12th Accident Compensation Seminar 2009 Rising to the Challenge

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Institute of Actuaries of Australia

Uncertainty-based framework for setting industry premium rates in workers' compensation

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(presented by Ivan Lebedev)

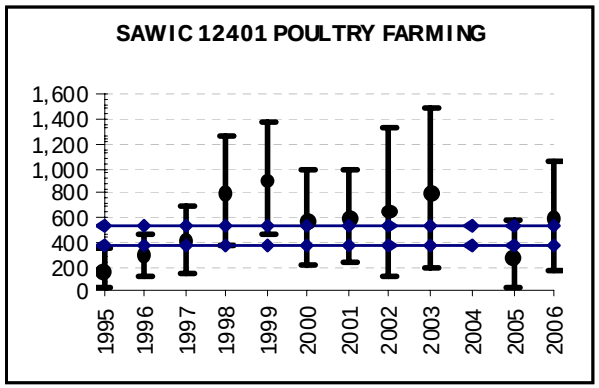
Outline

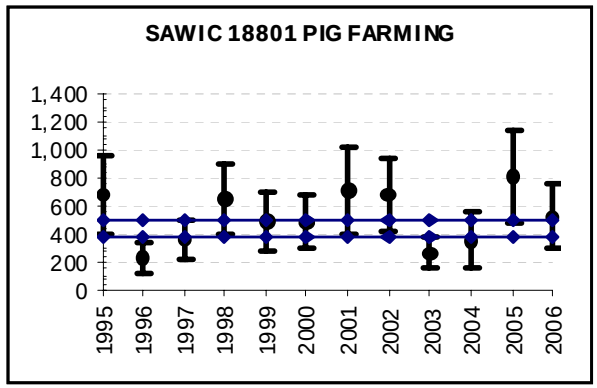
1. Quick tour of industry-level experience
2. Industry rate calculation as a mathematical problem
3. Statistics of industry cost-ratios
4. Adaptive period length idea
5. Minimum disturbance approach for dealing with sparse data
6. Summary and conclusions

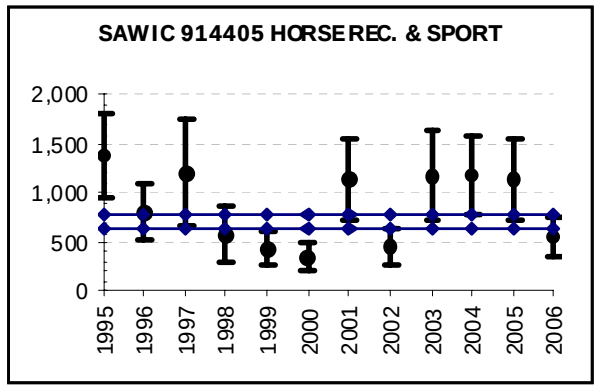
Tour of industry-level experience

- Cost ratio = (two-year paid-up cost of claims)/(remuneration/52*AWE)
- Claim costs are capped at $72,000

