.6	0.0001	0.000840	$	495,516
119	2	0.0001	0.000930	$	534,184
120	10	0.0001	0.003594	$	2,126,902
121	0.1	0.0002	0.000520	$	312,917
122	0.3	0.0002	0.000580	$	332,023
123	0.5	0.0002	0.000618	$	357,032
124	0.6	0.0002	0.000627	$	352,195
125	0.7	0.0002	0.000663	$	373,210
126	0.75	0.0002	0.000668	$	376,636
127	0.8	0.0002	0.000709	$	392,699
128	0.85	0.0002	0.000727	$	423,445
129	0.9	0.0002	0.000700	$	416,540

130	0.95	0.0002	0.000716	$ 405,441
131	1	0.0002	0.000758	$ 441,279
132	1.05	0.0002	0.000736	$ 437,041
133	1.1	0.0002	0.000735	$ 448,321
134	1.15	0.0002	0.000770	$ 444,982
135	1.2	0.0002	0.000791	$ 446,313
136	1.25	0.0002	0.000801	$ 447,375
137	1.3	0.0002	0.000808	$ 468,470
138	1.6	0.0002	0.000906	$ 524,106
139	2	0.0002	0.001019	$ 586,532
140	10	0.0002	0.003718	$ 2,156,576

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