COMMENT
A Topic of Interest

How to Extrapolate the Yield Curve

INTRODUCTION

One of the most fundamental concepts in actuarial practice is the time value of money. For any work where future cash flows are allowed for, such as reserving or pricing, it is natural to discount to present values so that an appropriate amount of money can be set aside today, allowing for future investment returns.

It is widely accepted that for claims reserving, liabilities should be discounted using the prices of the ‘risk-free’ assets available in the financial markets. This means that the present value of a liability cash flow should be set equal to the market price of a basket of risk-free assets that provides a matching cash flow.

Despite this general agreement, for some time there has been considerable debate on some practical aspects of the principle. This debate gained intensity following the global financial crisis of 2007/08 which saw large increases in the price of risk-free assets and correspondingly, large decreases in risk-free interest rates. Issues of debate include:

• What are the best instruments to use to determine risk-free interest rates?
• Should the risk-free rate include an ‘illiquidity premium’?
• What should be done when the liabilities being valued extend beyond the term of available market instruments?

In the Australian context, where we have a deep and liquid market in AAA rated Commonwealth Government Bonds, the Australian Prudential Regulation Authority (APRA) has made it clear that it regards these bonds as the best instruments to use to determine the risk-free interest rate. Further, for general insurance liabilities at least, APRA do not allow the inclusion of an illiquidity premium. However, the issue of what should be done when the liabilities being valued extend beyond the term of available market instruments has, in our opinion, not been fully addressed in any current Australian regulations. Further, actuaries operating in the Australian market have adopted a wide range of approaches to this issue and this has led to inconsistent valuations of long-term liabilities across entities. This issue is particularly relevant for Australia where the term of the longest dated government bond is currently around 15 years – in many other countries government bonds are available for terms of up to 30 years or more.

The aim of this article is to review the issues relevant to yield curve extrapolation. Figure 1 shows the three key features required in an extrapolation:

1. Deciding on a starting point for the extrapolation of the known yield curve.
2. Finding an appropriate long-term rate, which we refer to as the ‘Unconditional Forward Rate’ (UFR).
3. The speed and path of moving from the fitted curve to the UFR.

PHILOSOPHICAL AND REGULATORY CONSIDERATIONS

There are two philosophical approaches to yield curve extrapolation. The first emphasises market consistency, by which we mean that the yield curve is a genuine attempt to predict the yields on long dated government bonds, were they to exist. The liability estimate therefore aims to accurately quantify the cost of transferring that liability in the current market conditions. The second approach emphasises liability stability. This will tend to estimate the extrapolated yield curve so that it is more stable over time. This reduces the business impacts of volatility in the liability estimate, particularly when offsetting financial investments do not exist.

Our reading of current regulatory and professional standards in Australia is that, while not being completely explicit, the market consistent approach is to be preferred. In our research we have attempted to extrapolate the yield curve using this market consistent approach.

WHAT IS THE LONGEST MARKET FORWARD INTEREST RATE WE CAN ESTIMATE RELIABLY?

A common starting point for extrapolation is the term of the last available market instrument. In this section we consider reasons why this may not always be appropriate.

The most common approach to yield curve fitting is to estimate the forward rate curve using a smooth parametric fit to the observed prices. These fits contain a small amount of error as the pricing ‘noise’ of individual bonds lead to small departures from a smooth shape.
Figure 2 shows the estimation of the yield curve at 31/12/2011, using an exponential spline method commonly used for fitting such curves. Bond yields were available going out to duration 15 years. We have estimated the error of this fit along the curve by looking at the errors associated with the fit. Up until a term of 10 years, the 90% confidence interval is around 10-15 basis points in width. But by year 13 it has spread to 25 basis points, and reaches about 40 basis points by year 15.

Figure 2: Estimation error in the forward rate curve, 31 December 2011

While there are a number of reasons for this widening, we have found the pattern in the confidence intervals to be similar whenever the yield curve is estimated. Thus our first recommendation is that we should not be overly reliant on forward rate estimates made at the long end of the fitted forward curve, in particular the last two years of the observable range. In the example above, we would start the extrapolation at duration 13 years.

