Actuaries Summit
get involved, get ahead
20 – 21 May 2013
Hilton Sydney

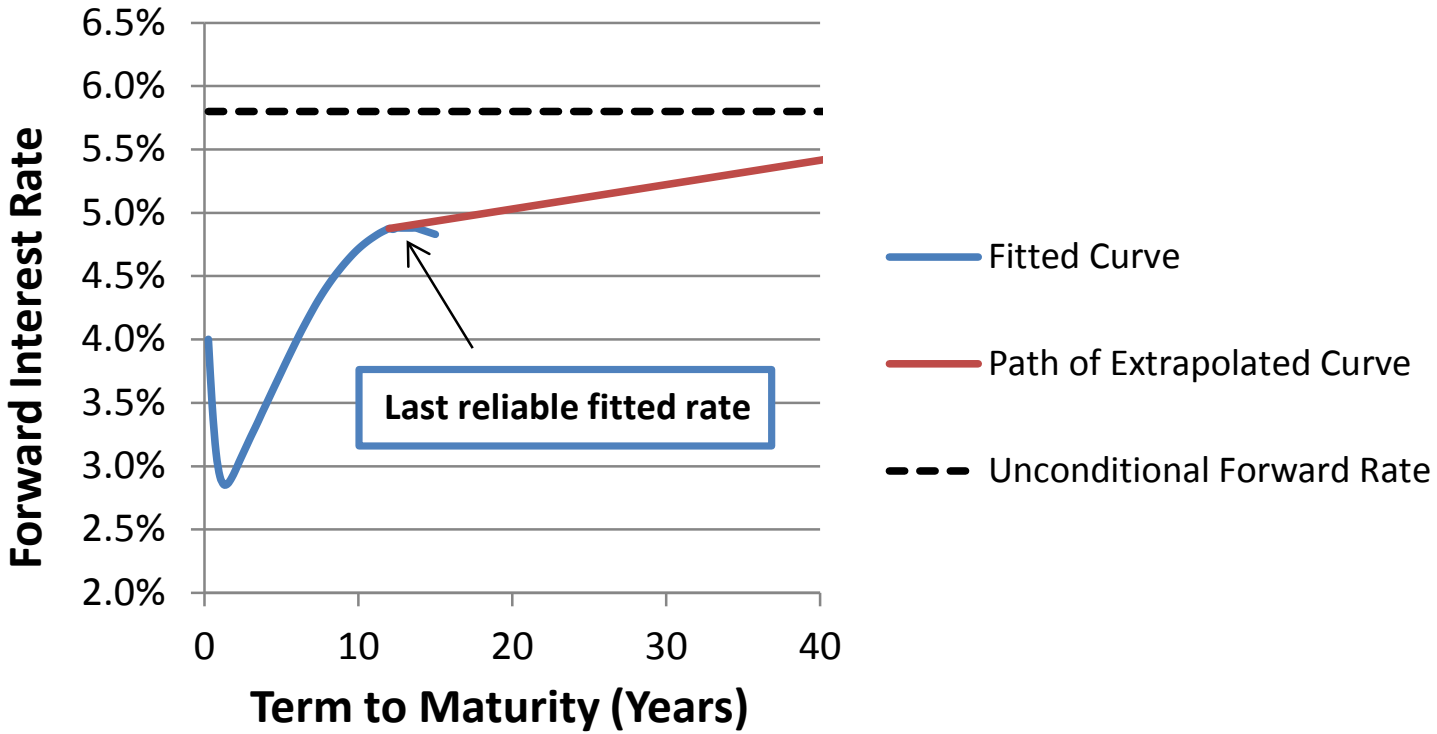
Actuaries Institute

A topic of interest – how to extrapolate the yield curve

Peter Mulquiney & Hugh Miller

© Taylor Fry Pty Ltd

Three main issues for extrapolation



Philosophical and Regulatory Considerations

Two philosophical approaches

- Emphasis on **market consistency** at a point-in-time
- Emphasis on **liability stability** across time

Regulatory and professional considerations

- **AASB 1023** – silent on the issue
- **GPS 310 & PS300** have similar requirements

IAA PS300 – General Insurance Business

8.2.2 Legislative and/or regulatory requirements may prescribe whether Claim Payments are to be discounted. The Member must consider the purpose of the valuation and document whether the future Claim Payments are to be discounted. Discount rates used must be based on the redemption yields of a Replicating Portfolio as at the valuation date, or the most recent date before the valuation date for which such rates are available.

8.2.3 If the projected payment profile of the future Claim Payments cannot be replicated (for example, for Classes of Business with extended runoff periods), then discount rates consistent with the intention of Paragraph 8.2.2 must be used.

Regulatory and professional considerations

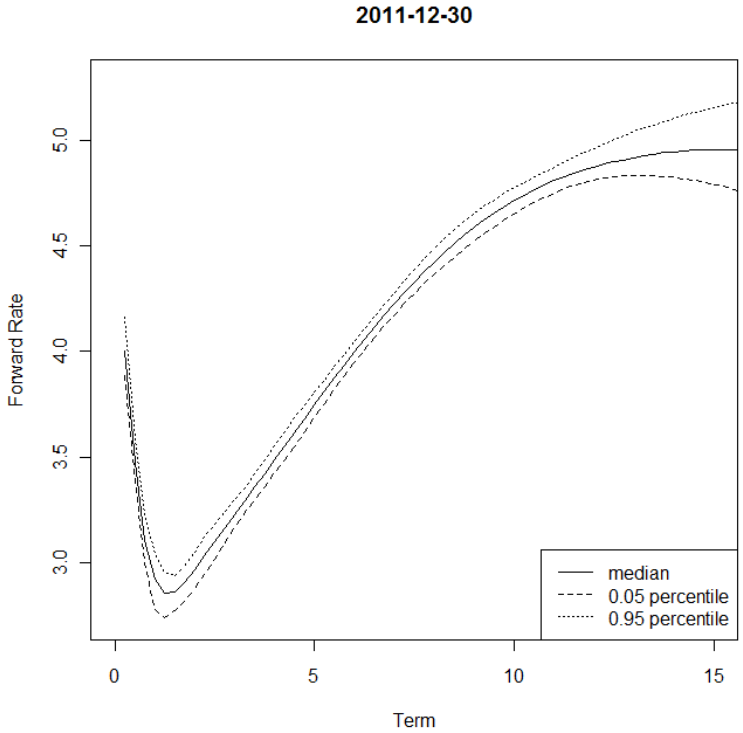
- **APRA Draft GPS 320 - Discount rates**

