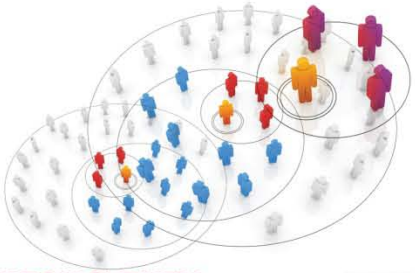
Institute of Actuaries of Australia

BIENNIAL CONVENTION 2011

BEYOND THE MANDATE



10 – 13 APRIL 2011 • HILTON SYDNEY

Replicating Portfolios and Risk Management

Presented by Joshua Corrigan, Charles Qin

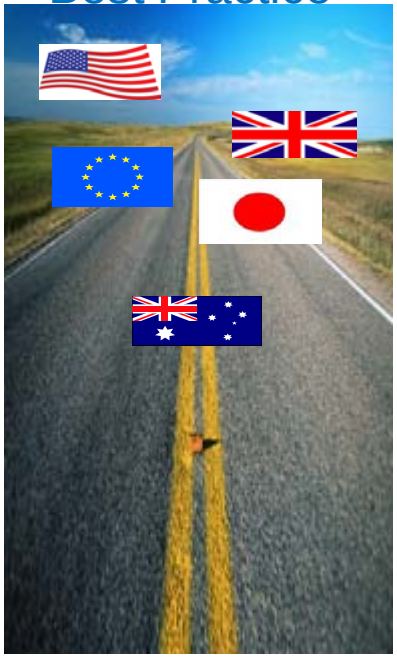
Co-authored by Gene Leynes, Chris Onken

Contents

Contents

Risk Management

Best Practice

Dynamic Hedging		Real World Replication
Static Hedging		Replicating Portfolios
Matching		Market Consistent
Traditional ALM		Risk Neutral
Asset Liability Segregation		Real World Prudent

Historic / Old World

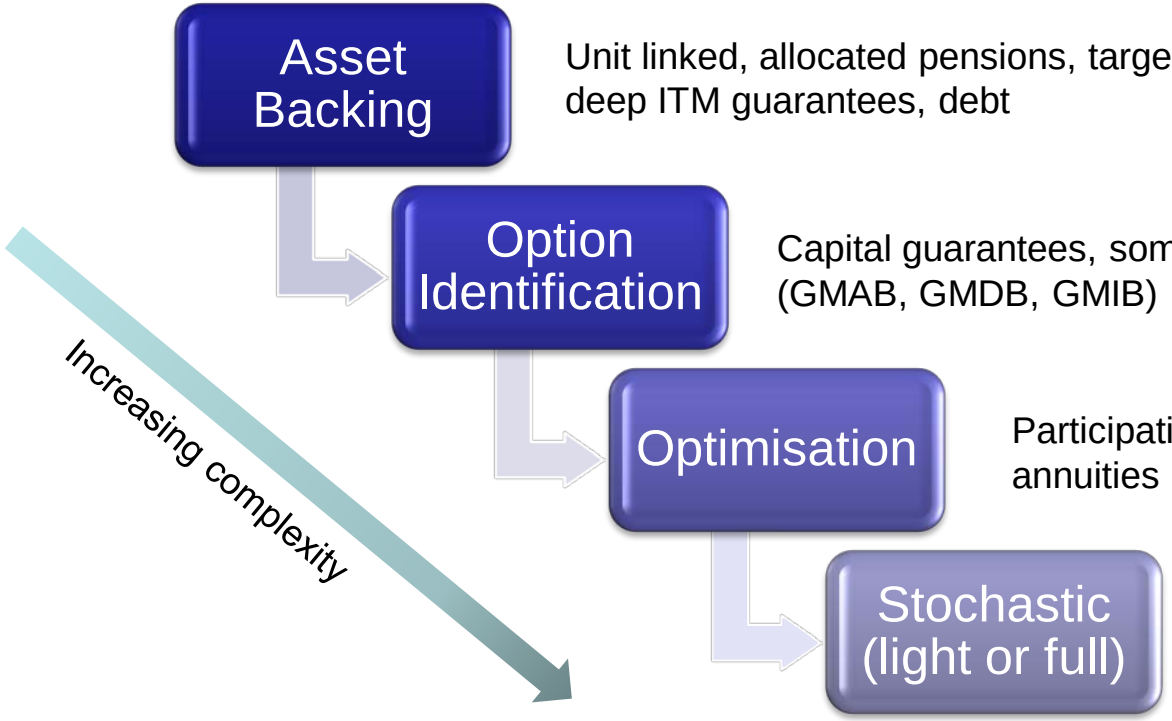
Capital Management
APRA Pillar 2: ICAAP Requirements

