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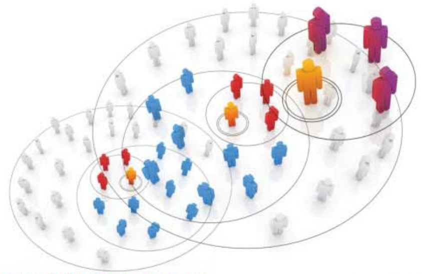
BEYOND THE MANDATE



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Heterogeneity of Australian Population Mortality, and Implications for a Viable Life Annuity Market

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Agenda

- Introduction
- Model Specification
  - Frailty Model
  - Markov Aging Model
- Data
- Results
  - Model Fitting
  - Impact on Annuity Rates
- Conclusion

Background

- Risks in an insured portfolio are heterogeneous
  - Ignoring heterogeneity: adverse selection
  - Risk rating using risk factors
- Life Annuity
  - Limited use of rating factors
  - Demand for annuity products
  - Understanding heterogeneity
    - Pricing and adverse selection in annuity business
    - Underestimation of mortality improvement at higher ages

Research Objective

- Quantifying mortality heterogeneity of Australian population
  - Frailty model (Vaupel et al. 1979)
  - Markov aging model (Lin and Liu 2007)
- Projection of mortality rates at higher ages, taking into account heterogeneity
- Impact of allowing for heterogeneity on life annuity pricing
  - Difference in annuity prices between heterogeneous lives
  - Impact on annuity business

Frailty Factor

- An unobserved mortality risk factor, fixed at birth
- Mathematically defined in terms of force of mortality:

$$\mu(x, z) = z \cdot \mu(x, 1)$$

- Assumed form of standard force of mortality and frailty distribution
  - Standard force of mortality $\mu(x, 1) = \alpha \cdot e^{\beta x}$
  - Frailty distribution
    - Gamma
    $$f_Z(z) = \frac{\lambda^k}{\Gamma(k)} \cdot z^{k-1} \cdot e^{-\lambda \cdot z}$$
    - Inverse Gaussian
    $$f_Z(z) = (\frac{\delta}{\pi})^{\frac{1}{2}} \cdot e^{\sqrt{4\delta\theta}} \cdot z^{-\frac{3}{2}} \cdot e^{-\theta z - \frac{\delta}{z}}$$

Frailty Model

- Distribution of frailty at age x
  - Gamma distribution

$$f_{Z|X}(z|X=x) = \frac{(\lambda(x))^k}{\Gamma(k)} \cdot z^{k-1} \cdot e^{-\lambda(x)\cdot z}$$

with

$$\lambda(x) = \lambda + H(x,1), \text{ and } E[z] = \frac{k}{\lambda(x)}, \quad Var[z] = \frac{k}{(\lambda(x))^2}$$

  - Inverse Gaussian distribution

$$f_{Z|X}(z|X=x) = (\frac{\delta}{\pi})^{\frac{1}{2}} \cdot e^{\sqrt{4\delta\theta(x)}} \cdot z^{-\frac{3}{2}} \cdot e^{-\theta(x)\cdot z - \frac{\delta}{z}}$$

with

$$\theta(x) = \theta + H(x,1), \text{ and } E[z] = (\frac{\delta}{\theta(x)})^{\frac{1}{2}}, \quad Var[z] = \frac{1}{2}\sqrt{\frac{\delta}{(\theta(x))^3}}$$

Maximum Likelihood Estimation

- Mean frailty approach
  - Assumes the average force of mortality is the cohort force of mortality

$$\bar{\mu}_x = \mu(x,1) \cdot \bar{z}_x$$

  - Does not take into account the impact of heterogeneity on the variability
- Normal approximation for sample mean mortality rates
  - The observed cohort is a sample of size Ex of the population
  - According to CLT, the observed sample mean force of mortality is normally distributed
    - Gamma:

$$E[\hat{\mu}_x] = \mu(x,1) \cdot \frac{k}{\lambda + H(x,1)}, \quad Var[\hat{\mu}_x] = \frac{(\mu(x,1))^2 \cdot k}{E_x \cdot (k + H(x,1))^2}$$

    - Inverse Gaussian

$$E[\hat{\mu}_x] = \mu(x,1) \cdot \left(\frac{\delta}{\theta + H(x,1)}\right)^{\frac{1}{2}}, \quad Var[\hat{\mu}_x] = \frac{(\mu(x,1))^2}{2 \cdot E_x} \sqrt{\frac{\delta}{(\theta + H(x,1))^3}}$$