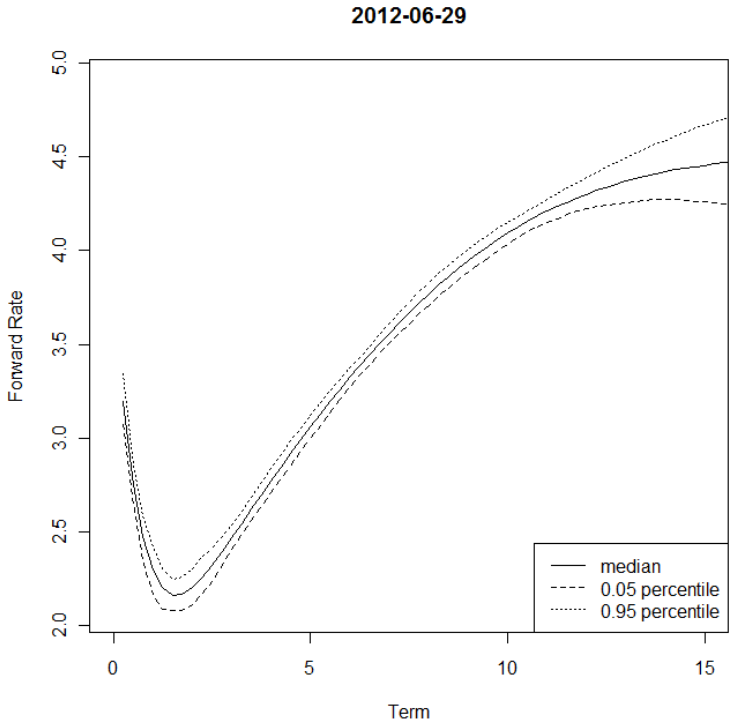
"Where the term of the insurance liabilities denominated in Australian currency exceeds the maximum available term of CGS, other instruments with longer terms and current observable, objective rates are to be used as a reference point for the purpose of extrapolation. **If there are no other suitable instruments or the Appointed Actuary elects to use an instrument that does not meet this requirement, the Appointed Actuary must justify the reason for using that particular instrument in the insurer's ILVR.** Adjustments must be made to remove any allowances for credit risk and illiquidity that are implicit in the yields of those instruments."

- It seems to us that the intention of both APRA's standards and PS300 is for yield curve extrapolation to be performed on a **market consistent basis**.

What is the longest market forward interest rate we can estimate reliably?

Estimation error in the forward rate curve





Longest reliable forward rate - conclusions

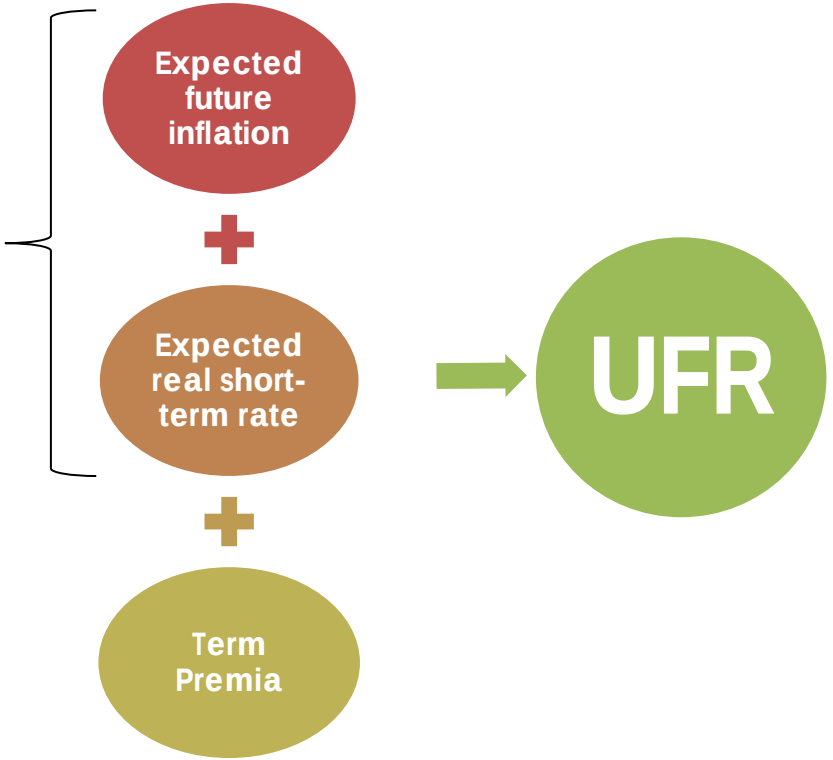
- We should not be overly reliant on forward rate estimates made at the long end of the fitted forward curve, in particular the last 2 years of the observable range.

- At present it appears that terms of around 13 years would be an appropriate point to start an extrapolation in Australia.

What is the ultimate very long-term “unconditional” forward interest rate?