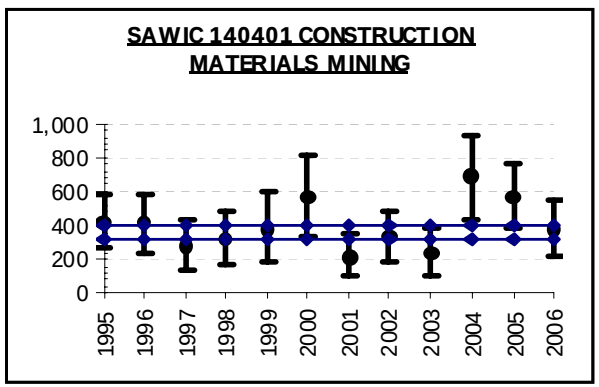
Agriculture, Forestry and Fishing

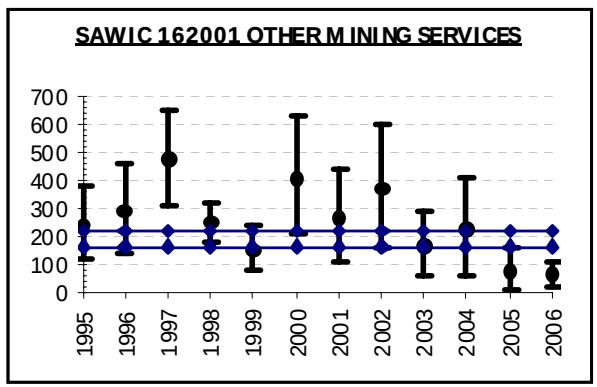


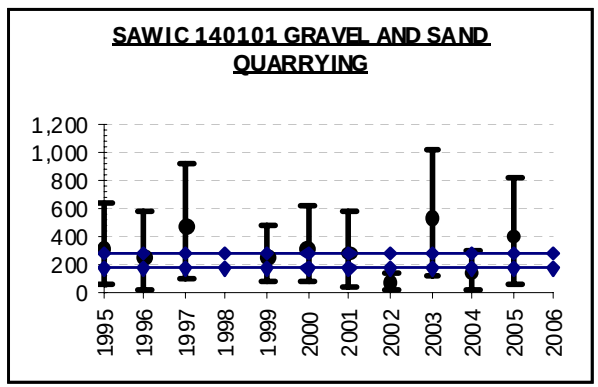




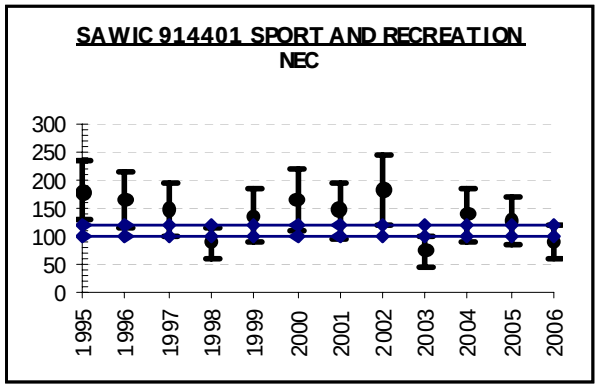
Mining

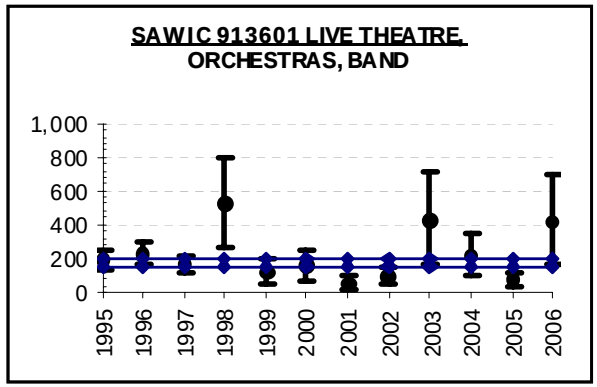


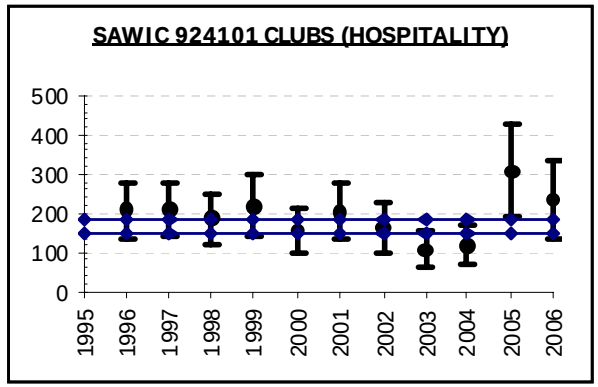




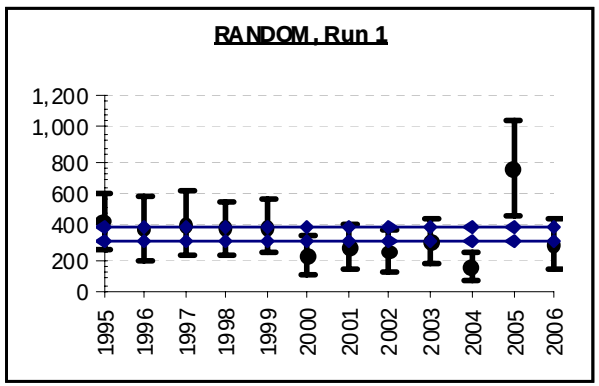
Recreation Personal Other

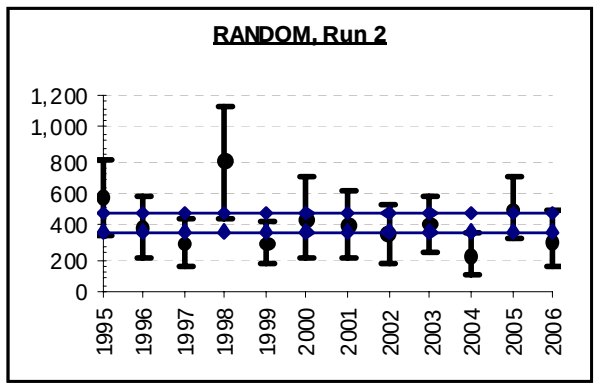


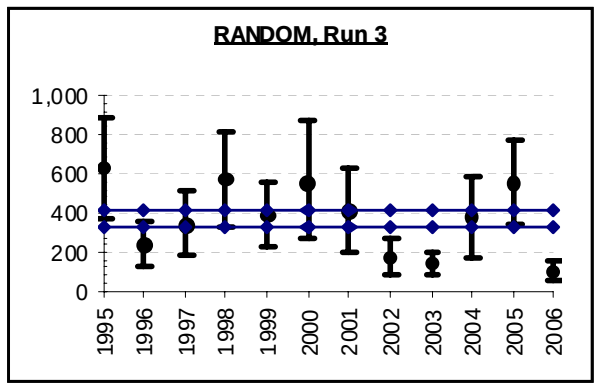




Generated using Poisson-distributed ($\lambda = 50$) number of claims and claims costs from the Scheme-average distribution







Industry-level experience

- No systematic trends are apparent
- Visually indistinguishable from realisations of a stochastic process with constant parameters

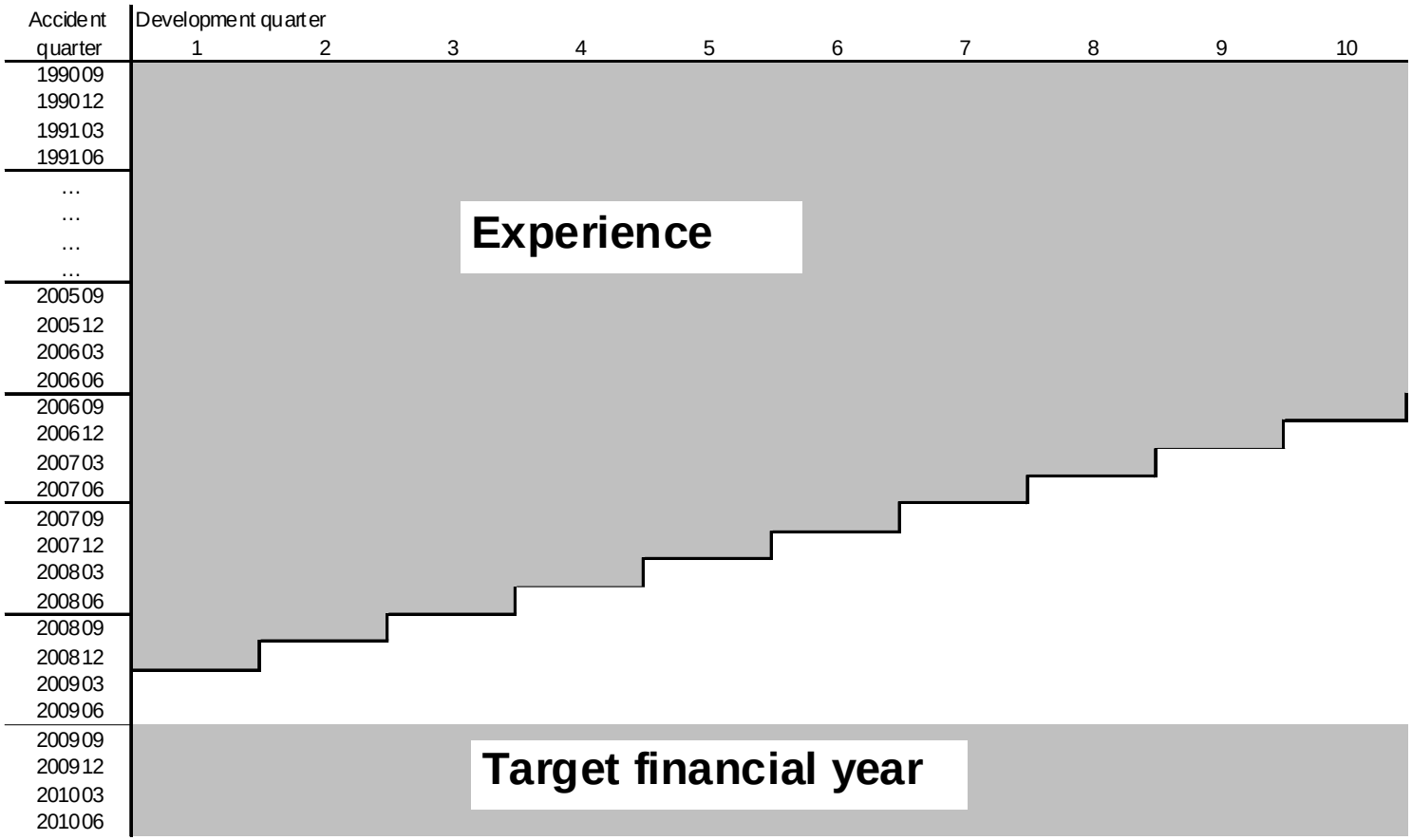
Mathematical formulation

- Need to apportion total target premium collection between all industries in proportion to their relativities.
- Relativities are a measure of relative riskiness.
- Riskiness is measured by Cost-ratio=Cost of claims/remuneration

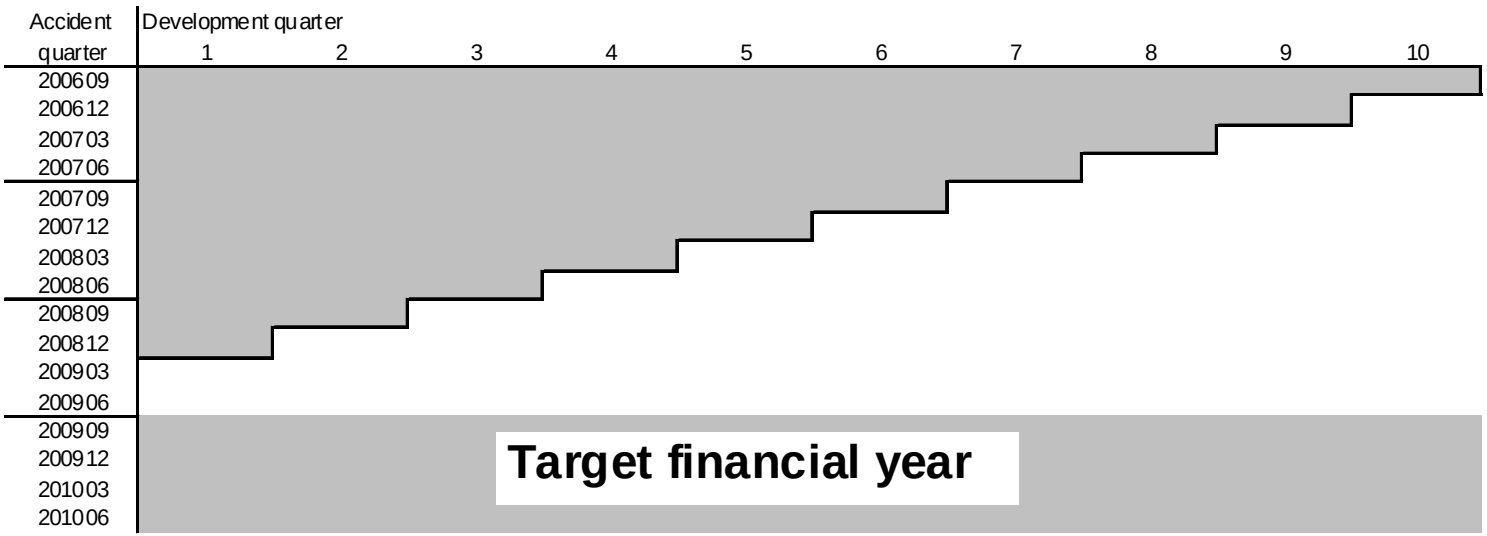
Mathematical formulation

- Approximate number of FTE units U=Remuneration/(52xAWE)
- s=cost of claims
- Observed cost ratio=s/U
- Underlying cost ratio=E[S]/U
- The aim is to estimate the **underlying** cost ratio

What cost of claims?