"To maintain at all times a level and quality of capital commensurate with the level and extent of risks to which the insurer is exposed from its activities"

"To have adequate systems and procedure for identifying, measuring, monitoring and managing the risks arising from its activities on a continuous basis, and for assessing the capital needed in relation to those risks"

Frequent A&L valuations → Computational Limitations → Fast valuations needed → RPs

Four Replicating Approaches



Unit linked, allocated pensions, target date funds, annuities, deep ITM guarantees, debt

Capital guarantees, some simple variable annuities (GMAB, GMDB, GMIB)

Participating business, some variable annuities

Most variable annuities

Contents

Definition

- A replicating portfolio is **an asset portfolio** consisting of:
  - **Real assets with liquid market prices**
  - **Real illiquid assets**
  - **Theoretical assets**

  that replicates the scenario-dependent cash flows of an asset or liability

Theoretical static assets → Theoretical dynamic assets → Real world dynamic hedging

Theoretical Framework

Principle of No Arbitrage

- If two instruments or portfolios have identical cash flows under all economic conditions...
- then the prices of the two instruments must be equal

Implementation of Replicating Portfolios

- If the cash flows of a portfolio of assets and the liabilities are identical under the base and stressed economic scenarios...
- then the market value of the liabilities must equal that of the asset portfolio

Illustrative Example 1

1.00 unit

1.5-years fixed coupon bond

6% semi-annual coupon

0.03 units

6-months ZCB

0.03 units

12-months ZCB

1.03 units

18-months ZCB

Illustrative Example 2

1.00 unit
Equity index forward
Forward price at *K*
Maturity *T*

1.00 unit
European equity call option
Strike *K*
Maturity *T*

-1.00 unit
European equity put option
Strike *K*
Maturity *T*

Contents

Different Methodologies

The OLS (ordinary least square) model

- Asset selection, optimisation, diagnostics

The GAM (generalised additive model) model

- GAM fitting, segmentation, asset selection, diagnostics

The equity replication model

- Parametric fitting, piecewise replication, diagnostics

The OLS Model

Asset Selection

- Applicable to each individual time step
- Select candidate assets based on the liability profiles

Optimisation

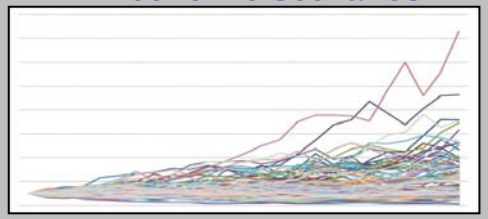
- Applicable to liability cash flows or the net present values
- Choose asset portfolio to minimise overall error, for both base and stressed scenarios, subject to constraints

Diagnostics

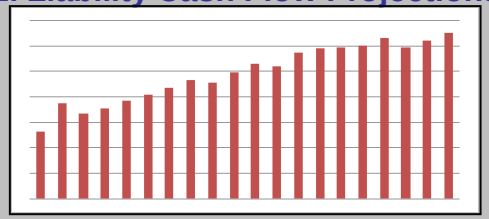
- Goodness of fit measured with standard statistics, e.g. R^2
- Out-of-sample tests performed with additional economic scenarios

The OLS Model

1. Economic Scenarios



2. Liability Cash Flow Projections



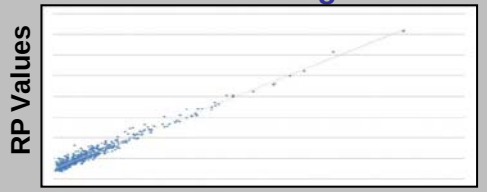
3. Candidate Asset Selection

Liability Cash Flows	Candidate Assets
Guaranteed interest rates	Zero-coupon bonds
Maturity payments	Stock derivatives
Annuities	Swaptions

4. Optimisation

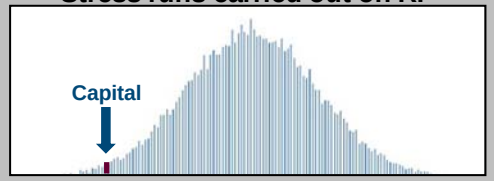
$$\begin{cases} \min\limits_{x_k \forall k} \sum_{t=1}^{T} \sum_{s=1}^{S} \left(cf^{s}_{Liabilities}(t) - \sum_{k=1}^{N} x_k cf^{s,k}_{Assets}(t) \right)^2 \\ \text{Subject to: RP Total Value} = \text{Simulated FV Liabilities} \end{cases}$$