Components of the "unconditional" forward rate

Expected future short-term (nominal) interest rate



Components of Term Premia

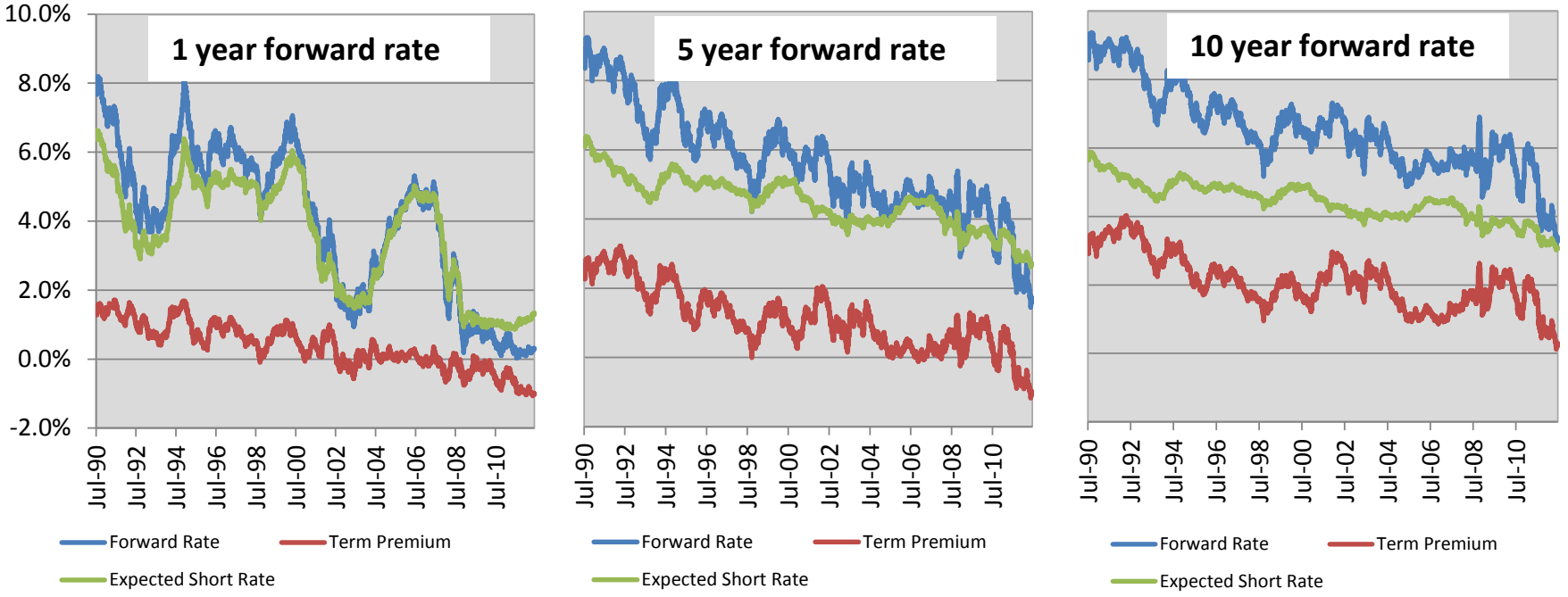
Term Premia - difference between the forward rate and the expected future short-term interest rate. Has the following components:

- **risk premia** - Investors demand a premium for locking into long-term investments. It is compensation for holding long term bonds whose value will fluctuate in the face of interest rate uncertainty, exposing the holder to mark to market losses.

- **term preference** - Demand for long-term government securities from large institutional investors can drive down long-term forward rates because these long-term bonds offer a closer match to liabilities.

- **convexity effects** - Fixed income securities have positive convexity which means that the capital gains and losses from equal sized interest rate swings are not matched – the gains will be greater than the losses. This effect can theoretically cause very-long duration bonds to trade at higher prices (lower yields).

The time-varying behaviour of term premia



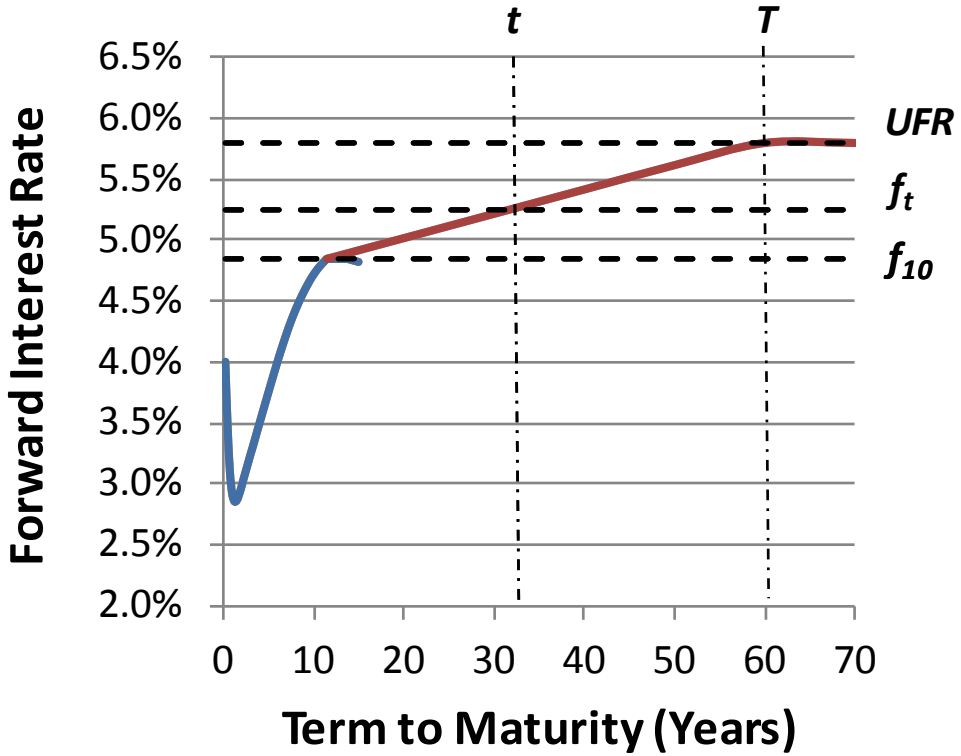
Suggested UFR

Component	Rate
Expected future short-term interest rate	
Expected future inflation	2.5%
Expected real interest rate	2.0%
	4.5%
Term Premium	
Risk Premium	1.5%
Convexity adjustment	-0.2%
	1.3%
Unconditional forward rate	**5.8%**

How quickly should we return to the UFR?

International evidence

How quickly should we return to the UFR?