Markov Aging Model

- Aging process can be modeled in terms of changes in physiological functions
- Studies in human body functions reveal that functional variables decline roughly linearly after age 30
- Physiological age: represent the degree of aging in human body
  - Change in physiological age represents the decline in human body function
  - High physiological age represents higher probability of dying
  - Mathematically, transition is random in nature

Markov Aging Model

- Markov process with n transient states and 1 absorbing death state, describing the aging process of human beings

$$\Lambda = \begin{pmatrix} -(\lambda_1+q_1) & \cdots & 0 & 0 & \cdots & 0 \\ \cdots & \cdots & -(\lambda_k+q_k) & \lambda_k & \cdots & 0 \\ 0 & \cdots & 0 & -(\lambda+\gamma+\alpha e^{\beta(k+1)}) & \cdots & 0 \\ 0 & \cdots & 0 & 0 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 0 & \cdots & -(\alpha+e^{\beta n}) \end{pmatrix}$$

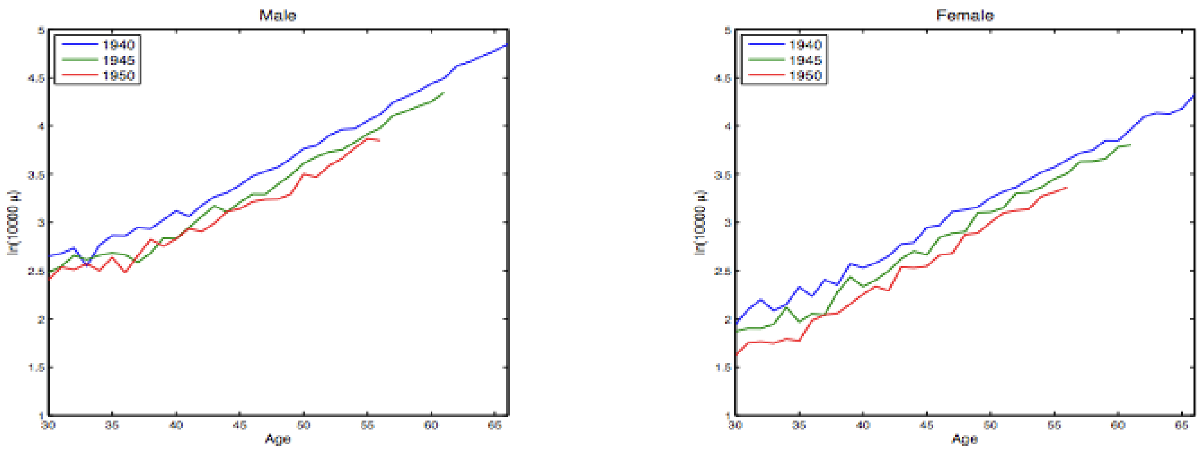
- $\lambda_i = \lambda$ for $i = 5, 6, \ldots n$
- Death rates for $i = 5, 6, \ldots, n-1$

$$q_i = \begin{cases} \gamma + \gamma_1 + \alpha e^{\beta i} & : \text{ for } i_1 < i < i_2 \\ \gamma + \alpha e^{\beta i} & : \text{ otherwise} \end{cases}$$

- Time to death follows phase-type distribution with $\hat{S}(t) = \alpha exp(\Lambda t)e$
- $\hat{q}_x = \frac{\hat{S}_x - \hat{S}_{x+1}}{\hat{S}_x}$
- Weighted least square estimation: $\sum_x (q_x - \hat{q}_x)^2 \cdot w_x$

Data for Frailty Model

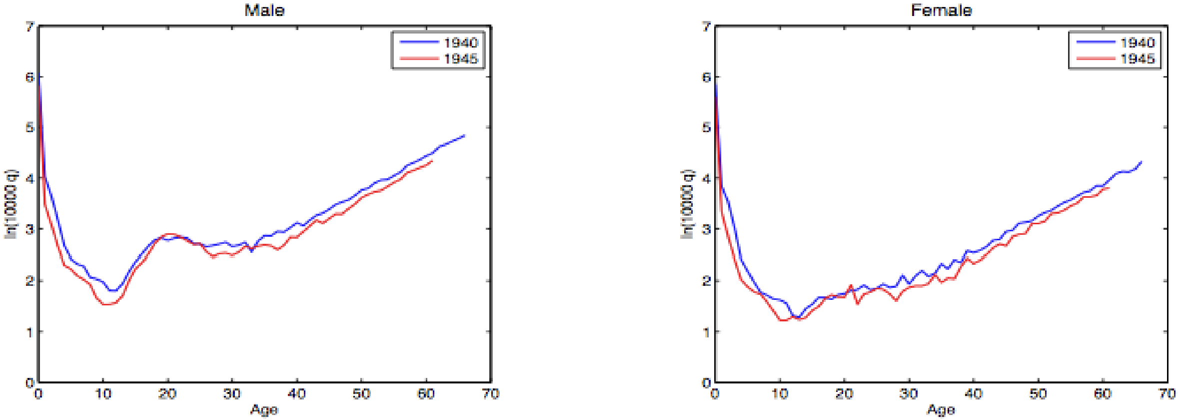
Figure: Observed Cohort Force of Mortality: Log Transform