5. Validation & Diagnostics



6. Solvency & Capital Requirements

Stress runs carried out on RP



The GAM Model

GAM Fitting

- Fit GAM curves to (base & stressed) liability cash flows at each time step
- Apply optimisation, which balances between both goodness of fit and smoothness (robustness)

Segmentation

- Segment / discretise the GAM curves
- Can be achieved via clustering or optimisation

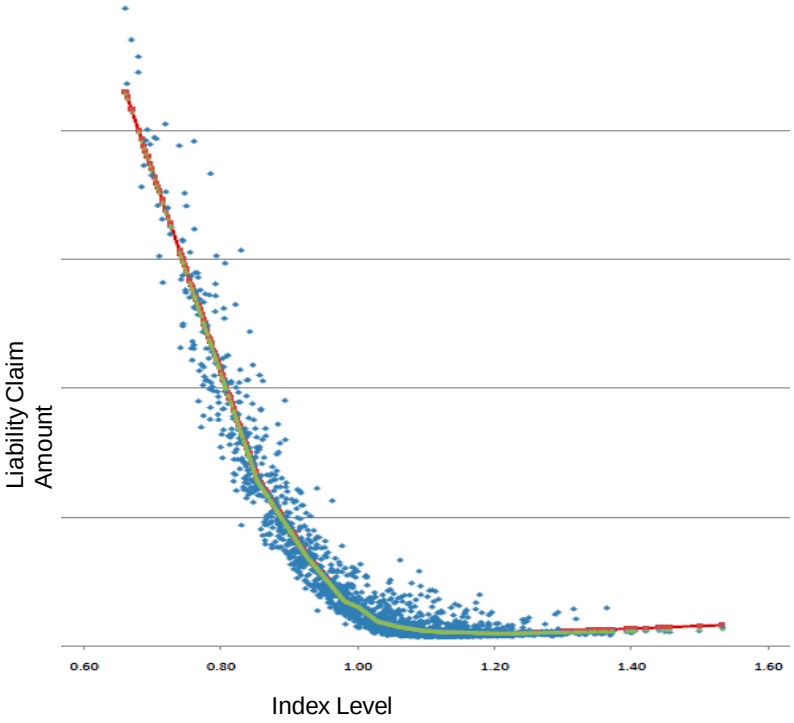
Asset Selection

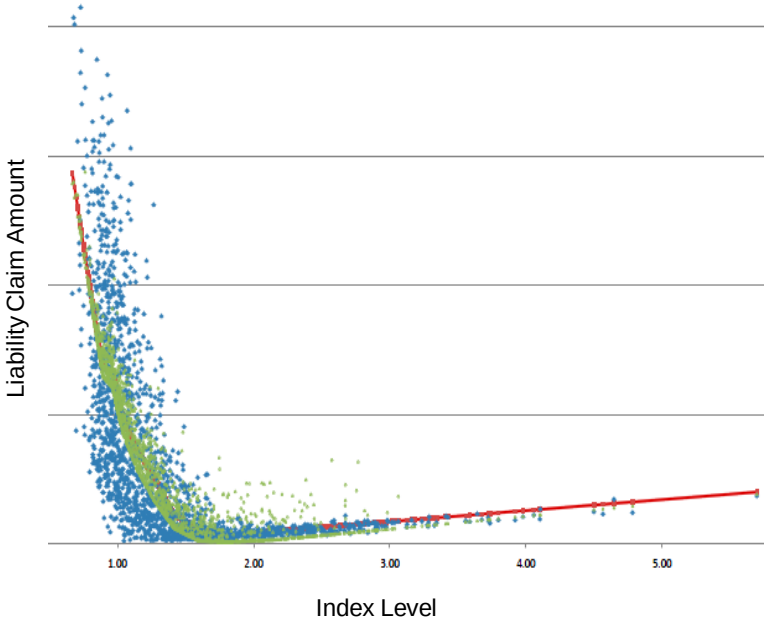
- Applicable to each individual time steps
- Choose appropriate replicating assets based on the segmented curves
- Determine final assets via optimisation on the GAM curves

Diagnostics

- Goodness of fit measured with standard statistics, e.g. generalised cross validation (GCV) and R^2
- Out-of-sample tests performed with additional economic scenarios

The GAM Model





The Equity Replication Model

Parametric Fitting

- Models net assets (assets less liabilities) at next time step as a multivariate polynomial of pre-selected risk factors
- Determine coefficients via optimisation, by utilising both normal and extreme economic scenarios