International evidence

- Longer bond rates (up to 30 years) available for a number of countries:
  - UK
  - USA
  - Canada
- These can give considerable insight into the behaviour of the yield curve
- Data since 1998 used (from when inflation expectations stable)

Regression based approach

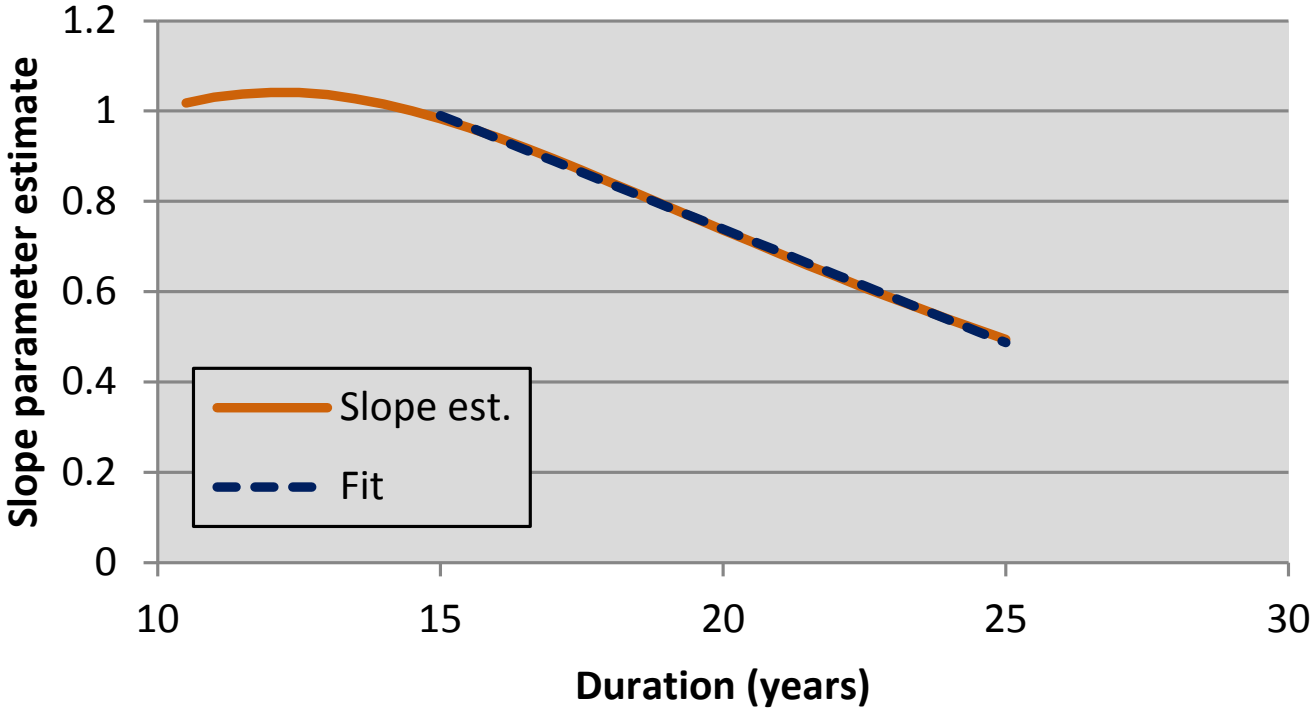
- Consider regression equation:

$$f_{s+t} = \alpha_{s:t} + \beta_{s:t} f_s$$

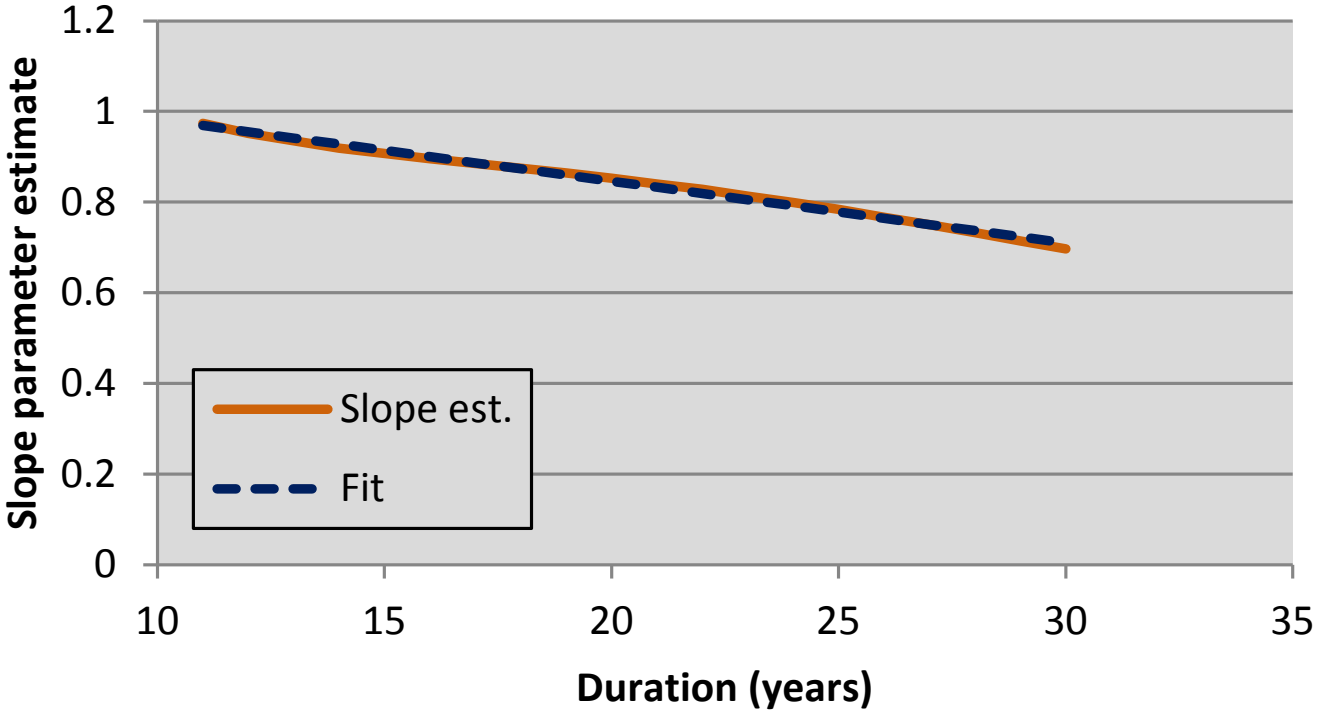
- The $\alpha_{s:t}$ concern the expected rates and term premia
- The slope coefficients $\beta_{s:t}$ should give a good indication of progress to a UFR

- E.g When we regress f_{20} against f_{10} for the UK data we get $f_{20} = 0.78\% + 0.74 f_{10}$. Varying t gives insight into how to extrapolate the curve

