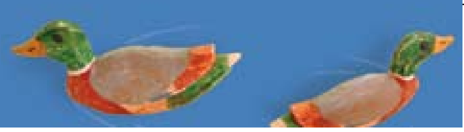
Two Ducks Swimming 22



with Swoodsy

Easy as a, b, CD

The compact disc (CD) is an optical device used to store digital data. As many actuaries and ordinary people alike know, CDs can store enormous amounts of information. (As an aside, I enjoy the fact that CDs read from the inside to the outside – the opposite of vinyl records – which goes to show that sometimes the conventional wisdom needs to be turned inside out!) However their universal acceptance and application today also reflects another important characteristic: their durability. This feature is a testimony of the true genius of CDs: the mathematics.

An early issue with CDs was the inability to retrieve data when a number of data points became corrupted. It is fair to say that this issue has been overcome and I recall witnessing a practical experiment that confirmed just how much so. A former colleague of mine – let's call him Warren Dickham* – scratched a CD to see what effect it would have on the quality. With every iteration, the damage inflicted was more extreme and more extensive. For the final test he physically cut sectors from the CD and we were in awe that it continued to play. It was almost impossible to stop the whiny, almost hypnotic tones of Ginuwine's insipid song *Pony*.

So how can CDs retain their integrity despite such treatment? The answer is oversampling. Not the musical kind but rather the mathematical kind.

Reed-Solomon error correction is a code that works by oversampling a polynomial constructed from the data points. You may recall from high school that 2 points are sufficient to define a line, 3 points are sufficient to define a binomial and so on. By evaluating the polynomial at more points than is necessary, Reed-Solomon error correction allows the receiver to recover the original polynomial even when there are 'bad' data points. This enables the ongoing recovery of data from CDs that have data corruption due to imprint errors, wear and tear, scratches or even whole sectors removed.

In addition to CDs, other applications of Reed-Solomon error correction include electronic encryption, barcoding, broadcast systems and data transmission. Indeed it has been used in satellite transmissions since the 1960s, including the Gemini, Voyager, Galileo, Cassini, Pathfinder and Rover missions. I believe this was to send images and communications data back to earth, rather than to enjoy the melodic musings of Ginuwine and his homeys. But you never know.

If you're horny / Let's do it / Ride it / My pony – Ginuwine

* *This name has been changed to hide questionable taste in music.*

Easy as 1, 1, 2, 3 (*AA* 128 – Solution)

Does there exist a Fibonacci number ending in 999? Prove your answer.

The Fibonacci series is defined by $F_n \equiv F_{n-1} + F_{n-2}$. As $F_1 = F_2 = 1$, we deduce that $F_0 = 0$, $F_{-1} = 1$ and $F_{-2} = -1$.

Now consider the Fibonacci series in modulo 1000. By the pigeonhole principle, there must be at least two identical pairs of successive numbers in every $1000^2 + 1$ pairs. Therefore suppose that $F_{n+k} \equiv Fn$ and $F_{n+k+1} \equiv F_{n+1}$, where $k \leq 1000^2 + 1$. (In practice, k = 1500 is sufficient to find identical pairs.)

Working backwards, $F_{n+k-1} \equiv F_{n+k+1} - F_{n+k}$ and $F_{n-1} \equiv F_{n+1} - F_n$. Hence $F_{n+k-1} \equiv F_{n-1}$. Similarly, $F_{n+k-2} \equiv F_{n-2}, \ldots, F_k \equiv F_0$, $F_{k-1} \equiv F_{-1}$ and $F_{k-2} \equiv F_{-2} \equiv -1$.

Now if $F_{k-2} \equiv -1 \bmod 1000$, that means it ends in 999.

The following readers impressed to the nines:

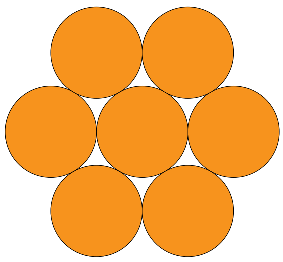
Chris Wood	**Julian Widdup**
Felix Tang	**Francis Ratna**
Charles Qin	**Corey Plover**
Angelo Hysnelaj	**Alan Brown**

Marina Batliwalla (who generously credited her brother-in-law **Nadir Godrej**)

The winner of this *Two Ducks* puzzle was **Francis Ratna**, who scored $150 to spend at the restaurant of his choice, courtesy of the Institute. Being based in New York, this might afford him a bagel or two. Easy.

Touching you, touching me

It is as easy as pi to demonstrate that any given circle can touch up to 6 circles of the same radius, as shown.



For any given sphere, what is the maximum number of spheres of the same radius that it can touch?

For your chance to win the sensational prize of $150 to spend at the restaurant of your choice (generously provided by the Institute), send your solution by email to <twoducks@actuaries.asn.au> or by regular mail to *Two Ducks* via the Institute. ▲

Stephen Woods
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