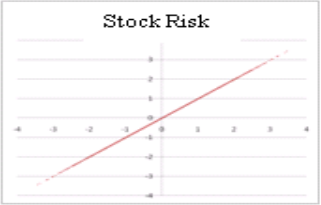
Piecewise Replication

- Construct a portfolio of instruments to replicate each term of the multivariate polynomial via optimisation
- Combine with polynomial coefficients to establish replicating portfolio for the entire liability profile

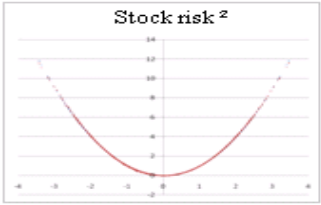
Diagnostics

- Goodness of fit measured with standard statistics, e.g. R^2
- Out-of-sample tests performed with additional economic scenarios

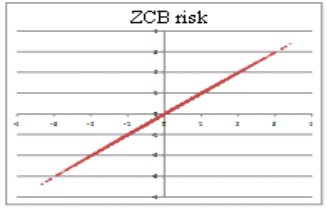
The Equity Replication Model



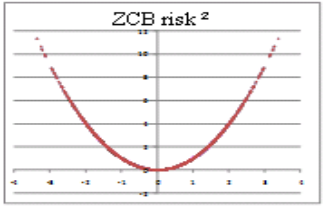
Sub portfolio consisting of **cash** and **stocks** (**equity derivatives** for convexity adjustment)



Sub portfolio consisting of **equity put** and **call options**



Sub portfolio consisting of **cash** and **zero-coupon bonds**



Sub portfolio consisting of **zero-coupon bonds**, **interest rate caps** / **floors**

GAM Preferred Method

- The **GAM model** is our preferred method, based upon years of implementation and experience.
  - Not impacted by **outliers in data** and **more robust** (compared to the standard model)
  - Allow **replication of individual liability items** (compared to the equity replication model)
  - GAM framework well understood and is **supported by most standard statistical softwares**
  - **Cross terms** may be **difficult to replicate** under the equity replication approach
  - **Optimisation** problem more **plausible** (compared to the standard model)