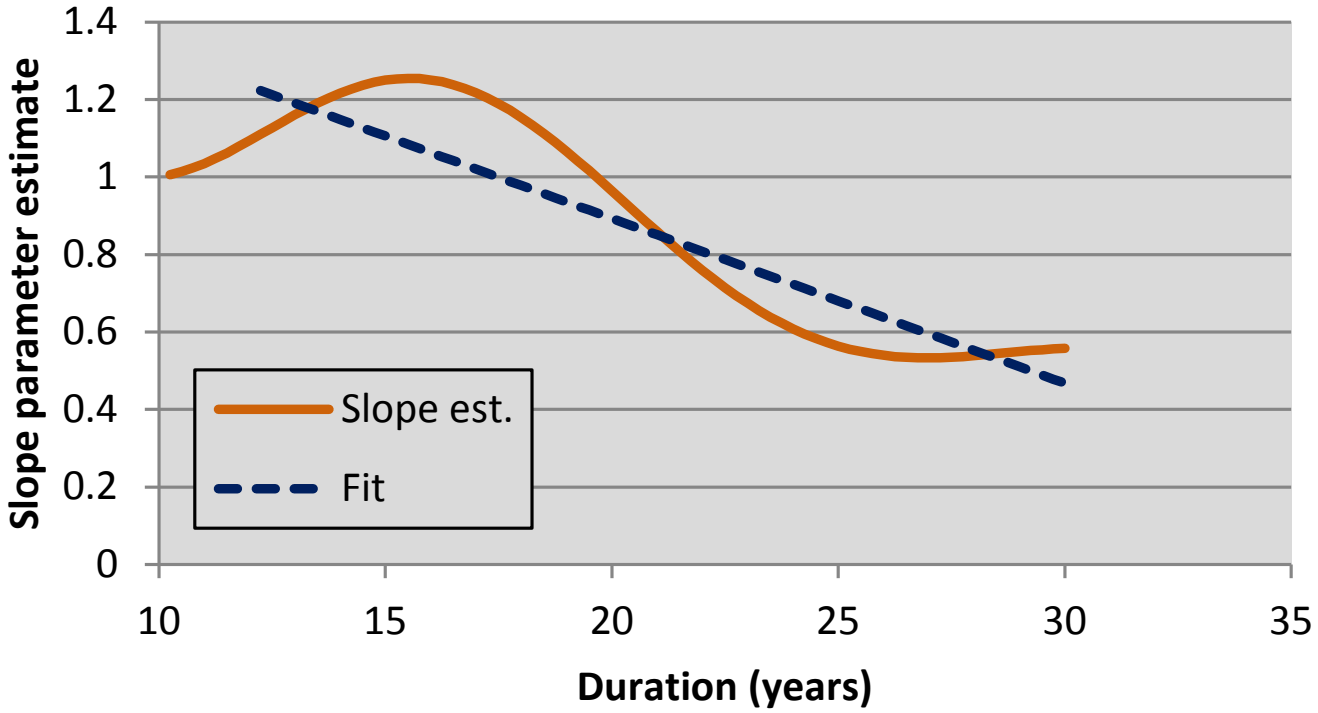
UK regression results



US regression results



Canada regression results



We extrapolate to when the slopes hit zero

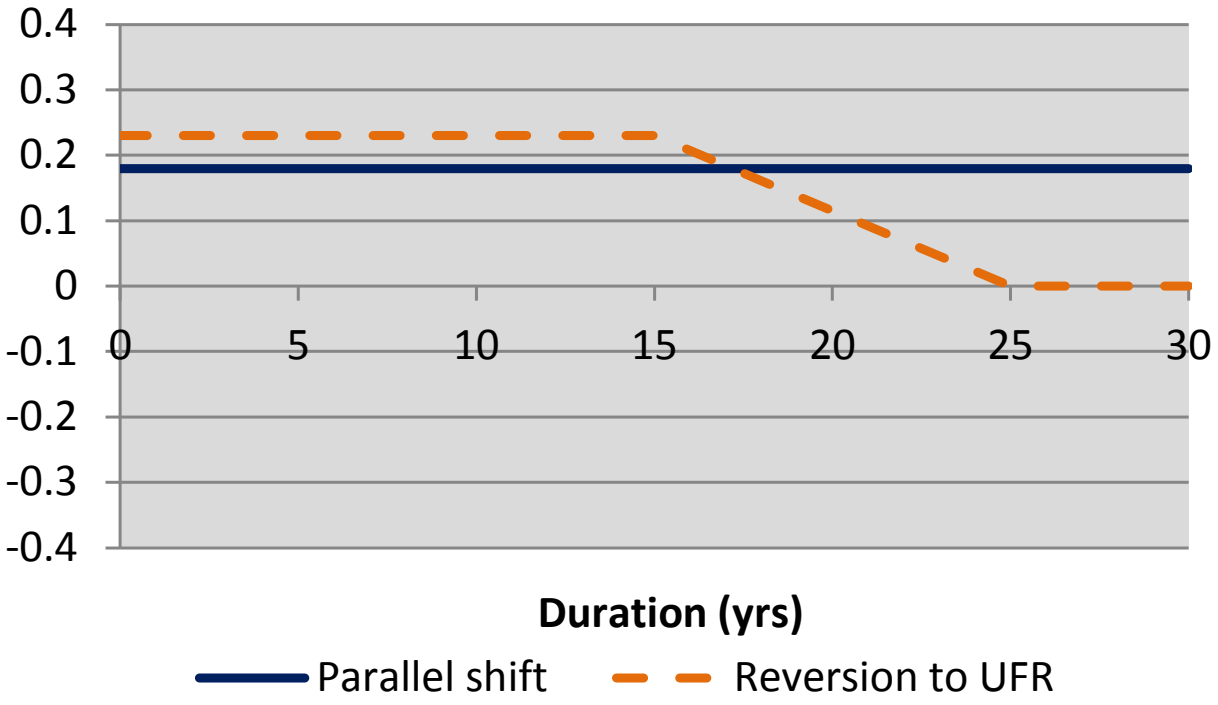
Country	*Duration decay starts*	Duration when reach UFR	*95% confidence interval*
US	10	82	(55, 168)
UK	15	34	(31, 40)
Canada	10	41	(35, 47)