‘Burning cost’

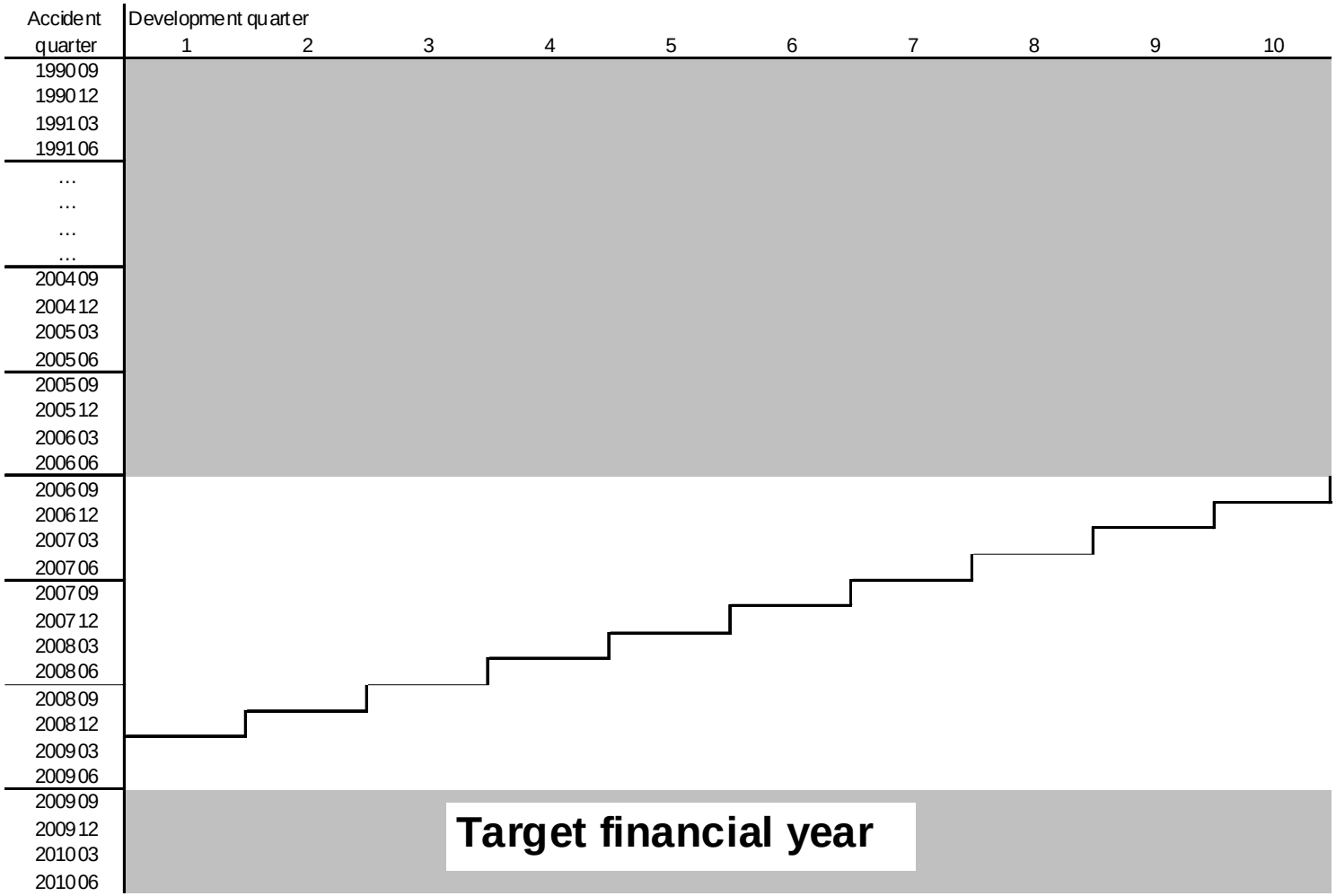


	DevYr						
AccYr	1	2	3	4	5	Paid-up cost	Exposure
200406	50	93	95	55	46	339	19,356
200506	52	101	78	69		300	19,408
200606	52	96	86			234	19,890
200706	51	94				145	19,754
200806	53					53	19,589
Total						1,071	97,997

- Contribution of different accident years is different
- Cost ratio as shown=1,071/97,997=1.09%
- Imagine that remuneration in 2008 increased by 50%. New cost ratio=

  (1,071+26.5)/(97,997+9,795)=1.02%

Fixed paid-up period length

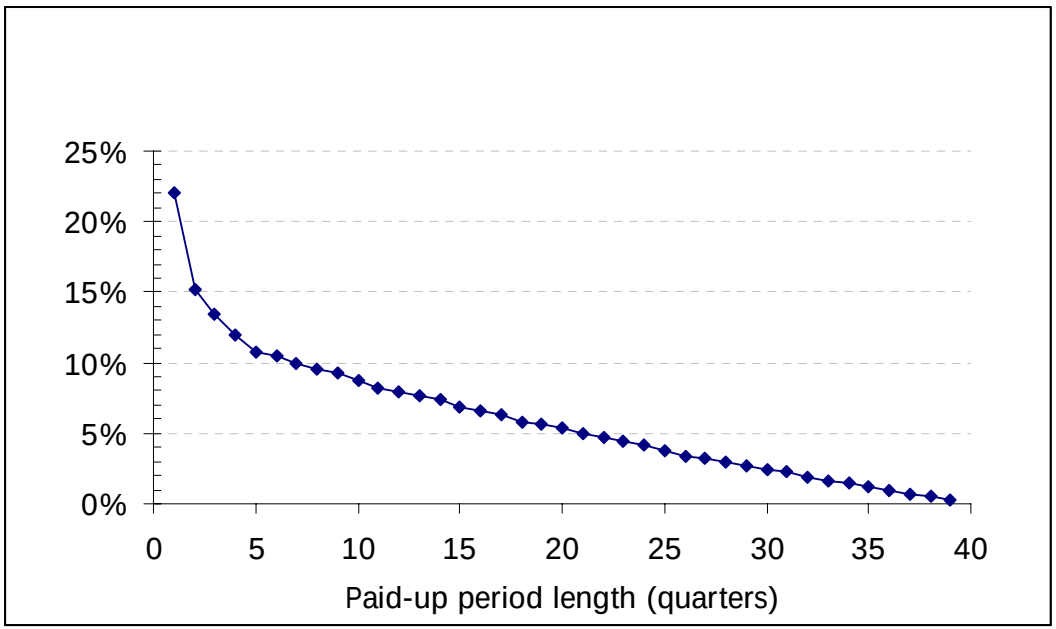


Fixed paid-up period length

- Advantages:
  - No bias in response to changing exposure
  - Contributions of all accident periods are equal
  - Mathematically tractable!!!
- Disadvantages:
  - Gap between latest accident period and target financial year

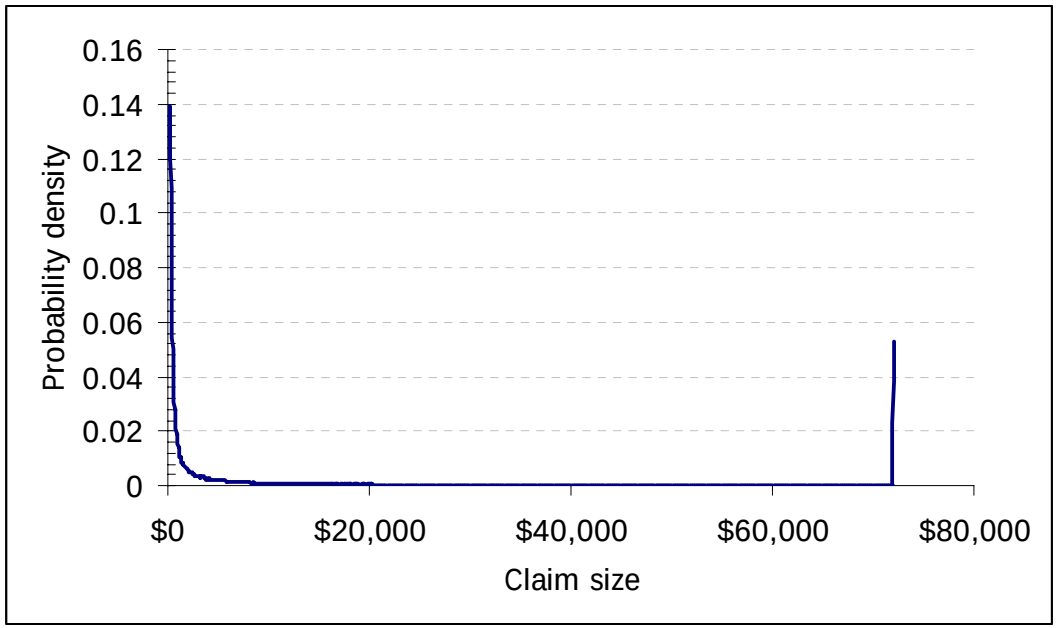
What is the optimal paid-up period length?