Contents

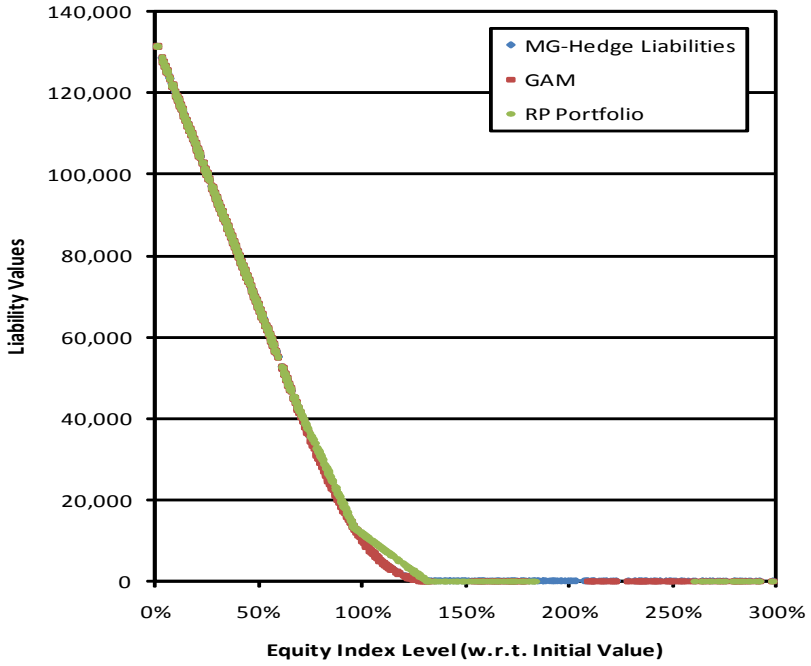
Case Studies

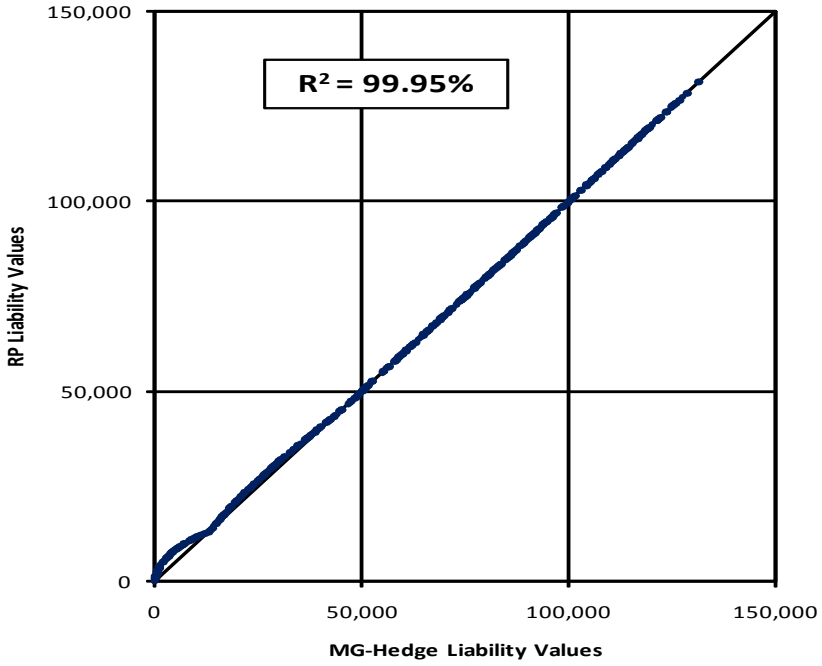
Simple Guaranteed Minimum Accumulation Benefit (Simple GMAB)	• To illustrate the validity of the concept • Product guarantees a minimum investment return over a fixed investment horizon without the real-world features
Complex Guaranteed Minimum Accumulation Benefit (Complex GMAB)	• To assess the effectiveness of the replication of a real-world investment guarantee • Product guarantees a minimum investment return over a fixed investment horizon
Range Forward (a.k.a. Cylinder, Tunnel or Collar)	• To illustrate some shortfalls of the GAM approach • Consists of a short European put option and a long European call option

Case Study – Simple GMAB

- A portfolio of 10 male policies each with $100,000 of account value
- Age profiles range from 35 to 85
- Death benefit (GMDB) available
- Term of GMAB = 20 years
- Range of guaranteed balances from $70,000 (out-the-money) to $130,000 (in-the-money)
- Deterministic lapse rates
- No roll-ups nor ratcheting of the guaranteed benefits

Case Study – Simple GMAB





Case Study – Simple GMAB

Liabilities	MG-Hedge Values	RP Values
Bond		6
European Options		56,118
Total Base Values	**56,933**	**56,124**
-20% Equity Stress	75,561	74,424
Capital Strain	**18,629**	**18,300**

Delta	MG-Hedge Values	RP Values
Delta	**773**	**761**

Rho	MG-Hedge Values	RP Values
0-1 Year	3	4
2-5 Year	13	13
6-10 Year	21	21
11-20 Year	91	90
21-30 Year	0	0
Parallel	**128**	**127**

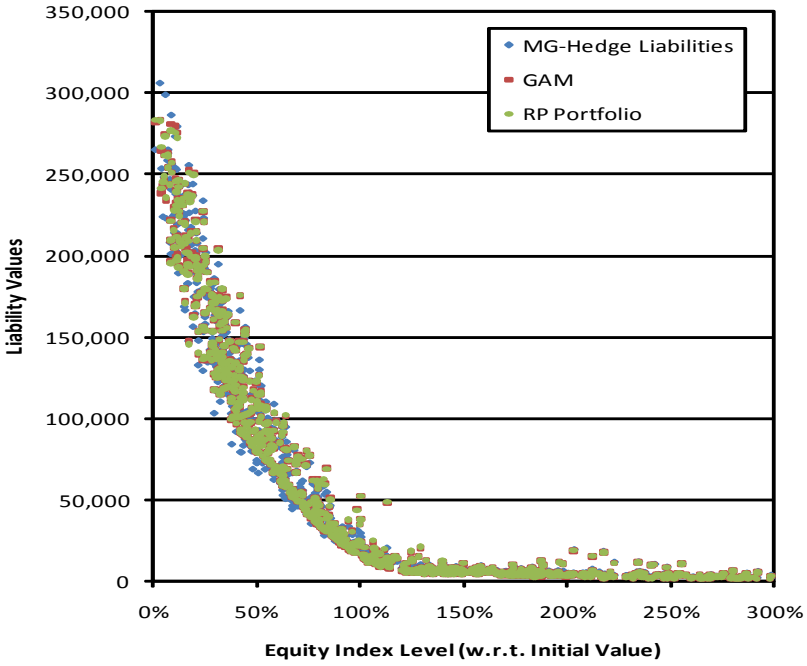
Vega	MG-Hedge Values	RP Values
1-2 Year	284	174
3-4 Year	176	289
5-6 Year	228	303
7-8 Year	417	299
9-10 Year	1352	1293
Parallel	**2,457**	**2,359**