Alternative analysis: Principal components analysis (PCA)

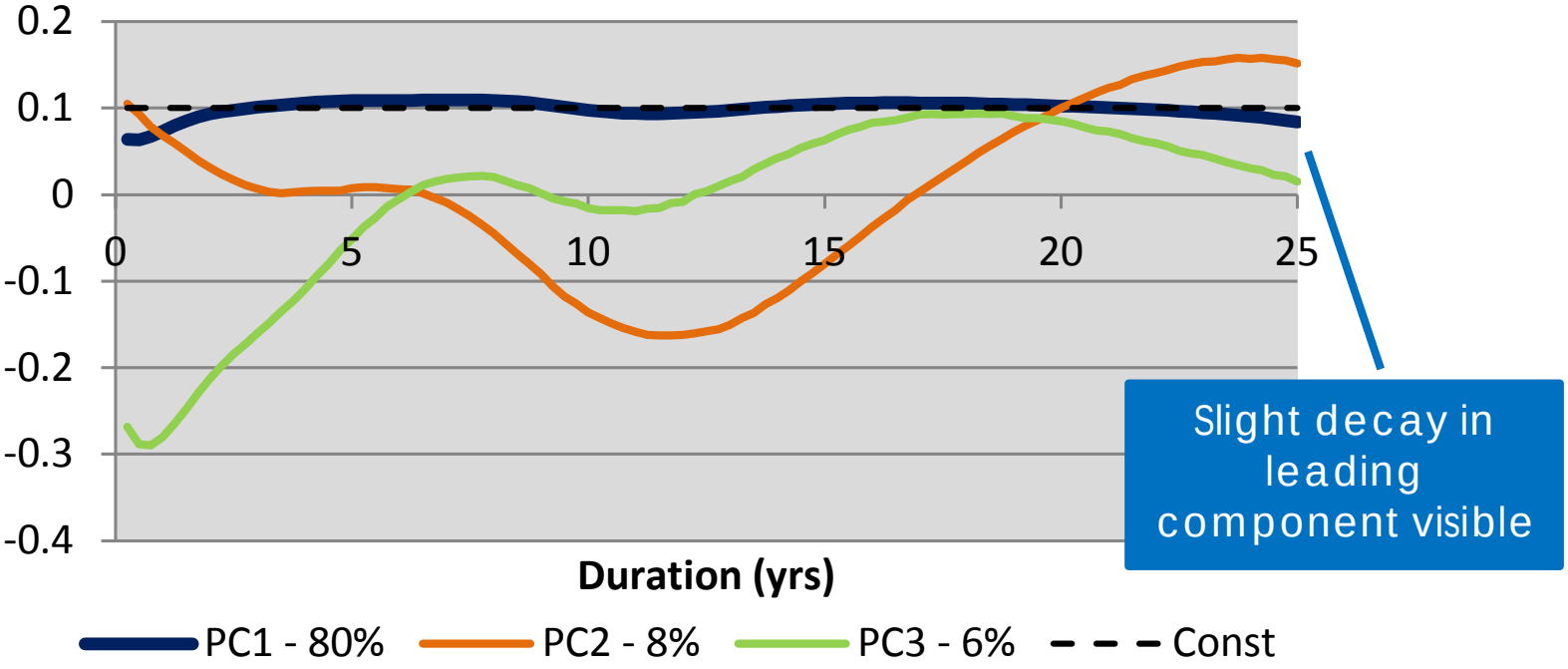
- Principal components analysis attempt to capture maximal variation in a system
- In our case, how does the yield curve tend to move away from its average shape?
- The shape in the tail reveals how "fixed" it is relative to the rest of the curve

