

UNORTHODOX ECONOMICS AND MATHEMATICS FOR THE ACTUARIAL SYLLABUS

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Quantitative
Strategies

Rationale: Risk culture

- New areas of practice such as enterprise risk management
- Many economic ideas not included in the standard actuarial syllabus are relevant to ERM
- Especially for understanding ***risk culture***
 - The “soft” subjects that are really “hard”
- Not just the actuarial syllabus needs expanding
- Research agenda?

- The ideas presented here are eclectic and personal
 - Not the result of a formal research program

Topics

- Why don't workers hire capital?
- How do societies avoid Tragedy of the Commons?
- Fairness, corruption, and bad faith
- Adam Smith and his two economies
- Constructivist and ecological rationalities
- Complex probabilities

Workers hiring capital? Who's the boss?

- In naïve economics we could just as easily see workers hire capital as capital hiring workers, but we almost always see the latter.
- What do the various theories that explain this observation imply about the risk management culture?
 - Bowles 2004 – Chapter 10

Who's the Boss?

- All contracts are incomplete to some extent
 - So, in a firm, those with control rights have power over other people's money, assets, and labour
- Why are these control rights given to suppliers of capital, not labour?

Who's the Boss?

Standard explanations

- Risk averse workers want wage employment as a form of insurance
- Appointing a boss allows better monitoring of labour-effort
- Control goes to those with most to lose (capital providers)
- Suppliers of labour have lower access to credit

Who's the Boss?

Power

- A problem with the above is that control rights are limited
 - Q. When the boss says jump, why does the employee jump?
 - A. The boss has power over the employee

Power:

Bowles p345

- For B to have power over A
 - B can gain by affecting A's actions through threatening sanctions on A
 - A lacks this capacity with respect to B
- In general, in non-clearing markets, those on the short-side have the power
 - Capital can sack managers
 - Managers can sack staff

(Note: Banks are on top because they don't have to lend to equity)

Risk Management Implications

- Power is easily abused but is omnipresent in a firm
- People are sensitive to appropriate use of power:
 - resent the abuse of power
- Power must be exercised carefully

Risk Management Implications

- If subordinates gain power managers can't threaten
- CRO must monitor ways in which employees can gain subversive power in the hierarchy
 - This is actually a Marxist analysis

Risk Management Implications

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Tragedy of the Commons: Risk appetite

- Elinor Ostrom has studied the strategies used by societies to overcome the Tragedy of the Commons
- Risk appetite is a common resource of a firm that could be subject to such a Tragedy
- What can be learnt from other resource extraction situations?

Tragedy of the Commons

Observations – Ostrom p 237

- Challenge 3 common assumptions:
 - Don't have central authorities
 - Resource appropriators can overcome the temptation to over-harvest
 - Designing rules to change incentives is difficult

“Instead of central direction, what is needed are policies that enhance the accuracy and reliability of information, that provide low-cost conflict resolution, and that develop the authority to govern resources at multiple levels.”

Tragedy of the Commons

Design Principles - Ostrom p271

- Define the boundaries of the resource and who can use it
- Clarify the relationship between the benefits received and the contribution to costs
- Enhance participation of key decision makers

Design Principles

Ostrom p271 (ii)

- Who are the monitors and are they appropriately incentivised?
- What are the sanctions and are they proportional to the damage caused?
- What are the local and regional conflict resolution mechanisms?

Design Principles

Ostrom p271 (iii)

- Are there functional and creative efforts by local appropriators that should be recognised?
- How do we create multiple-layer, polycentric systems that can be dynamic, adaptive, and effective?

Threats to Robust Governance

- Rapid exogenous change
- Transmission failures from one generation to the next
- Programs relying on blueprint thinking and easy access to external funds
- Corruption and opportunistic behaviour

Threats to Robust Governance (ii)

- Lack of large-scale institutional arrangements for: information; fair and low cost conflict resolution; educational facilities; help when major problems occur

Fairness

Trust

- People seem to positively value fairness, which comes in many economic guises
 - Note the plain English usages!
- We enjoy punishing those who do not act fairly
- Trust and trustworthiness are confounding factors when trying to measure fairness
- Reciprocity is, perhaps, even more fundamental

Corruption, bad faith

- These are an evolutionarily stable strategy in a population
 - We will always have them
 - May even be non-rational response
 - Contribute strongly to damage of booms and busts
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- Akerlof & Shiller, V. Smith

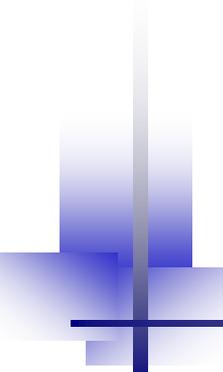
Adam Smith: Moral Sentiments and the Wealth of Nations

- His famous phrase from WoN: “the propensity to truck, barter, and exchange one thing for another”
- Applies to ToMS just as well – a marketplace of favours, gifts, assistance
- Reciprocity is fundamental to both

Rationality in Economics

Vernon Smith

- Constructivist rationality:
 - Modelling rational individual actions
 - designing social systems (inc. markets)
- Ecological rationality:
 - Adaptive human decision making
 - Group process of discovery in natural social systems (evolutionary)



Constructivist Ecological

- Both forms are needed in understanding economic behaviour
- Constructivist approaches need to be informed by ecological rationalities.