Cross-funding as a function of paid-up period length



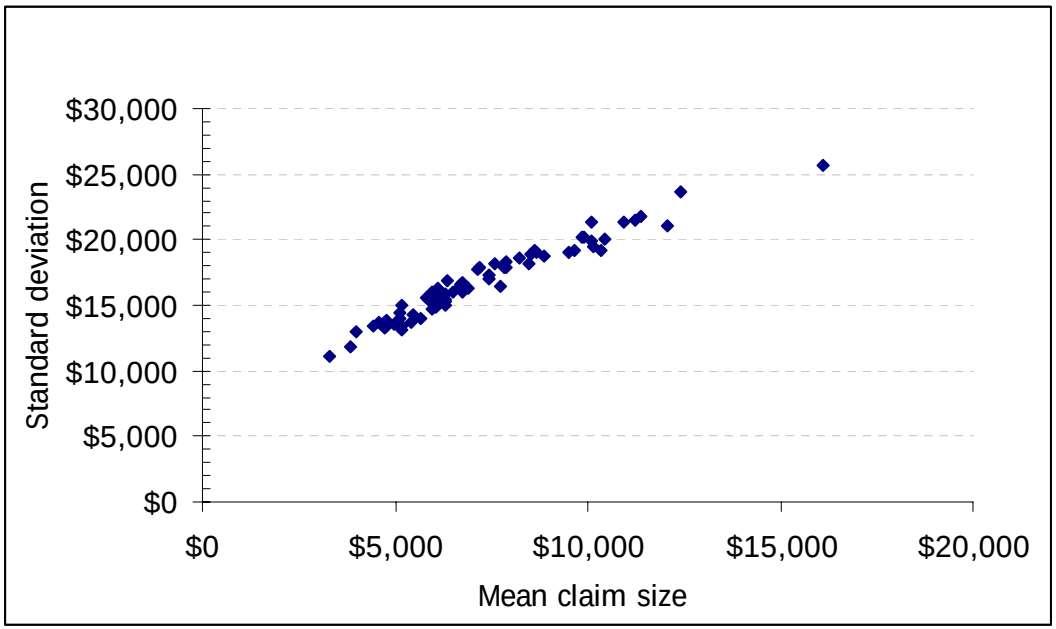
L=9 quarters is a sensible choice

Claim size distribution – combined



- Mean=8,160 Median=600
- StDev=18,340 CV=2.23

Claim-size distribution – at industry level



- Standard deviation ~ mean

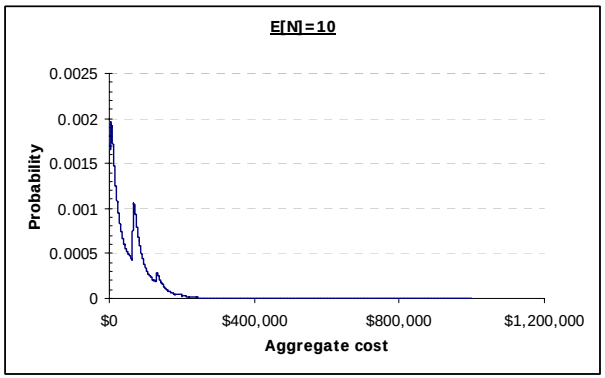
Aggregate cost of claims

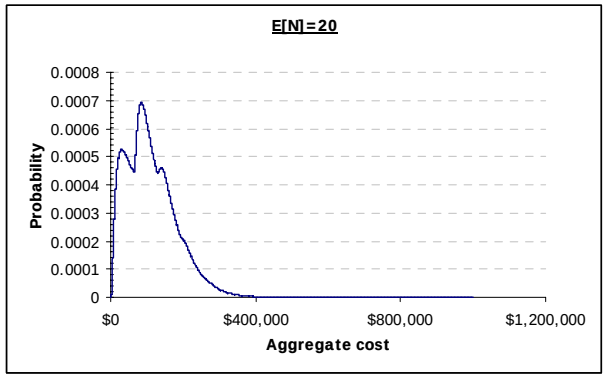
$$S = \sum_{i=1}^{N} X_i \qquad N \sim Poisson(fU)$$

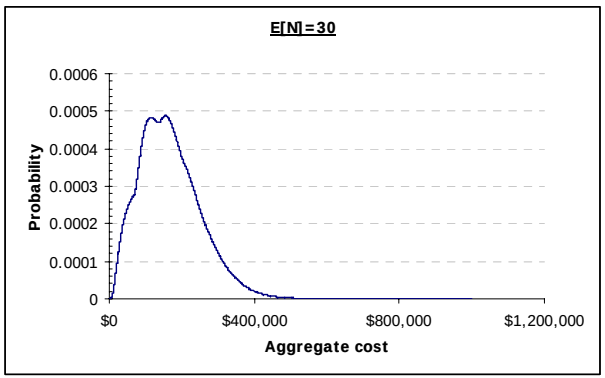
- X – cost of single claim, empirical distribution
- f – claim occurrence rate per FTE unit
- U – approximate number of FTE units