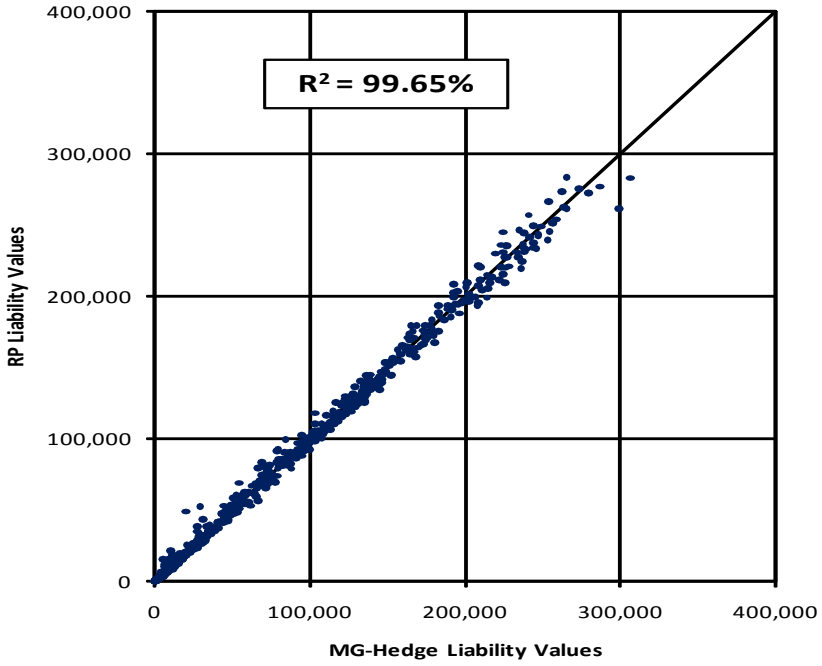
- Replicating portfolio consists of only European options and zero-coupon bonds
- Excellent replication of base liability values
- Minimal errors in the sensitivities to key risk variables
- Very accurate capital strain from equity stress

Case Study – Complex GMAB

- A portfolio of 10 male policies each with $100,000 of account value
- Age profiles range from 35 to 85
- Term of GMAB = 20 years
- Range of guaranteed balances from $70,000 (out-the-money) to $130,000 (in-the-money)
- Dynamic lapse rates assumed
- Roll-ups (0% - 9% p.a.) and ratcheting of the guaranteed death benefits (GMDB)