Note: Cohort force of mortality is estimated by the central death rate for birth cohort 1940, 1945, and 1950, both male and female

Data for Markov Aging Model

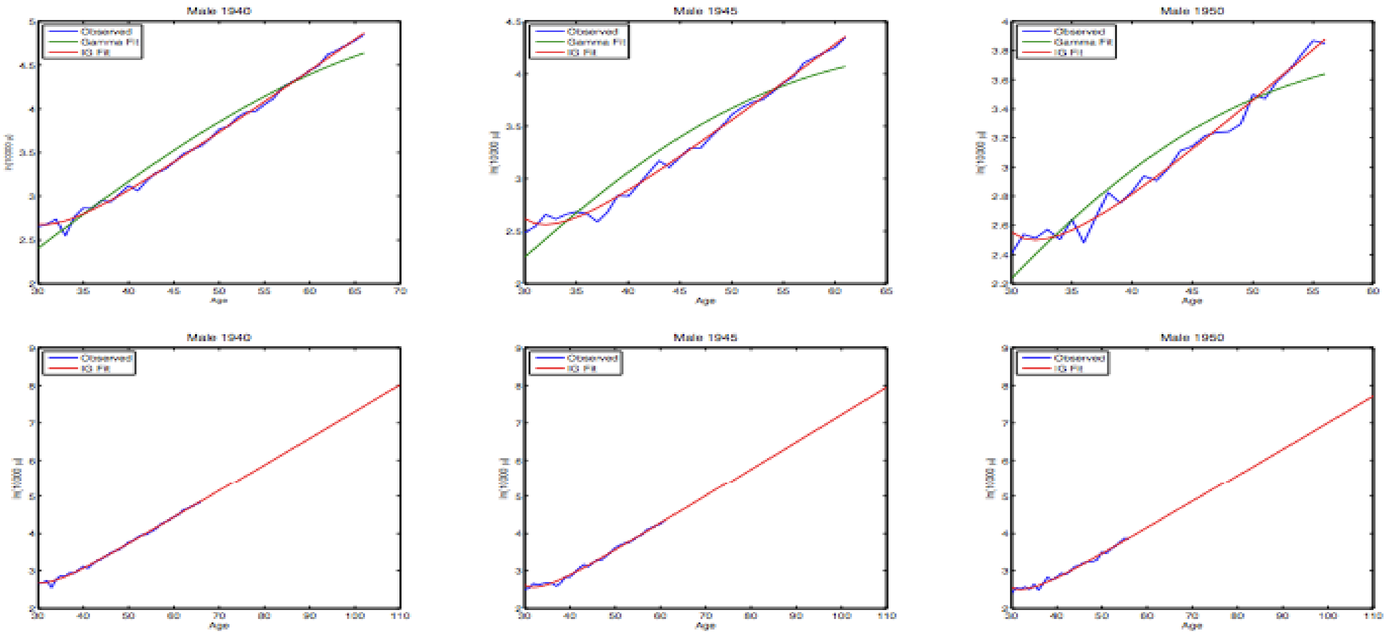
Figure: Observed Cohort Death Probability: Log Transform



Note: Observed death probability is estimated from central death rate $q_x = \frac{m_x}{1+1/2m_x}$ for birth cohort 1940, and 1945, both male and female