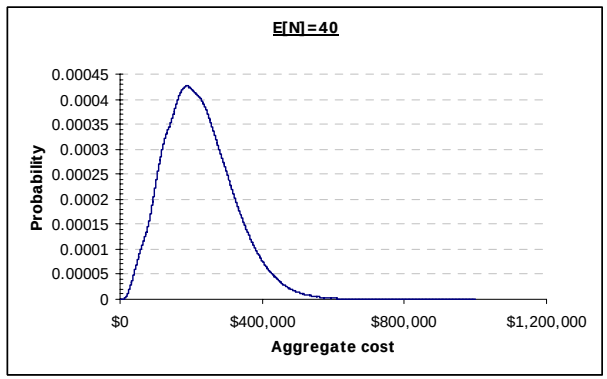
Small Poisson parameter

- Use Panjer's recursion









Large Poisson parameter

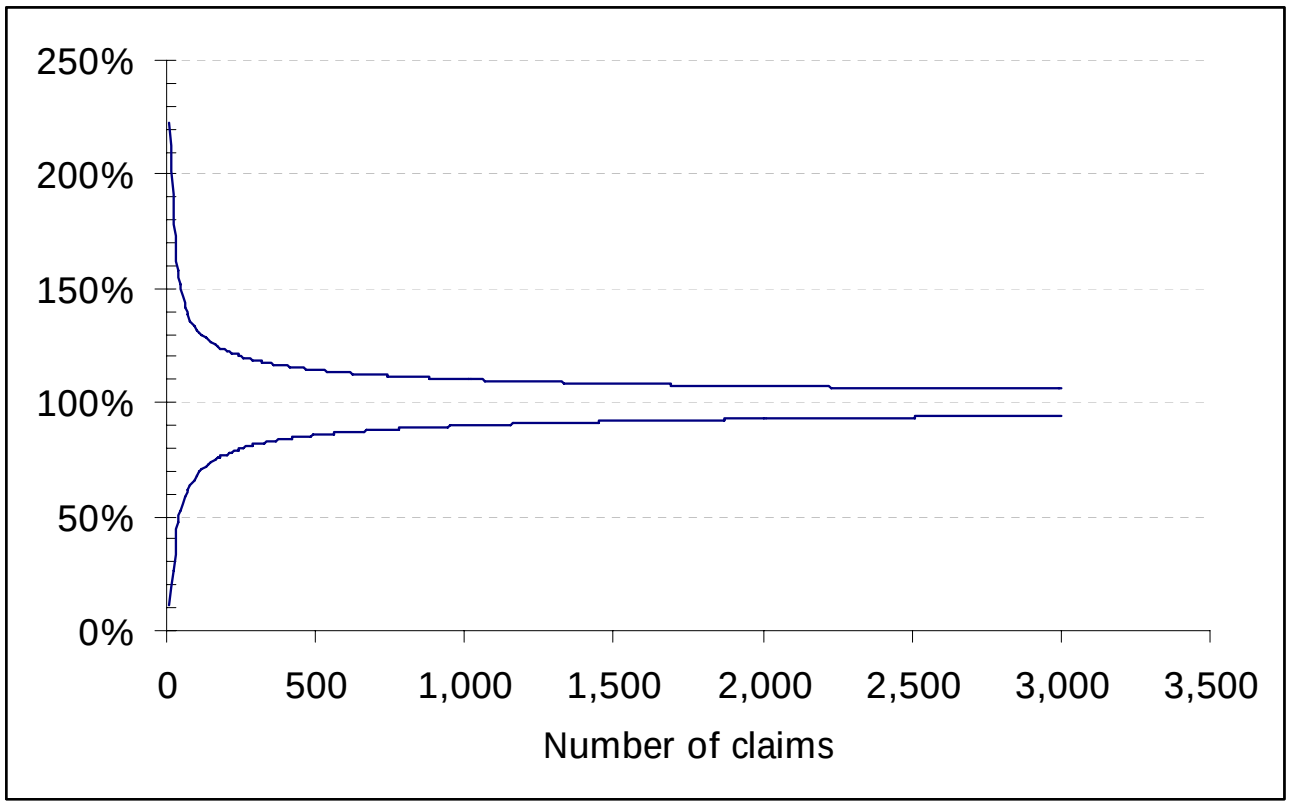
$$E[S] = E[N]E[X],$$

$$Var[S] = E[N]Var[X] + Var[N](E[X])^2$$

$$CV[S] = \frac{\sqrt{Var[S]}}{E[S]} = \frac{\sqrt{fU(Var[X] + (E[X])^2)}}{fU\,E[X]} = \frac{1}{\sqrt{fU}}\sqrt{1 + (CV[X])^2}$$

$$CV[S]_{est} \approx \frac{\sqrt{1 + 2.3^2}}{\sqrt{n}} = \frac{2.5}{\sqrt{n}}$$

Combined accuracy estimate



10th-percentile/mean and 90th-percentile/mean

How much experience do we have?

Number of claims in 2006 accident year

Number of claims	Number of industries
<10	160
10 to 19	75
20 to 29	33
30 to 39	33
40 to 49	22
50 to 99	48
100 to 199	29
200 and more	15
Total	415

How much experience do we have?

Number of claims in 2002-2006 accident years

Number of claims	Number of industries
<10	81
10 to 19	37
20 to 29	33
30 to 39	23
40 to 49	29
50 to 99	69
100 to 199	66
200 to 299	38
300 to 399	26
400 to 499	9
500 to 599	7
600 to 699	7
700 to 799	9
800 to 899	4
900 to 999	6
1000 and more	15
Total	459

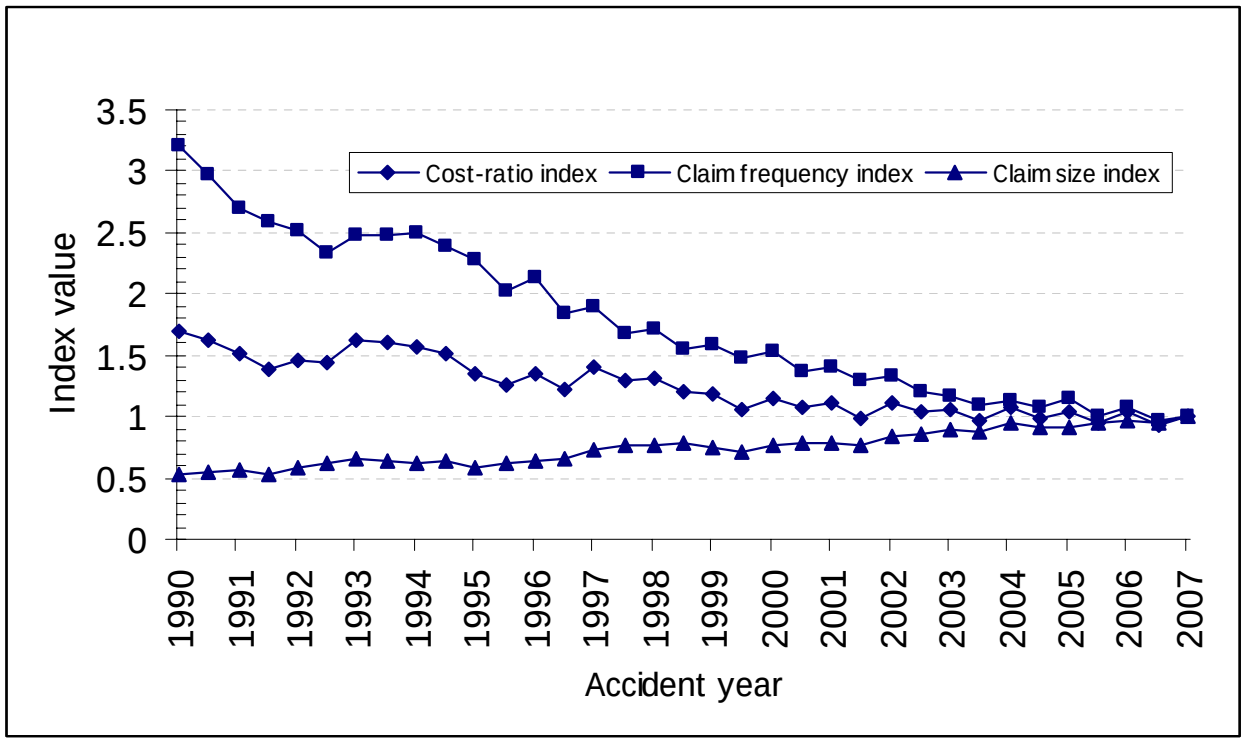
Adaptive experience period

- Aim to achieve +/-10% accuracy at 80% significance level => need at least 1,000 claims
- Go as far back as needed to get them

