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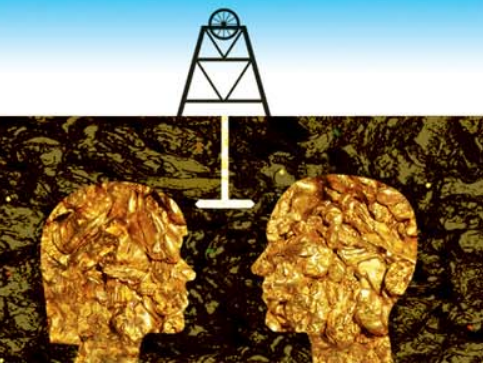


Stochastic Solvency Testing in Life Insurance

Genevieve Hayes

Deterministic Solvency Testing

- Assets > Liabilities
- In the insurance context, the values of the insurer's assets and liabilities are uncertain.
- This uncertainty should be allowed for in any insurer solvency calculations, but historically it has been ignored.

Stochastic Solvency Testing

- Involves determining probability distributions for A(t) and L(t) (or just C(t)).
- Insurer must hold an amount of capital at the valuation date sufficient to satisfy a probability-based criterion.

Stochastic Risk Measures

- x% VaR: Determine C(0) such that:

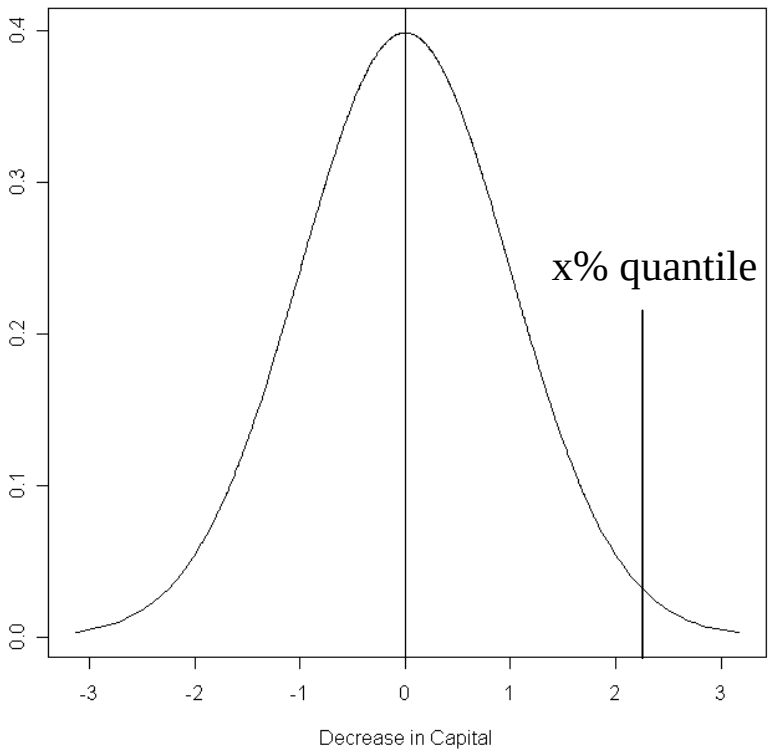
$$\Pr(C(t) > 0) = x\%$$

or $$\Pr(-\Delta C(t) < C(0)) = x\%$$

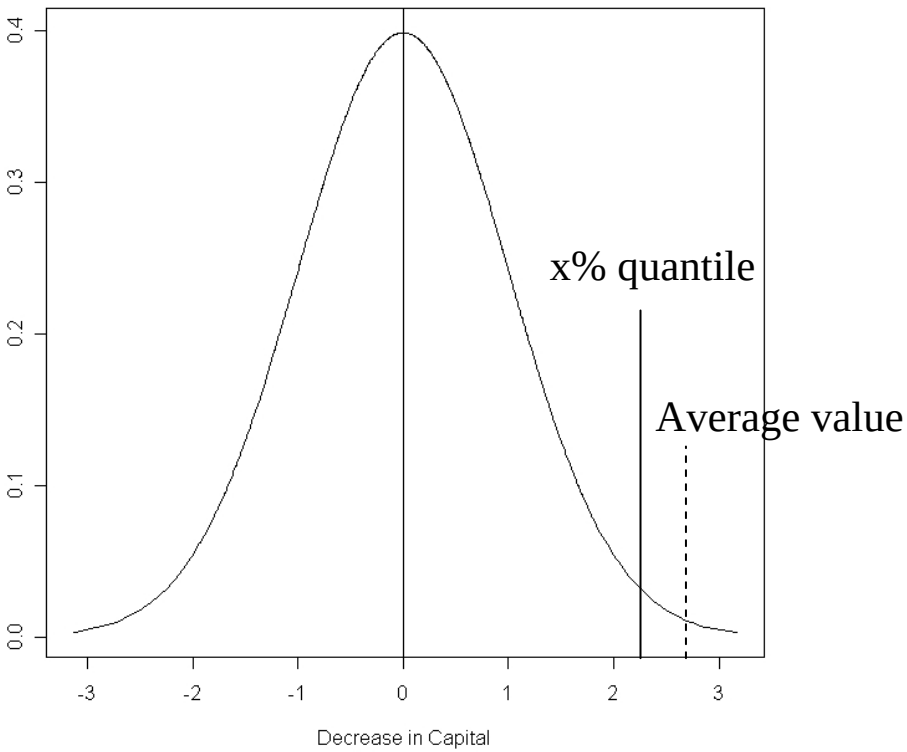
where $\Delta C(t) = C(t) - C(0)$

- x% TVaR = $E(-\Delta C(t) \mid -\Delta C(t) > x\%\ \text{VaR})$

Value at Risk (VaR)



Tail Value at Risk (TVaR)



Cost of Capital Risk Margins

- Allow for “the hypothetical cost of regulatory capital necessary to run off all of the insurance liabilities, following financial distress of the company”.
- Included so as to provide adequate risk compensation for a hypothetical insurer who may take over the portfolio in the future.

Australian LI Solvency Legislation

- Three actuarial valuation standards: LPS1.04, LPS2.04 and LPS3.04.
- No requirement is made for the actuary to use stochastic assumptions under any of these three standards.
- The probabilities of adequacy of the solvency and capital requirements are unknown.

Australian LI Solvency Legislation

- According to Karp (2002): “the solvency risk criterion was set at a 5% probability of assets falling below liabilities within any of the next three annual balance dates”.

Australian GI Solvency Legislation

- If the internal model based approach is used, the insurer must hold sufficient capital such that the insurer's probability of default over a one year time horizon is reduced to 0.5% or below.

International Association of Actuaries

- A reasonable period for the solvency assessment time horizon is one year.
- The amount of required capital must be sufficient with a high level of confidence (eg. 99%) to meet all future obligations.
- The most appropriate risk measure for solvency assessment is the TVaR.

Data – Policy Types

- Type 1: Whole of life/endowment insurance.
- Type 2: Unbundled policies (capital guaranteed and investment-linked).
- Type 3: Level term insurance.
- Type 4: Yearly-renewable term insurance.