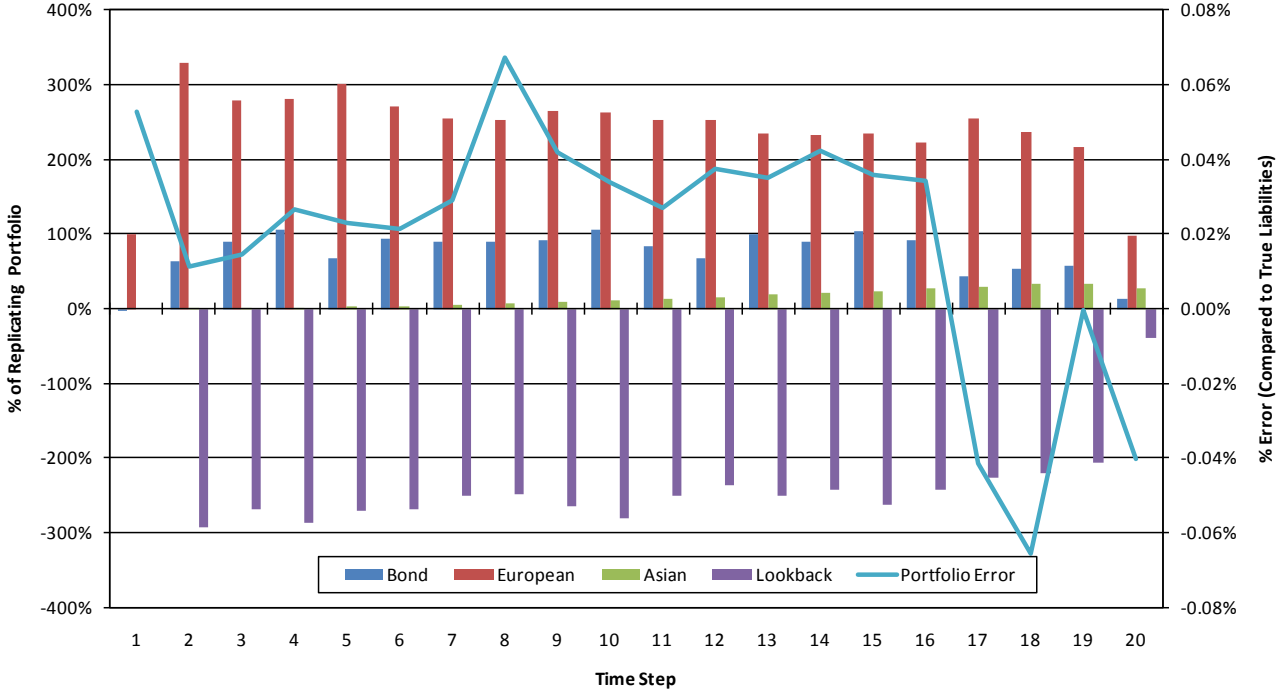
Case Study – Complex GMAB





Case Study – Complex GMAB

Case Study – Complex GMAB



Case Study – Complex GMAB

Liabilities	MG-Hedge Values	RP Values
Bond		116,948
European Options		368,834
Asian Options		22,067
Lookback Options		-346,121
Total Base Values	**164,800**	**161,729**
-20% Equity Stress	192,287	188,423
Capital Strain	**27,487**	**26,694**

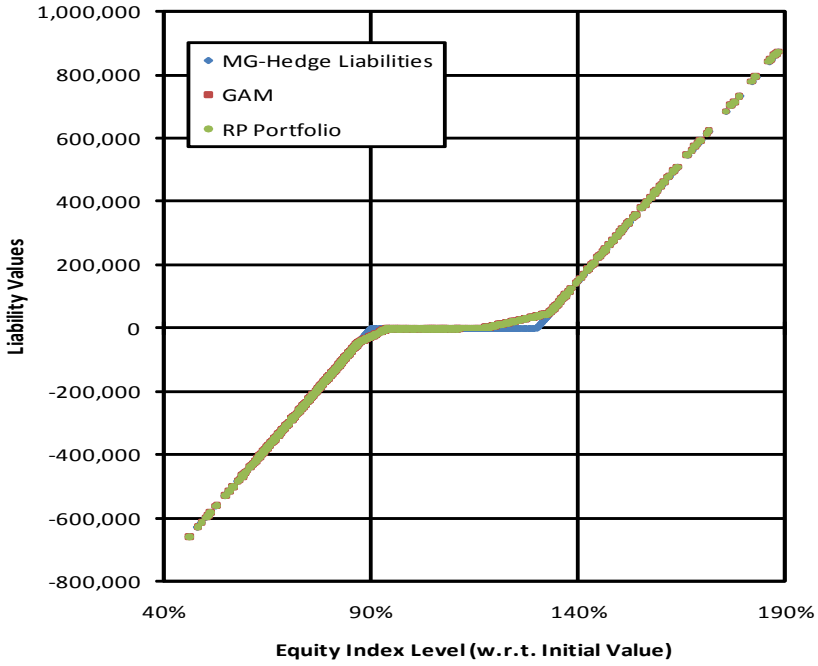
Delta	MG-Hedge Values	RP Values
Delta	**1,026**	**1,007**

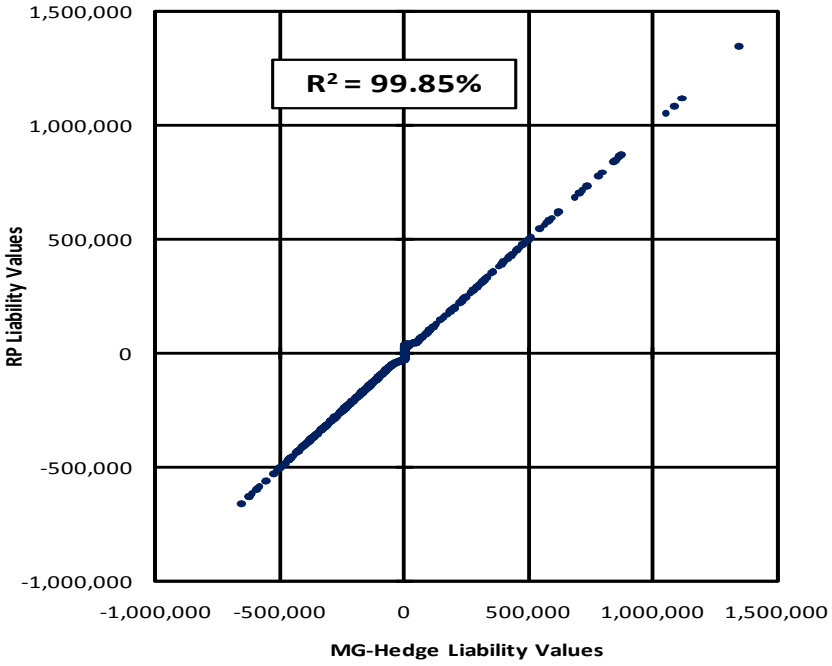
Rho	MG-Hedge Values	RP Values
0-1 Year	3	0
2-5 Year	20	17
6-10 Year	80	89
11-20 Year	203	199
21-30 Year	0	0
Parallel	**306**	**306**