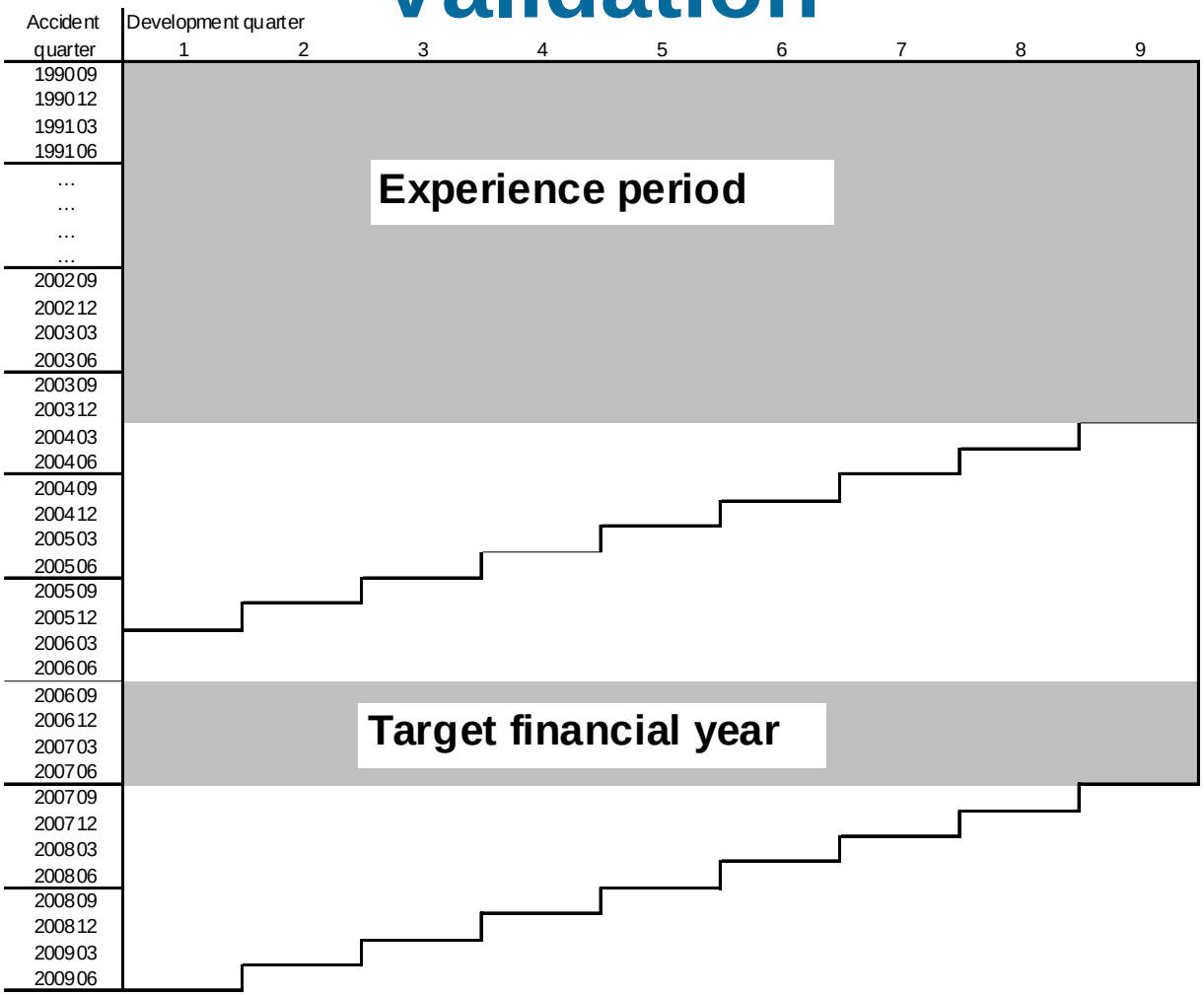
Adaptive experience period

Minimum experience period (y)	Number of industries	Proportion
1	0	0%
2	3	1%
3	1	0%
4	4	1%
5	4	1%
6	7	1%
7	11	2%
8	6	1%
9	11	2%
10	2	0%
11	4	1%
12	13	3%
13	6	1%
14	5	1%
15	16	3%
16	10	2%
All available experience	383	79%
Total	486	100%

Adjustments for claim frequency and claim size changes



Validation


Accident quarter
Development quarter
1
2
3
4
5
6
7
8
9
199009
199012
199103
199106
...
...
...
...
200209
200212
200303
200306
200309
200312
200403
200406
200409
200412
200503
200506
200509
200512
200603
200606
200609
200612
200703
200706
200709
200712
200803
200806
200809
200812
200903
200906
Experience period
Target financial year

Validation

- From past data, we predict claim occurrence rate and average claim size for each industry
- Given this, for each industry *i* one can calculate the P-value of the actual aggregate claims cost observed,

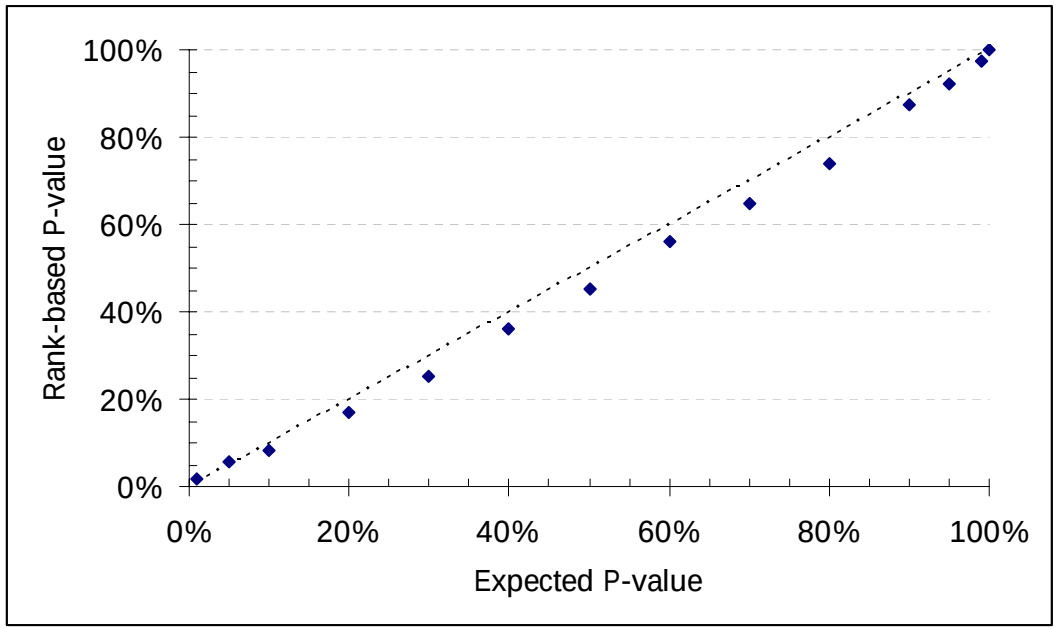
$$P(s_i)=Prob(S<s_i)$$

- Because of randomness, there will be a range of values of $P(s_i)$

Validation

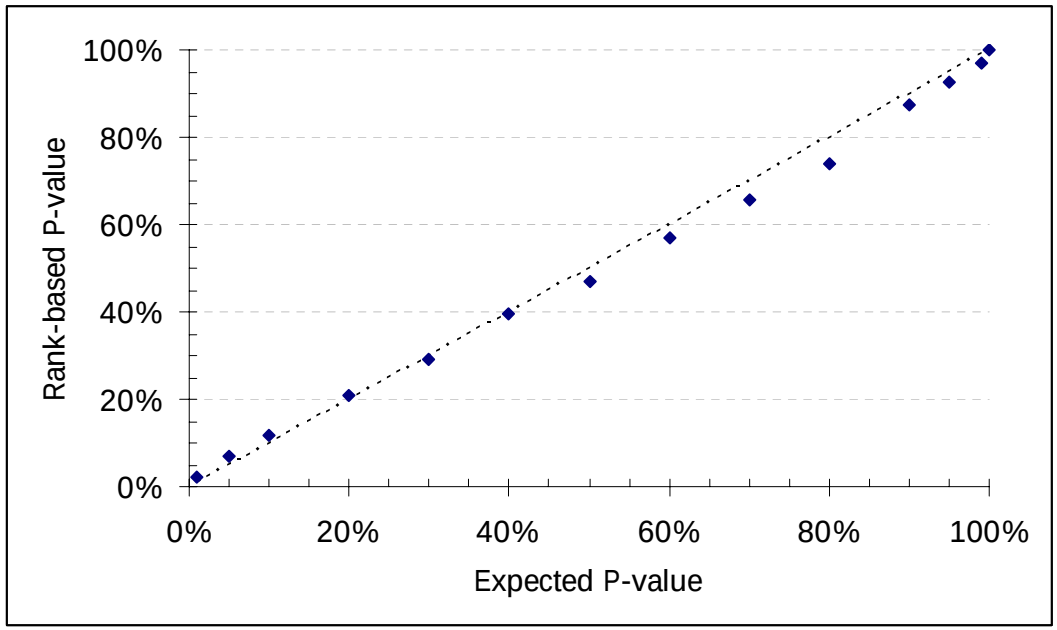
- If the model is perfect, than 5% of all industries will have P<0.05, 10% of industries will have P<0.1, etc.
- This can be checked with a quantile-quantile plot

Validation



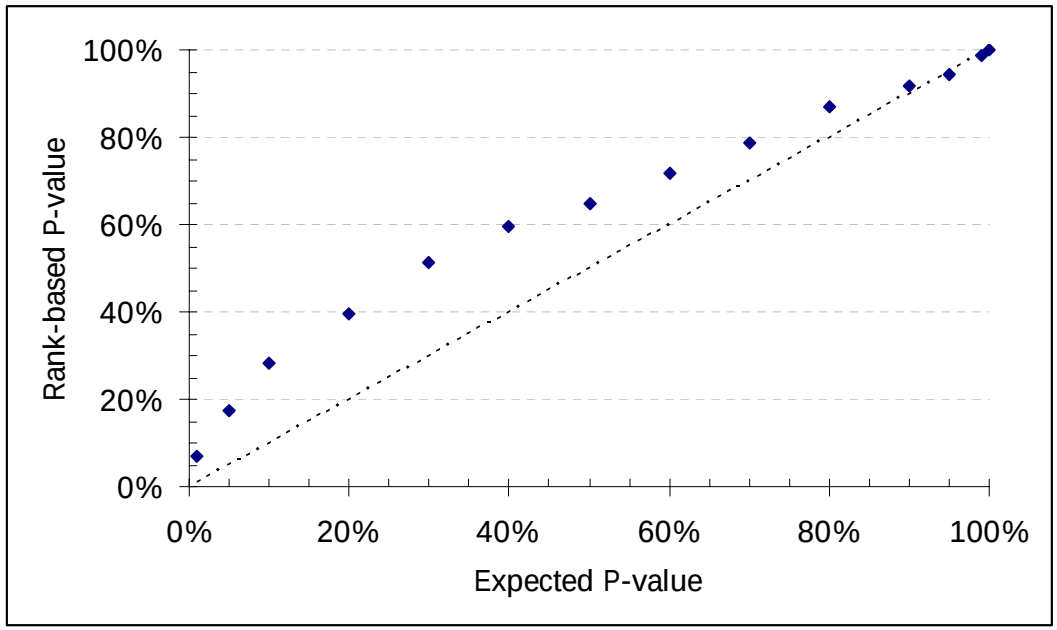
Quantile-quantile plot of observed claims costs in accident year 2007. There were 359 industries included in the rank calculation. Industries that had less than 40 claims over the entire experience period or Poisson parameter <1 were excluded.