WHAT IS AN APPROPRIATE VERY LONG-TERM ‘UNCONDITIONAL’ FORWARD INTEREST RATE?
The rational expectations hypothesis of the term structure of interest rates suggests, in its more general form, that long-term forward rates are the sum of the expected future short-term interest rate plus a constant ‘term premium’ that varies only by term but only slowly over time. Taking this to be true, the process of determining the ultimate very long-term ‘unconditional’ forward interest rate (‘UFR’) would involve making estimates of what these two components are. Unfortunately, neither component is particularly easy to estimate.

The problem of setting the expected short-term interest rates is usually split into determining the:

- expected future inflation; and
- expected future real short-term interest rate.

In relation to expected future inflation, the Reserve Bank of Australia has been very successful in targeting inflation and entrenched low and stable inflation expectations for at least the last 15 years. It seems reasonable then to adopt the mid-range of the bank’s current CPI target of 2-3% as our future inflation expectation. For expected real short-term interest rates a typical approach has been to look at historical averages for real cash returns across several countries. We believe a real short term rate average of 2% is fairly consistent with the conclusions in recent research.

Term premia are the differences between the forward rates and the expectation of the future short-term interest rates. They arise from:
- the premium demanded for locking in a long-term rate;
- duration preference, leading to different relative demand; and
- convexity effects, where gains from small interest rate changes on long-dated bonds tend to be larger than losses.

These premiums can also change markedly over time. Some recent work has estimated term premia are somewhere in the range of 1-2%.

We present our estimates of each of these components at the present time in the table below. Based on this, we believe that a UFR of about 5.8% seems reasonable for Australia.

Table 1: Components of the Unconditional Forward Rate for Australia in 2012

<table>
<thead>
<tr>
<th>Component</th>
<th>Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expected future short-term interest rate</td>
<td></td>
</tr>
<tr>
<td>Expected future inflation</td>
<td>2.5%</td>
</tr>
<tr>
<td>Expected future real interest rate</td>
<td>2.0%</td>
</tr>
<tr>
<td>Term premium</td>
<td></td>
</tr>
<tr>
<td>Risk Premium</td>
<td>1.5%</td>
</tr>
<tr>
<td>Convexity adjustment</td>
<td>-0.2%</td>
</tr>
<tr>
<td>Unconditional forward rate</td>
<td>5.8%</td>
</tr>
</tbody>
</table>

WHAT PATH FROM THE KNOWN CURVE TO THE UFR?
We now move on to the final and arguably most important aspect of yield curve extrapolation: what path should be set between the longest duration market rate and the unconditional forward rate and how long should we take to reach it? The speed of ‘reversion’ to the UFR is of practical importance; in the Australian context, if it returns quickly (by duration 20 years say), then long-tailed liabilities will be considerably more stable over time than if it returns slowly (e.g. by duration 100).

We have attempted to answer this question by looking at the relationship between medium and long duration bonds in some countries that do have longer dated bonds – USA, UK and Canada. Letting $f(s)$ be the instantaneous forward rate at term $s$, consider the simple linear regression equation:

$$ f(s+t) = \alpha_{s:t} + \beta_{s:t} f(s) $$

This gives the relationship between the forward rate at term $s+t$ and the forward rate at term $s$. The intercept $\alpha_{s:t}$ allows for the UFR as well as any (fixed) term premiums across the yield curve, and $\beta_{s:t}$ is the linear dependence of the forward rate at term $s+t$ on the forward rate at term $s$. The estimate of $\beta_{s:t}$ should be a good indicator of progress towards the UFR; if the slope parameter is close to 1, this implies the forward rate at $s+t$ is still moving in sync with the rate at $s$, so no reversion to the UFR has taken place. Note too that if the slope was consistently close to 1 as $t$ increases, this would
be strong evidence against any reversion to the UFR. Conversely, if the slope is close to zero then the forward rate at $s+t$ is largely independent of the rate at $s$, suggesting that it has reverted to a constant level.