- More detail in the paper

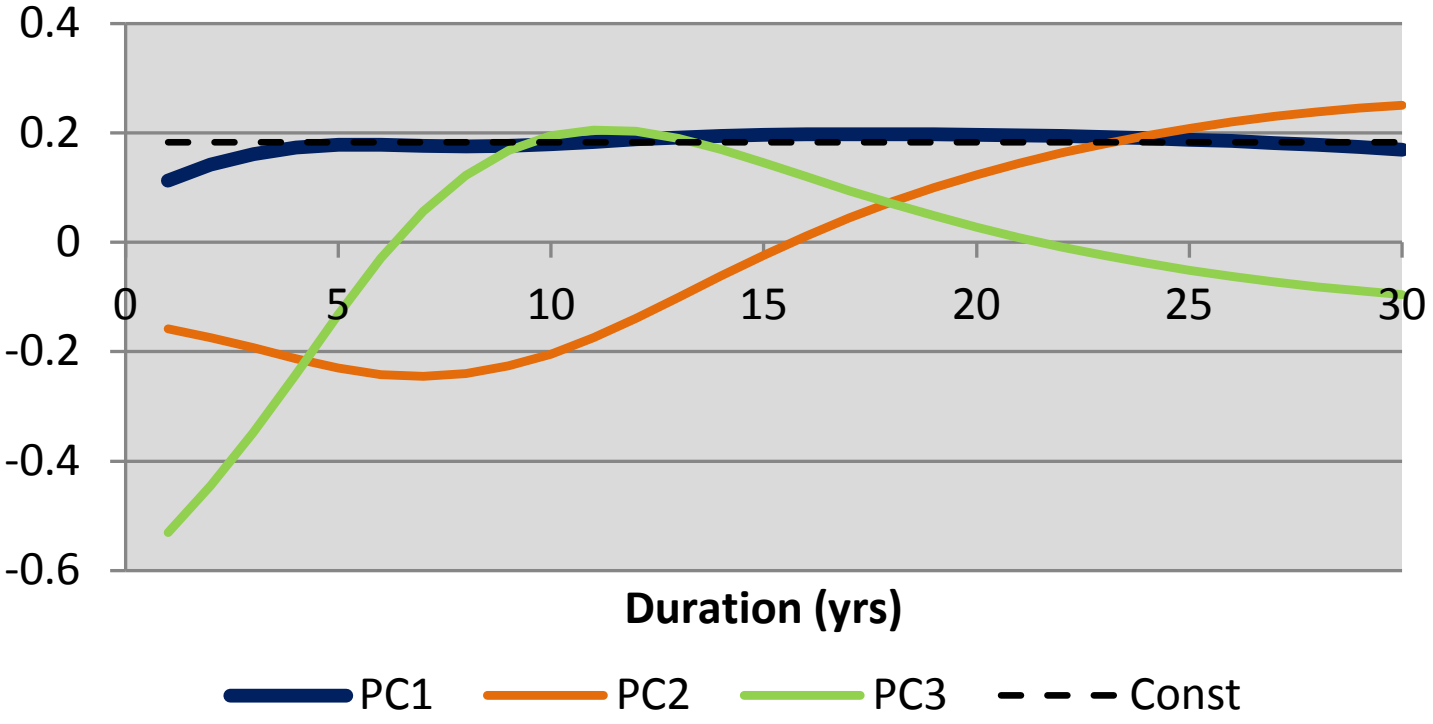
PCA – illustrative shapes



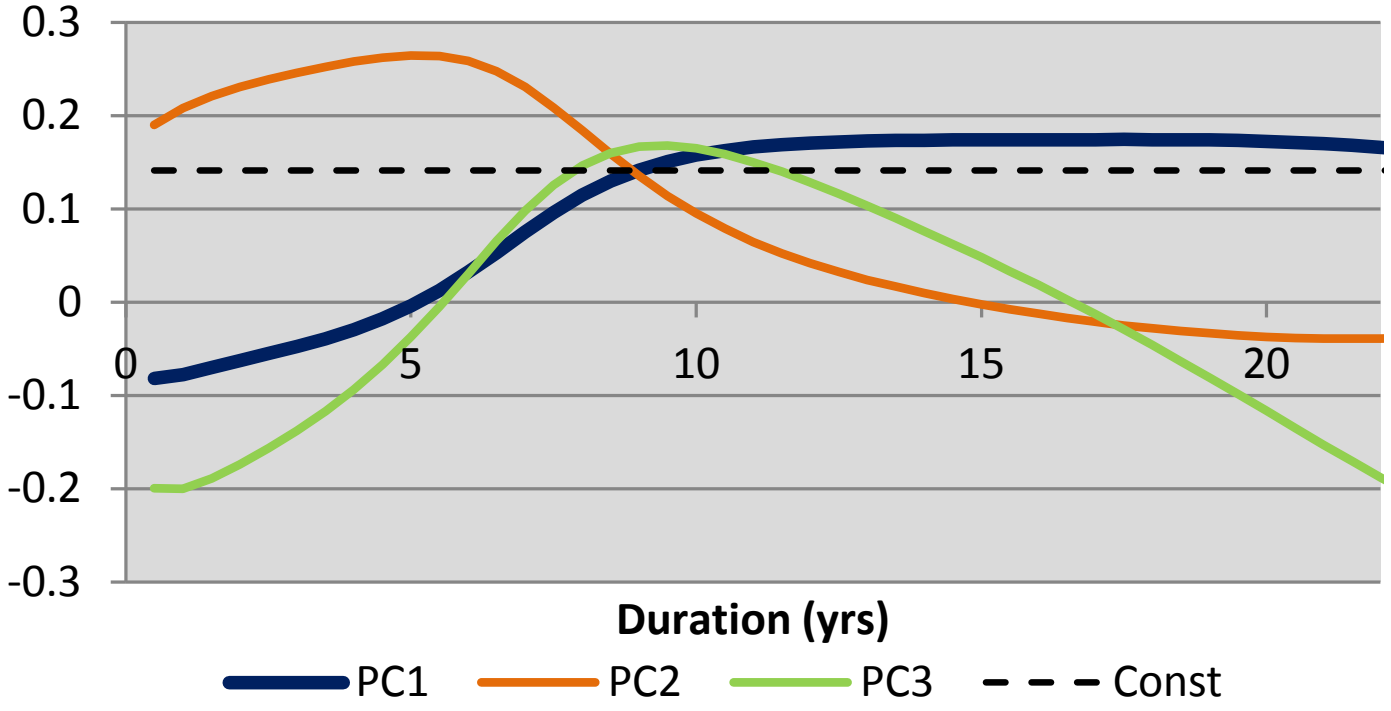
PCA - Canadian yield curve



PCA – US yield curve



PCA – UK yield curve



As before, we can extrapolate linearly to when the slopes hit zero

Country	Duration decay starts	Duration when reach UFR	95% confidence interval
Canada	16	64	(46, 101)
UK	15	84	(60, 122)
USA	17	110	(87, 147)

International evidence - conclusions

Based on the analysis of Canadian, UK and USA yields:

- A reversion occuring somewhere between duration 40 to 100 appears correct
- Duration 60 a reasonable point estimate (giving slightly more weight to regression results)

Note this is a very **slow** reversion to the mean UFR

Actuaries Summit
get involved, get ahead
20 – 21 May 2013
Hilton Sydney

Actuaries Institute

Long term yield extrapolations and hedging strategies

Hedging long term liabilities

What if we use **duration matching hedging strategies** for liabilities beyond the longest term assets:

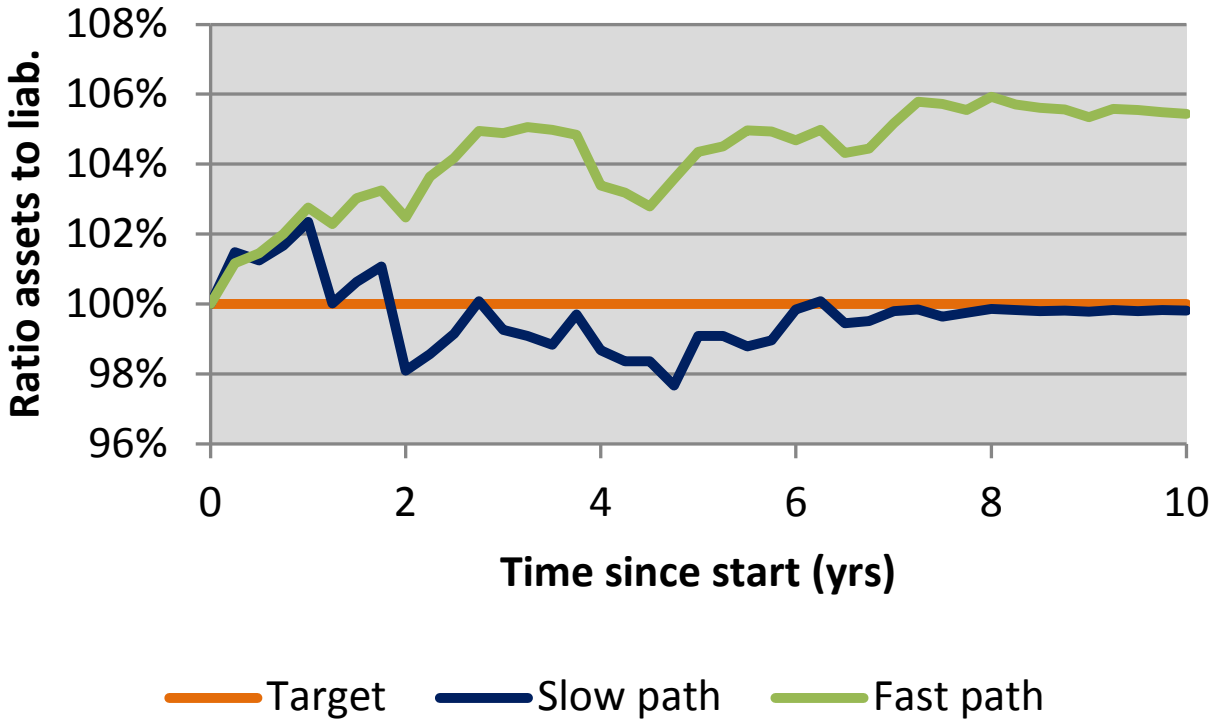
- Sell a short-term bond and Buy a long-term bond gives a duration greater than the long-term bond
- Matching a very long term liability requires a yield curve extrapolation to get its present value and duration
- Strategy is not well represented in actuarial literature

Does a slow reversion give better hedging performance compared to fast UFR reversion?

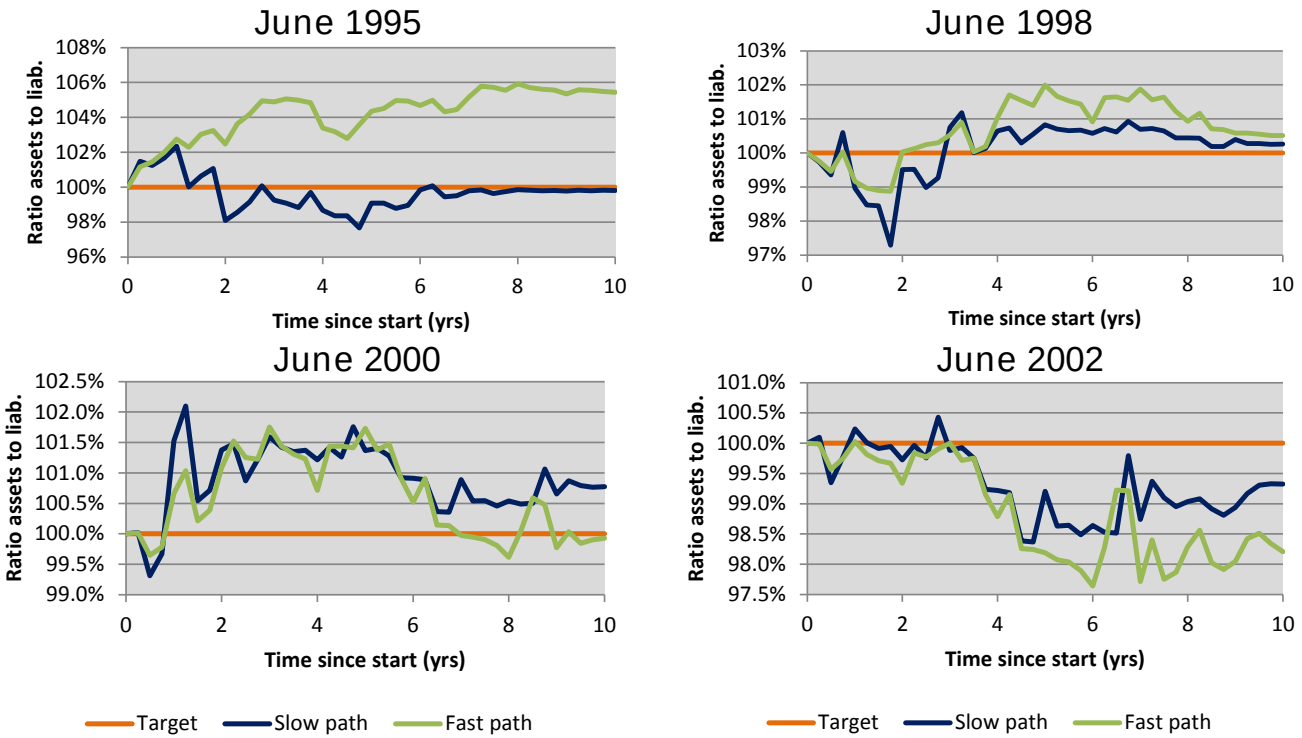
Testing on Australian data

- Suppose $100 liability due in 20 years
- Can invest in 4 and 10 year bonds
- Rebalance every quarter (close the 3.75 and 9.75 year bond positions, reinvest in 4 and 10 year bonds)
- Track hedging performance over time for slow and fast reversion assumptions
- The fast UFR assumption (returns to UFR linearly between 10 and 20 years) reduces the (modified) duration of the liability, so gives less aggressive short-long positions

Hedging performance, starting June 95



Hedging performance, various start dates



Hedging conclusions

- A slow reversion assumption gives **better hedging performance**, further evidence that it is superior to fast reversion
- The hedging strategy performs very well, with 1% gain/loss in each experiment
- Also, approach is far superior to simply investing only in the 10 year bond

Final conclusions & comments

Main conclusions

- Yield curve up to 2 years before longest dated bond can be estimated reliably
- Reasonable evidence for slow (~duration 60) reversion to a UFR in international markets
- Linear path plausible, other paths possible
- Long term risk free hedging possible, and favours market consistent slow reversion assumptions

Final comments

- Current Australian standards somewhat unclear in their philosophical approach to long term rate setting

- Slow reversion creates an (irreconcilable?) tension between market consistency and liability stability – worthy of further discussion