Validation



Quantile-quantile plot of observed claims costs in accident year 2007. There were 172 industries included in the rank calculation. Industries that had less than 40 claims over the entire experience period or Poisson parameter <20 were excluded.

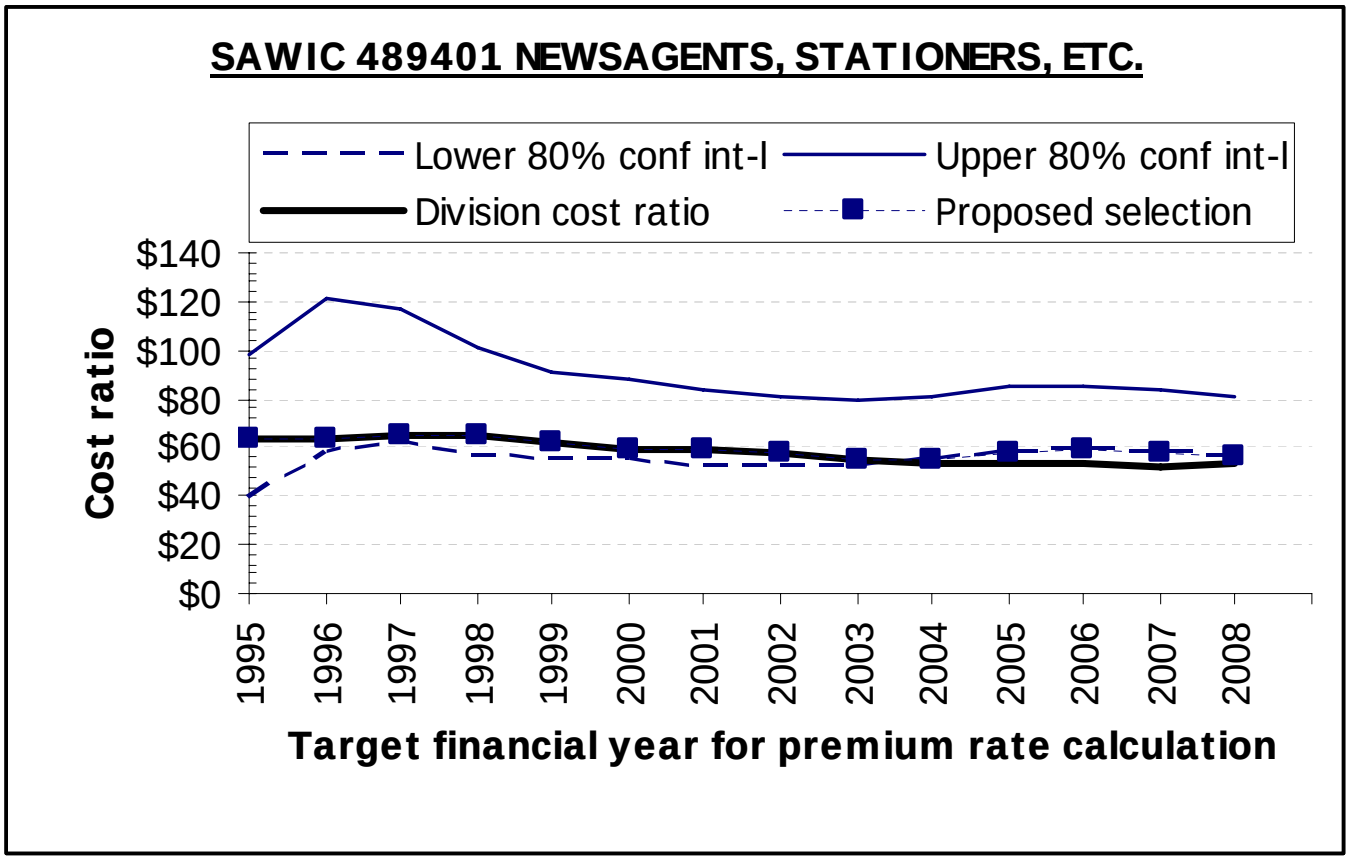
Validation – no adjustments for scheme-wide changes



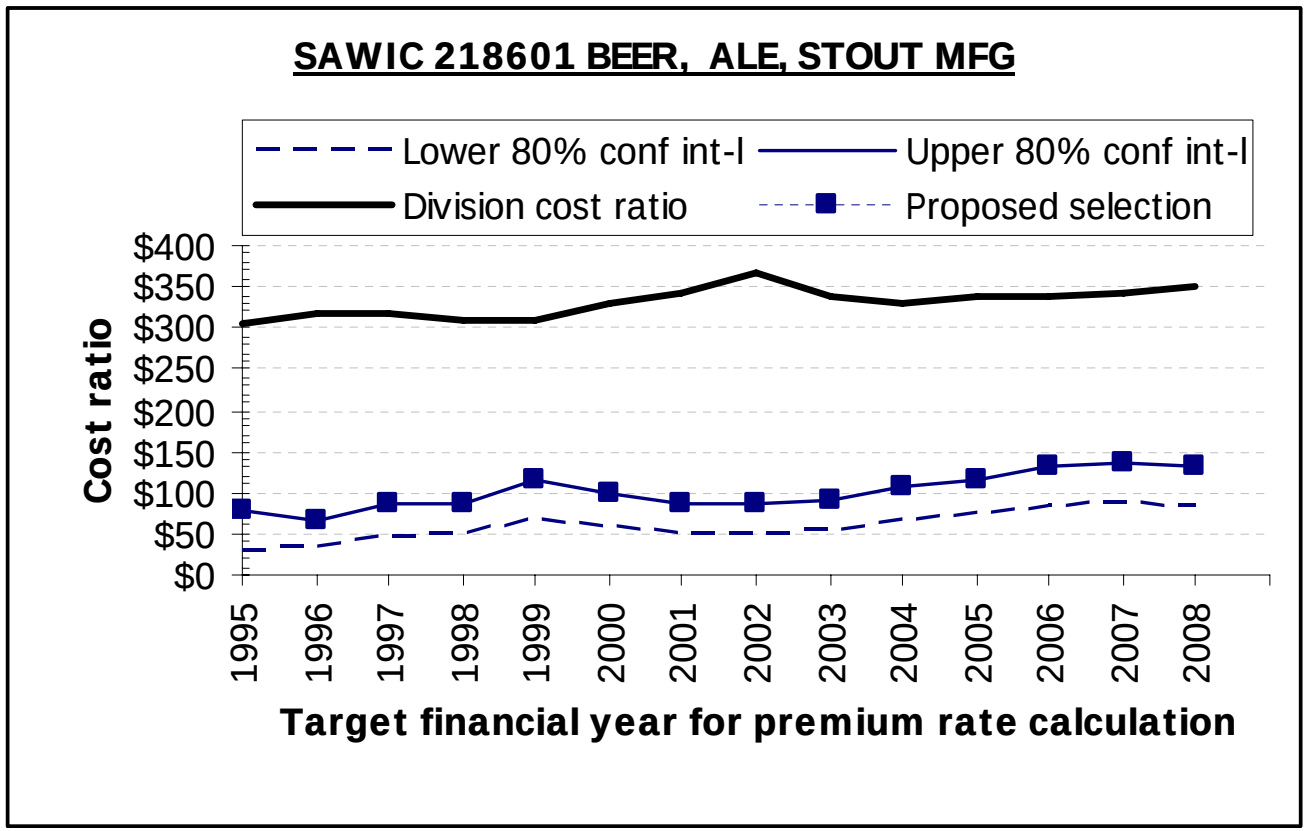
Minimal disturbance idea

- Set industry cost-ratio=industry division cost ratio, unless there is statistically significant difference.
- If they are significantly different, pick the value within the confidence interval and nearest to industry division cost ratio.