Vega	MG-Hedge Values	RP Values
1-2 Year	535	475
3-4 Year	370	721
5-6 Year	733	807
7-8 Year	889	820
9-10 Year	3403	3187
Parallel	**5,931**	**6,010**

- Asian and look-back equity options required for replicating dynamic lapses and ratchets
- Excellent replication of base liability values
- Minimal errors in the sensitivities to key risk variables
- Very accurate capital strain from equity stress

Case Study – Range Forward





Case Study – Range Forward

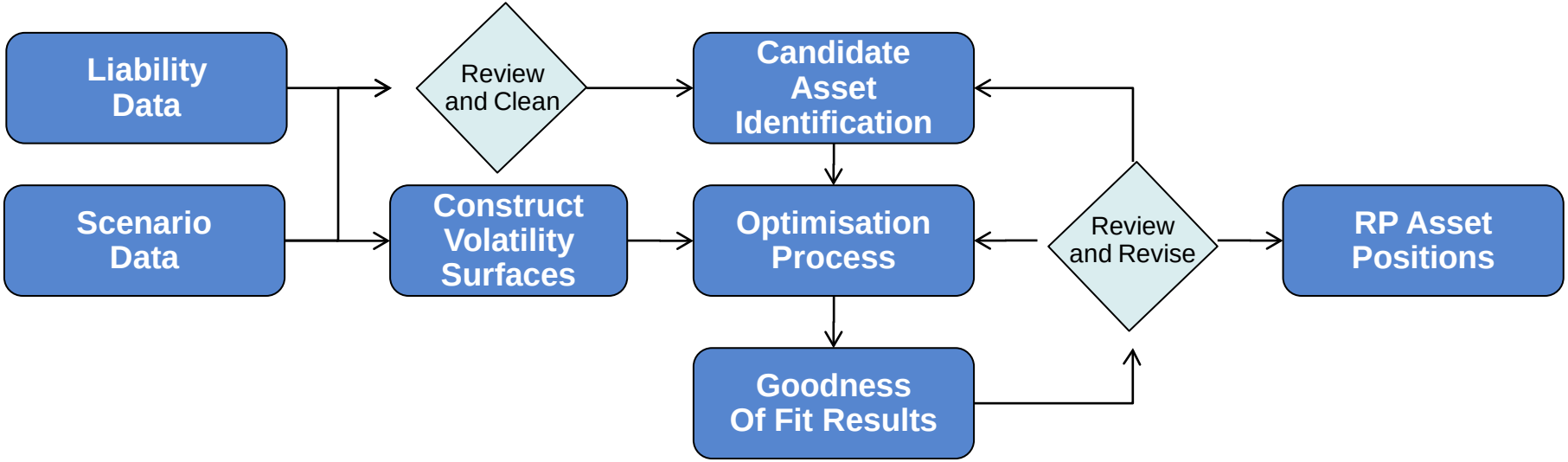
- GAM model smoothes input data
- More knots are required to produce unsmooth payoffs
- Replication still excellent – both fair value and risk sensitivities, i.e. replicating portfolio not unique!

Range-Forward	MG-Hedge Values	RP Values
Fair Value	-42,946	-41,580
Delta	-7,598	-7,699
Rho	-80	-81
Vega	-748	-737

Contents

Implementation Requirements

- **Systems**: data collection, pre-processing, optimisation
- **Processes**: data review, asset selection, optimisation design, results review, signoff



Challenges

- Business requirements
  - Scope: single / multiple blocks, full balance sheet
  - Characteristics: accuracy, frequency, level of detail
- Choice of most appropriate method
- Timelines of data dependencies
- Robustness versus fit trade-off
- Goodness of fit thresholds of acceptability

Expertise and experience can make all the difference

Conclusion

- **Replicating portfolios have numerous applications:**
  - Risk management: assessment, hedging,
  - Capital management: APRA Pillar 2 requirements
  - Internal MI and education
- **Suite of methods and tools available**
- **Implementation challenges exist but can be overcome with suitable experience and expertise**

Thank You and Questions

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