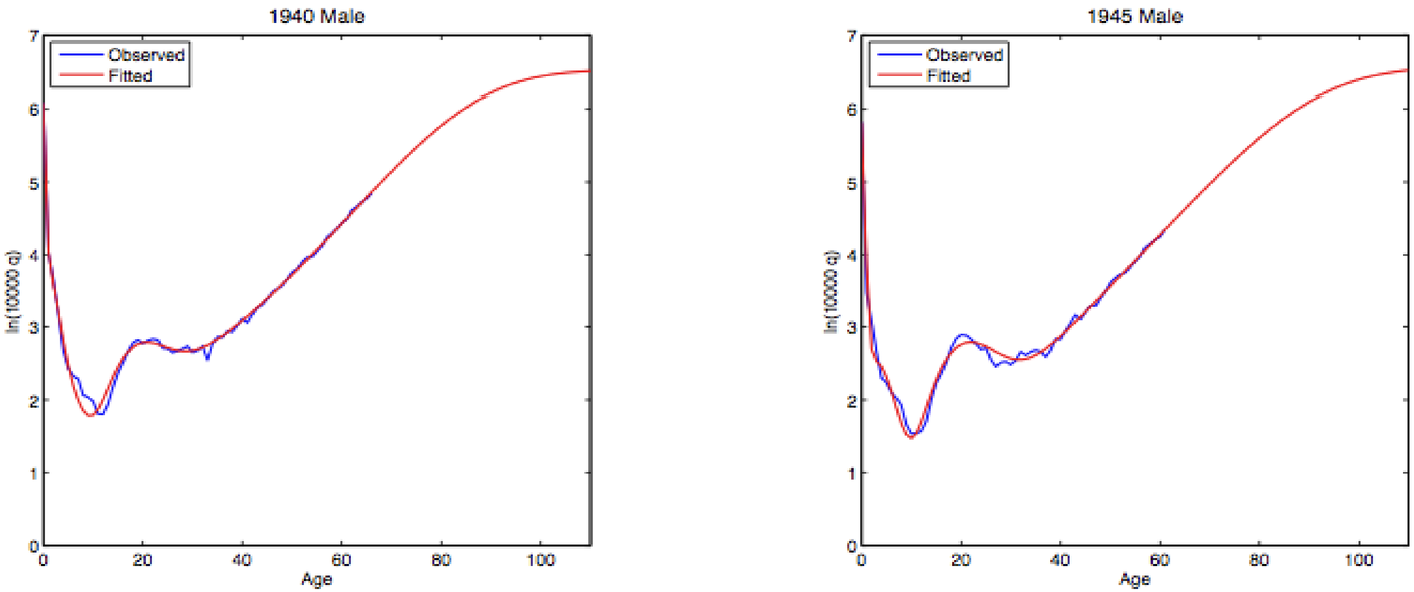
Fitting for Frailty Model

Figure: Observed v.s. Fitted Cohort Average Force of Mortality: Male



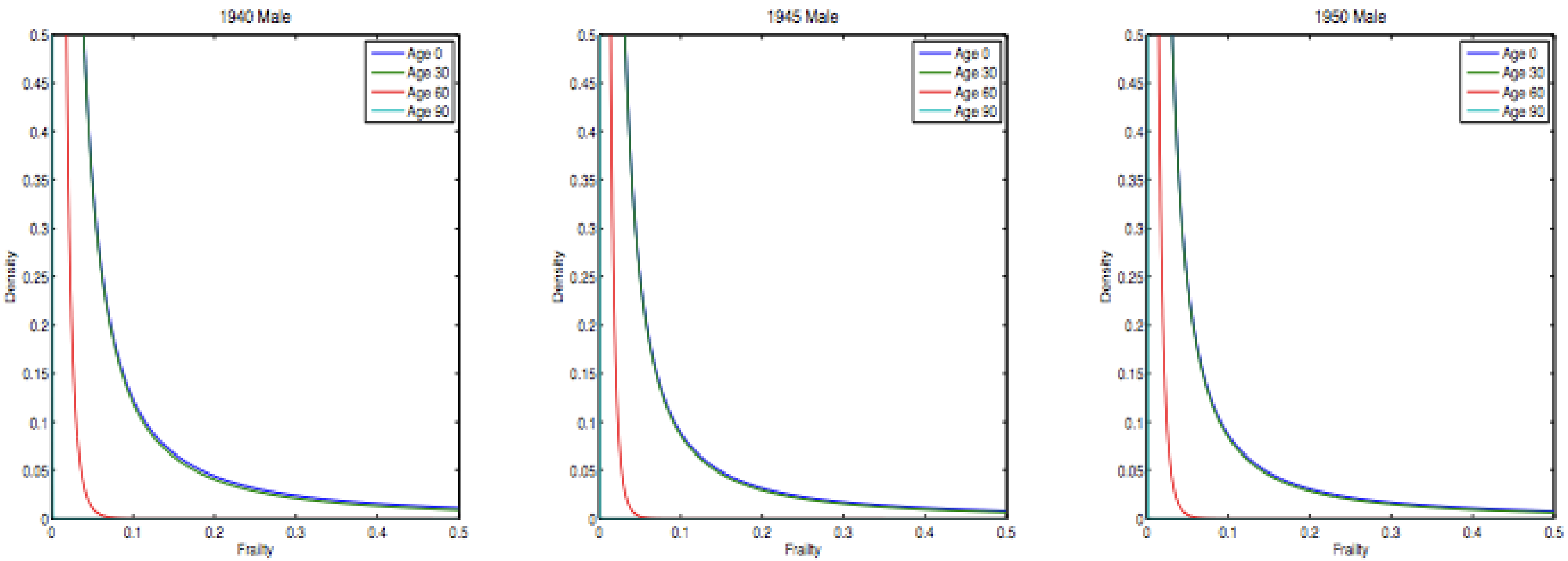
Fitting for Markov Aging Model

Figure: Observed v.s. Fitted Death Probability with Projection at Higher Ages



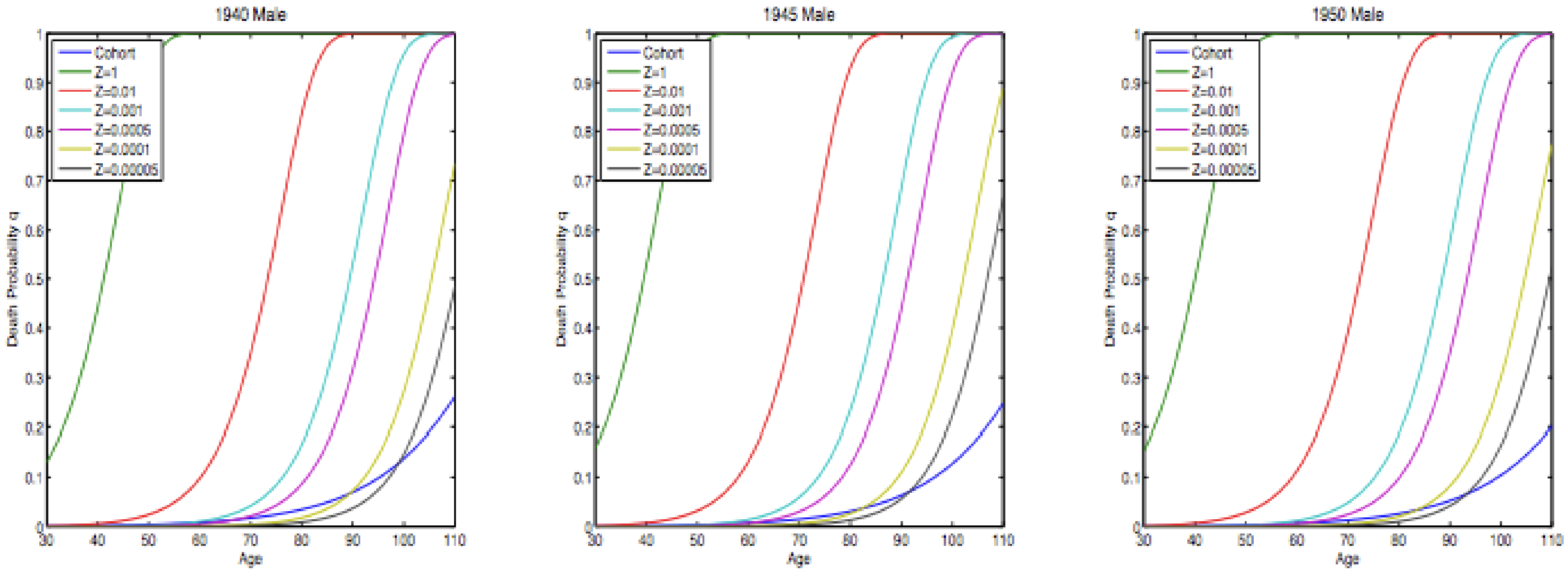
Distribution of Frailty

Figure: Distribution of Frailty at different ages



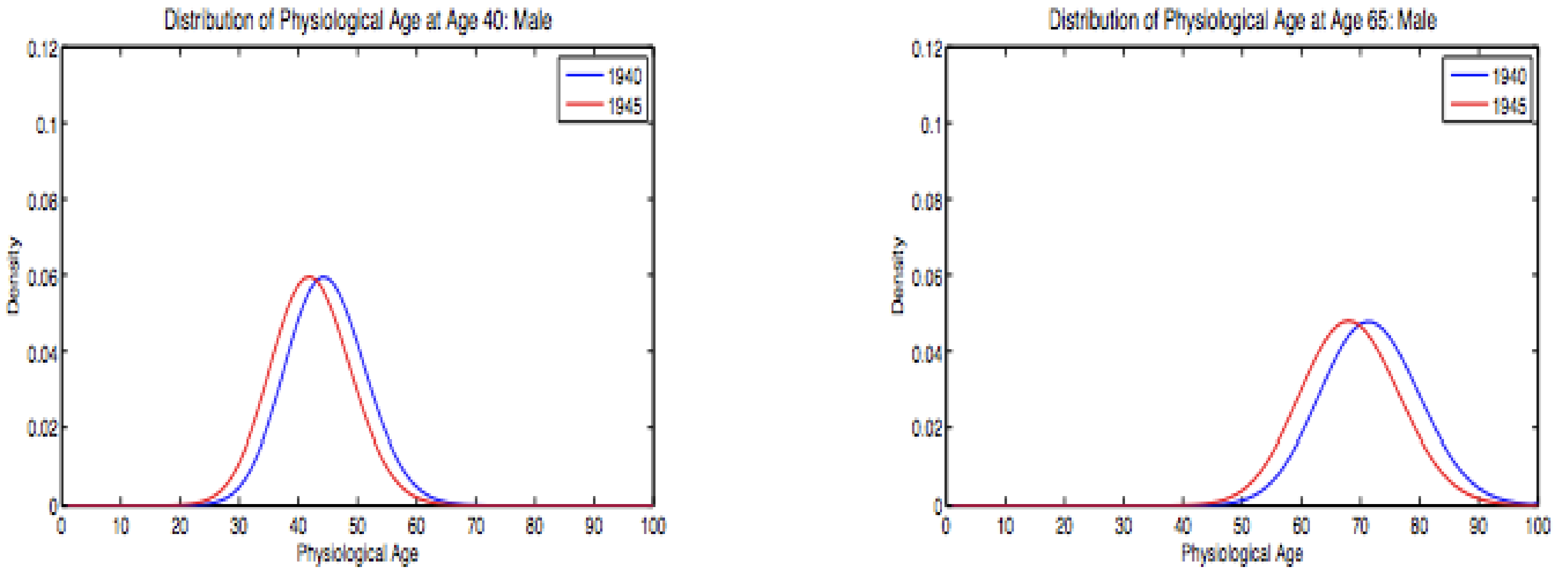
Mortality Rates of Individuals with Different Frailty

Figure: Mortality Rates of Individuals with Different Frailty: Male



Distribution of Physiological Age at Each Age

Figure: Distribution of Physiological Age at Different Ages: Male