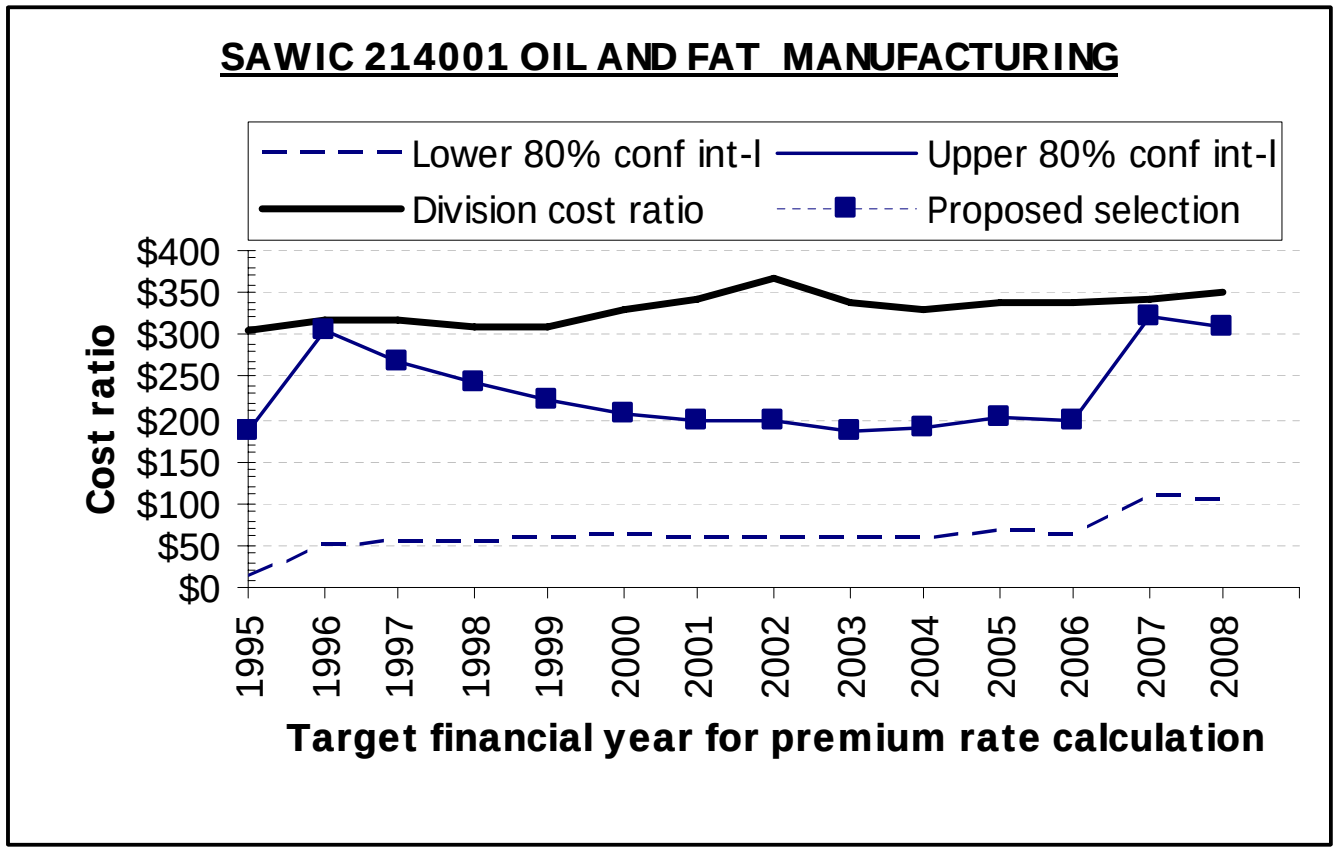
Minimal disturbance – example 1



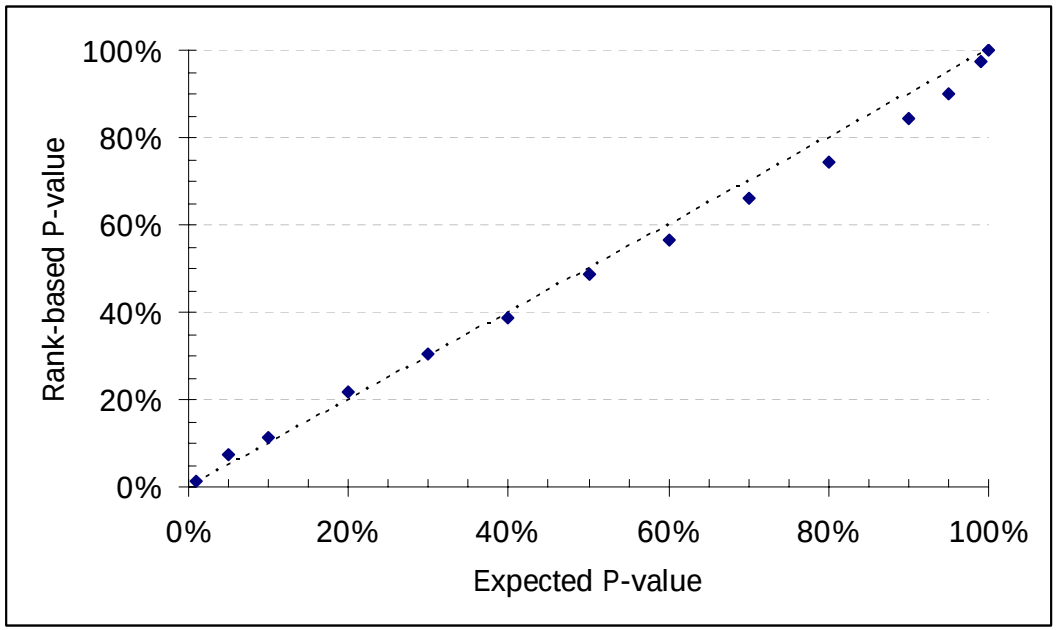
Minimal disturbance – example 2



Minimal disturbance – example 3

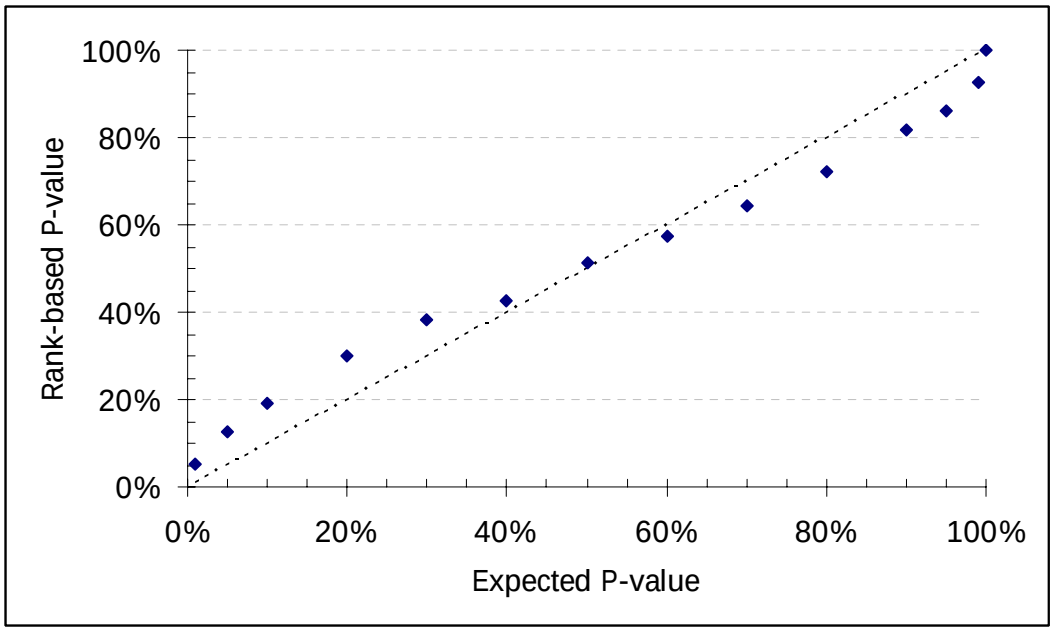


Validation of minimal disturbance method



Quantile-quantile plot for claim distribution parameters estimated from the minimum disturbance method. The minimum number of claims allowed for self-experience was set at 100.

Validation of minimal disturbance method



Quantile-quantile test when all industries are given industry group-average characteristics. It is seen that industry group-averages are, in general, poor estimators for individual industries.

Summary of results

1. Proposed an objective approach to select paid-up period length
2. Developed descriptive statistics of aggregate claims costs
3. Quantified the uncertainty of historical estimates
4. Proposed ‘adaptive experience period length’ method and validated it using quantile-quantile plots
5. Proposed ‘minimal disturbance approach’ for dealing with sparse data