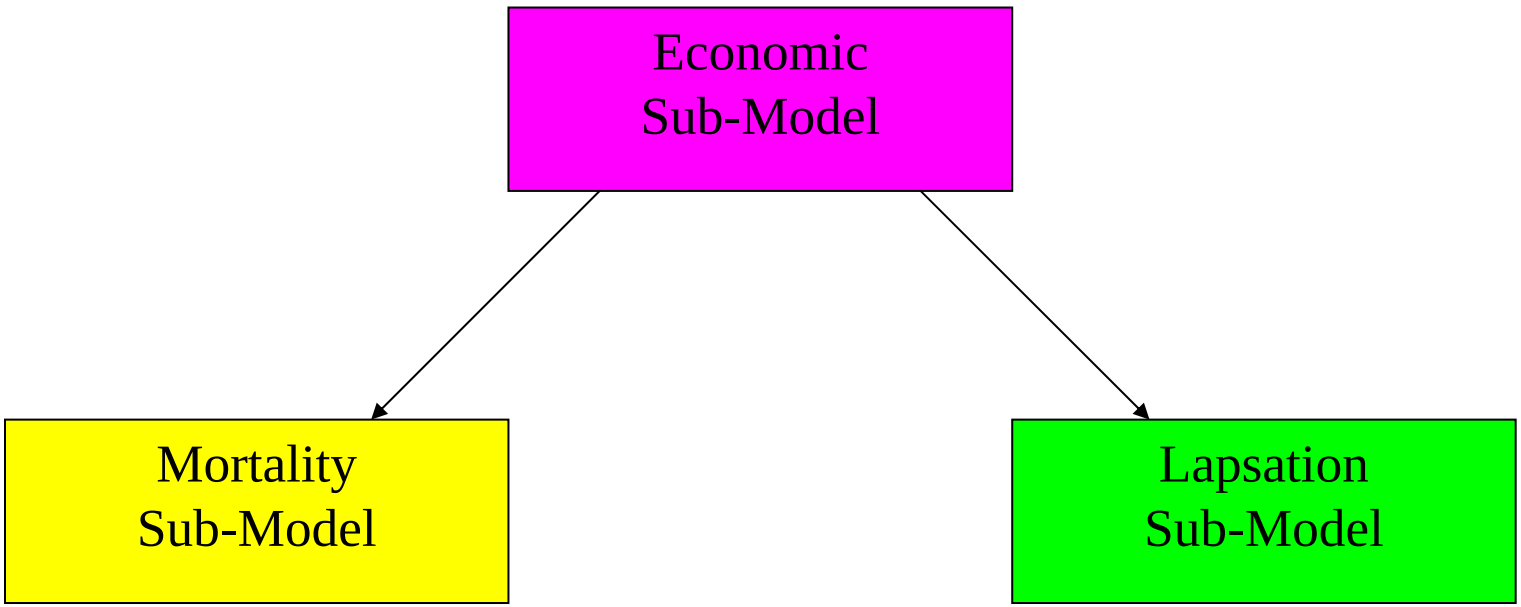
Stochastic Solvency Testing Model

- Compatible with the existing Australian valuation philosophy.
- Non-policy liabilities are ignored.
- Investment earnings, inflation, tax, expenses, mortality and policy discontinuance are all considered.
- Dependency relationships are considered.

Dependency Relationships

- Some evidence to indicate the presence of selective lapsation, but the evidence is inconclusive.
- Evidence to suggest a significant relationship exists between fluctuations in the short-term interest rate and mortality.
- Evidence to suggest a significant relationship exists between fluctuations in economic variables and lapsation.

Dependency Relationships



Stochastic Sub-Models

- Economic:
  - modified CAS/SOA model.
- Mortality:
  - Negative binomial distribution for Type 1 policies.
  - Poisson distribution for all other policy types.
- Lapsation:
  - Normal-Poisson model for all policy types.

Deterministic Capital Requirements

- Solvency Capital Requirement =
  LPS2.04 Solvency requirement – BEL

- Capital Adequacy Capital Requirement =
  LPS3.04 Cap. Ad. requirement – BEL

Stochastic Capital Requirements

- 99.5% VaR of the change in capital distribution over a one year time horizon.
- 99.5% TVaR of the change in capital distribution over a one year time horizon.
- 95% VaR of the change in capital distribution over a three year time horizon.
- 95% TVaR of the change in capital distribution over a three year time horizon.

Stochastic Asset Requirements

- Stochastic Minimum Asset Requirement (SMAR) =

  Best estimate liability

  + Cost of capital risk margin

  + Solvency capital requirement

- Similar to the minimum asset requirement under the Swiss Solvency Test.