Annuity Rates for Heterogeneous Individuals

Table: Annuity Rate for Individuals with Different Frailty

Frailty	Cohort	0.00005	0.0001	0.0002	0.0005	0.001	0.01
q_{65}	0.012	0.001	0.002	0.004	0.010	0.021	0.189
Whole Life Annuity	$14.31	$18.12	$16.36	$14.38	$11.49	$9.18	$2.59
Deferred Life Annuity	$2.36	$4.32	$2.94	$1.66	$0.47	$0.08	$0.00
F(z)		19.40%	38.26%	56.95%	76.49%	86.70%	99.59%

Table: Annuity Rates for Individuals with Different Physiological Age

Physiological Age j	64	68	73	77	81	94
q_{65}	0.006	0.008	0.011	0.015	0.019	0.047
Whole Life	$19.44	$18.31	$16.83	$15.63	$14.44	$11.15
Deferred Life	$5.34	$4.55	$3.63	$2.99	$2.46	$1.50
F(j)	19.47%	35.49%	59.01%	75.81%	87.83%	99.60%

Summary of Results

- Both models indicate that mortality heterogeneity for Australian population is significant, but the heterogeneity structure and estimated results are different
- Frailty model
  - Distribution of frailty is heavily skewed
  - Small proportion of high risks will die first: comply with observed causes of death pattern
  - Heavily depend on assumptions
  - Less practical since frailty factor is unobserved, hard to link it to real world observation
- Markov Aging Model
  - Heterogeneity increases with age
  - More practical since physiological age is easier to be linked with observed health conditions