Figure 3 shows the estimated slope parameter on US data since 1998, where $s = 10$ and duration = $s+t$ is varied. The results here are surprisingly clear, with a slow near linear reversion from 1 towards 0 starting at duration 10. This suggests to us that a linear shape of reversion is plausible, and that this reversion is very slow; when the 30 year forward rate is regressed against duration 10, the slope parameter is still around 0.7, suggesting a very slow reversion to the UFR.

Figure 3: Slope ($\beta$) coefficients for USA forward rates regressed against $f_{10}$

We can extrapolate the fit to estimate the duration at which the UFR is reached. Our results for each country are shown in Table 2. All estimates correspond to slow reversion, although there is significant uncertainty in estimation and variation between countries.

Table 2: Regression results for the linear extrapolation model

<table>
<thead>
<tr>
<th>Country</th>
<th>Duration decay starts (yrs)</th>
<th>Duration when reach UFR (yrs)</th>
<th>95% confidence interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>US</td>
<td>10</td>
<td>82</td>
<td>(55, 168)</td>
</tr>
<tr>
<td>UK</td>
<td>15</td>
<td>34</td>
<td>(31, 40)</td>
</tr>
<tr>
<td>Canada</td>
<td>10</td>
<td>41</td>
<td>(35, 47)</td>
</tr>
</tbody>
</table>

We also performed a principal components analysis of these yield curves as an alternative way of measuring how quickly long duration rates tend towards a value independent of the short to medium rates. This gave slightly longer estimates for reverting to the UFR than the regression approach. Our conclusions were that a linear shape of extrapolation to the UFR was plausible and that the speed of reversion was slowly.

**IMPLICATIONS FOR Hedging Balance Sheet Risk**

It is possible to use a duration matching strategy to hedge risks that are beyond the longest term assets available, if you are allowed to take short positions in assets and have an estimate for the risk-free rate beyond the longest term asset. Given that this hedging approach requires an extrapolated yield curve, a natural question is whether hedging performance is better or worse if the extrapolated yield curve is assumed to revert slowly or quickly to the UFR? We have tested the success of slow and fast reversion assumptions using historical data from the Australian Government Bond market.

Without going into too much detail, we attempted to hedge a 20 year liability with four and 10 year government bonds over a period of 10 years. A hedge was considered good if the ratio of assets and liabilities remained close over time. The assumed speed of reversion to the UFR in the yield curve extrapolation made a significant difference to the hedging performance, with the slow assumption consistently performing better – see Figure 4 for example.

Figure 4: Comparison of hedging performance, starting June 1995

This approach to long-term liability hedging appears fairly legitimate from both a theoretical and historical data perspective. It gave a hedging error of less than 1% in all experiments. The performance of the fast reversion assumption is inferior, giving further evidence that a slower reversion to UFR is closer to ‘truth’.

**CONCLUSIONS AND RECOMMENDATIONS**

We can summarise our findings relatively succinctly:

- The yield curve up to two years before the longest dated bond can be estimated reliably. For the last year or so, the noise and method of fit can cause significant (relative) error.
- There is reasonable international market evidence for reversion to a flat long-term forward rate. This rate is reached via extrapolation from the end of the observable yield curve.
- The rate of reversion is slow. We believe term 40 is about the minimum point to reversion based on the bond markets examined, with a central estimate closer to term 60.
- A linear shape of reversion is plausible, with other approaches possible.
- Long-term risk-free hedging is possible, at least for moderate term extrapolations of the yield curve.

We believe that these results make significant contributions to actuarial assumption setting.

Finally, we note that Joe Hockey has suggested introducing 50-year Australian Government bonds in the future. Perhaps there will be a day where such yield curve extrapolations are not necessary!

The full version of this paper was presented at the 2013 Actuaries Summit and can be downloaded from http://www.actuaries.asn.au/SUM2013/Program/Media.aspx