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Complex Probabilities

- **BASICS**

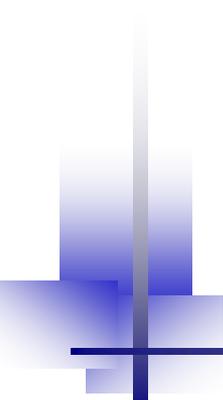
- We have n possible states of the world.
- \mathbf{p} is a state vector – n rows and 1 column – possibly subscripted by a time t where confusion is possible
- p_i is the probability of being in state i – a time t will be added to the subscript to remove ambiguity where needed
- \mathbf{A} is a transition matrix
- \mathbf{a}_{ij} is the probability of moving from state j to state i
- $\mathbf{p}_{t+1} = \mathbf{A}\mathbf{p}_t$
- There's no difficulty in extending this to a continuous framework, but for actuarial problems there is no advantage either.

Amplitudes

- My discussion here owes a lot to a lecture series by Scott Aaronson.
- p_i and a_{ij} are complex numbers, $x + iy$, x and y can be positive or negative.
- $|p_i| = x^2 + y^2$ This is the (standard) probability of observing state i
- $\sum |p_i| = 1$ i.e. you must be somewhere
- If \mathbf{p}_t and \mathbf{p}_{t+1} are state vectors with the above properties then what is the general form can \mathbf{A} (the transition matrix) take? It must be a *unitary matrix*, one where the inverse is equal to the conjugate transpose
- $\mathbf{A}^{-1} = \mathbf{A}^\dagger$
- $a_{ij}^\dagger = x_{ji} - iy_{ji}$

WHY BOTHER?

- Think about a regime shift model, with 2 regimes and the extreme case (purely for this exposition) where we move from one regime to the other with certainty in the next period, say a month.
- $A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
- What happens half-way through the month? What regime are we in? In some models this won't make sense, but in others it does. Anyway, in the standard stochastic model this can't be answered, there is no matrix **B** with
- $B^2 = A$
- But if we now move to measuring probability by amplitudes then we do have a solution.
- $B = 1/2 \begin{pmatrix} 1+i & 1-i \\ 1-i & 1+i \end{pmatrix}$

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- So, if we start in state 1

- $p = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

- $Bp = \frac{1}{2} \begin{pmatrix} 1 + i \\ 1 - i \end{pmatrix}$

- $|\frac{1}{2}(1+i)| = 0.5$

- So if we were able to measure the regime half-way through a period then we'd have a 50% chance of being in regime 1 and 50% of being in regime 2. This makes sense. The next point is where interpretation becomes difficult.
- If we don't determine the state of the world, i.e. which regime we're in, then we will be in regime 2 at the next period. If we do determine the regime halfway through the period then the process is reset, and at the end of the period we will have a 50% chance of being in regime 1 and 50% chance of being in regime 2; the amplitudes are as above.

Illiquid assets

- Suppose we have an illiquid unique asset, such as a large office building. Such an asset has no known price until a transaction occurs. Assume we have an econometric model for the price based on economy wide data, with an error term c . This variable takes two values, one positive and one negative, modelling a higher price than expected price and a lower than expected price respectively. Our dynamic model for c is based on Equation 1, a higher price modelled this year would be followed by a lower price for next year's modelled price.
- If we've just observed a high price, we would expect a low price if we saw a transaction in one year; what would we expect in 6 months' time? The answer is given in Equation 3, which implies a 50-50 chance of a high price vs a low price.
- But note that if we see a transaction in six months then it will be at either a higher or lower price than expected. The price change process, summarised by the matrix \mathbf{A} on a yearly basis or \mathbf{B} if we consider a six monthly time period, will commence again.
- If we wanted to we could calculate a daily movement matrix $\mathbf{A}^{1/250}$ and see what our model implies about the prices we would observe if a transaction occurred on the next day, or any day in the year.
- It is this power to move to finer and finer time intervals that is, in my opinion, the motivating factor for considering this model of probabilities.