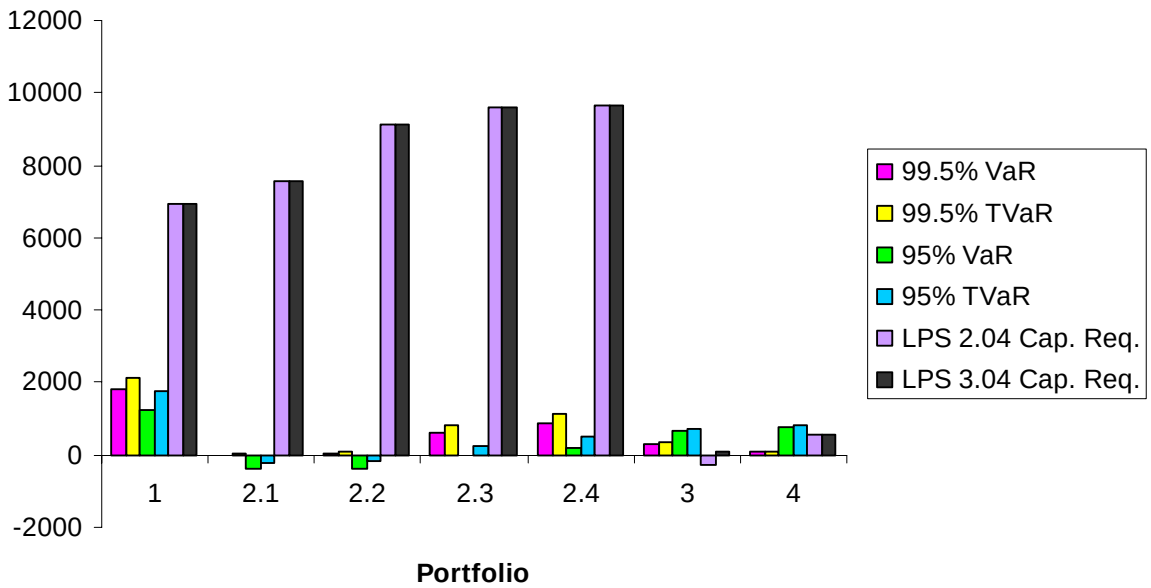
Model Asset Portfolios

- Composition of each of the Model Asset Porfolios:

	Portfolio 1	Portfolio 2	Portfolio 3	Portfolio 4
Equity	0%	25%	55%	75%
Property	0%	5%	5%	5%
Fixed Interest	70%	45%	30%	15%
Cash	30%	25%	10%	5%
Total	100%	100%	100%	100%

Base Case Simulation Results

- Capital Requirements per Policy for the Base Case Scenarios ($)



Base Case Simulation Results

- Levels of Sufficiency (on a VaR Basis) of the LPS2.04 and LPS3.04 Capital Requirements for the Base Case Scenarios

Liability Portfolio	Asset Portfolio	$-\Delta C(1)$		$-\Delta C_{min}(0,3)$	
		LPS2.04	LPS3.04	LPS2.04	LPS3.04
1	3	>99.99%	>99.99%	>99.99%	>99.99%
2	1	>99.99%	>99.99%	>99.99%	>99.99%
2	2	>99.99%	>99.99%	>99.99%	>99.99%
2	3	>99.99%	>99.99%	>99.99%	>99.99%
2	4	>99.99%	>99.99%	>99.99%	>99.99%
3	1	1.36%	72.45%	0.00%	1.19%
4	1	>99.99%	>99.99%	0.00%	0.00%

Sensitivity Analysis

1. Using a different economic sub-model.
2. Ignoring mortality over-dispersion.
3. Ignoring lapsation over-dispersion.
4. Ignoring mortality *and* lapsation over-dispersion.
5. Ignoring mortality *and* lapsation over-dispersion *and* dependency relationships.
6. Simplifying the formulae used to determine the mean mortality and lapsation rates.

Sensitivity Analysis Results

- SMAR are not significantly affected by ignoring mortality or lapsation over-dispersion or the dependency relationships between the sub-models.
- SMAR tend to be higher for Type 3 and 4 policies if calculated using an alternative economic sub-model.
- SMAR tend to be lower for Type 3 and 4 policies if calculated using simplified mean formulae.

Sensitivity Analysis Results

- In all cases, it is still true that:
  - for Type 1 and 2 policies, deterministic capital requirements are much greater than the stochastic capital requirements.
  - for Type 3 policies the LPS2.04 solvency requirements are less than the stochastic capital requirements.
  - for Type 4 policies, the deterministic capital requirements are greater than the 99.5% VaR and TVaR, but less than the 95% VaR and TVaR.

Implications

- For Type 1 and 2 policies, the LPS2.04 and LPS3.04 requirements are unnecessarily high.
- For Type 3 and 4 policies, the LPS2.04 and LPS3.04 requirements are too low.

Suggested Actions

- Increase the deterministic solvency requirements for portfolios containing Type 3 or 4 policies.
- Move from a deterministic solvency capital calculation regime to a stochastic regime.