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Beyond correlation: Recent developments in the modelling of claims dependencies

Benjamin Avanzi, School of Risk and Actuarial Studies, UNSW and Département de Mathématiques et de Statistique, Université de Montréal

Greg Taylor, Bernard Wong, School of Risk and Actuarial Studies, UNSW

Insights, 5 April 2016

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Significance of stochastic dependence

Practically speaking, dependence structures can have a major impact in several areas of the business of an insurance company (see also IAA, 2004, 2009), including

- determining actuarial reserves (IBNR): quantile, or central estimate (the mean) plus margin;
- determining a risk based capital for solvency assessment

In all cases, structures with less than perfect dependence will lead to diversification benefits, whose accurate estimation is crucial for

- capital efficiency (not underestimate);
- solvency of the insurance company (not overestimate).

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Modelling claim dependencies for the general insurance industry with economic capital in view: An innovative approach with stochastic processes

"The project will develop progressive methods to better represent the fine, complex structures driving the significant dependencies relevant to the Enterprise Risk Management of general insurers. [. . .]

Collaborative between, and jointly funded by Government, industry (Allianz, IAG, Suncorp) and academia

See also article in the Actuaries Magazine, August 2014 (Avanzi, Taylor, and Wong, 2014)

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At our last 'Insights' Session (October 2015), we concluded:

- Correlation depends on your model
- Correlation happens as a result of real phenomena
- The more of those phenomenons you can explain in your model, the less dependent your residuals will look like
- We wiped away all correlation from the AUSI dataset
- But what you can use to explain past data may not necessarily be available to explain (predict) the future
- Correlation is all but one way of specifying dependence

See also

- article in the Actuaries Magazine, September 2015 (Avanzi, Taylor, and Wong, 2015a),
- article in BusinessThink, December 2015 (BusinessThink, 2015),
- academic article in ASTIN Bulletin, in press (Avanzi, Taylor, and Wong, 2015b)

Correlation

Definition:

- We refer to 'Pearson correlation'
- This is a measure of linear dependence, which is symmetric around the mean
- Hence, it is tied to elliptical distributions (Normal, Student)
- In fact, it completely specifies the dependence structure of Normal and Student distributions

Why then, could there be a need to move beyond correlation?

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Risk margins

When calculating risk margins, usual practice would be to follow these steps:

1. Estimate the mean and variance of liability for each LoB;
2. Estimate the associated correlation matrix;
3. Hence estimate the mean and variance of the total liability across all LoBs;
4. Assume some convenient distribution for this total liability, usually log normal;
5. Calculate the 75-percentile from this distribution.

Frequent assumption for step [2] would be consistent with Bateup and Reed (2001) and/or Collings and White (2001).

Capital margins

When calculating (high percentile) capital margins, usual practice would be to follow these steps:

1. Estimate the distribution of liability for each LoB;
2. Assume a copula across the LoBs, most commonly t-copula;
3. Perform a multivariate simulation of liabilities for all LoBs;
4. Form the replicates of total liabilities across LoBs, and read off required percentile.

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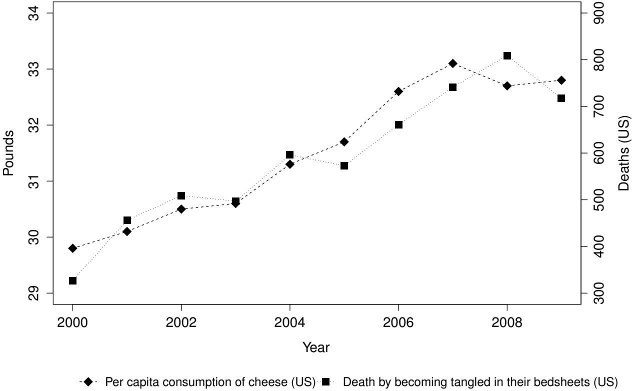
Assume high correlation when there should be none



Correlation = 0.95 !

Example from `http://www.tylervigen.com`

How pairwise independence can go wrong

Consider a sample of 20,000 observations of X, Y and Z. These observations yield the following correlation matrix:

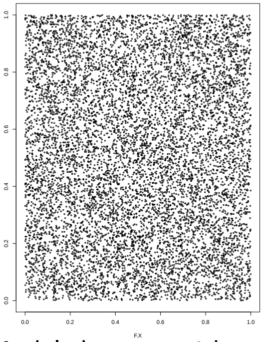
	X	Y	Z
X	1.00	-0.00	-0.01
Y	-0.00	1.00	-0.00
Z	-0.01	-0.00	1.00

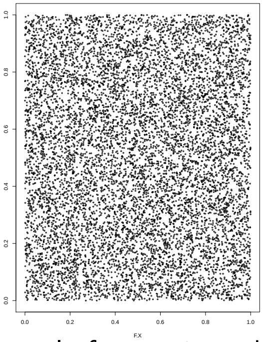
Furthermore, the three random variables look perfectly Normal, all with mean 5 and standard deviation 2.

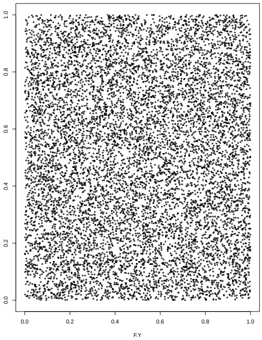
Let us look at their dependence structure

Empirical copulas (scatterplots of respective cdf's of data) of...

Y vs X Z vs X Z vs Y



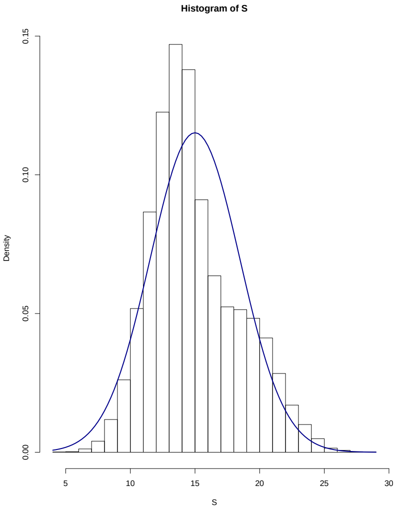




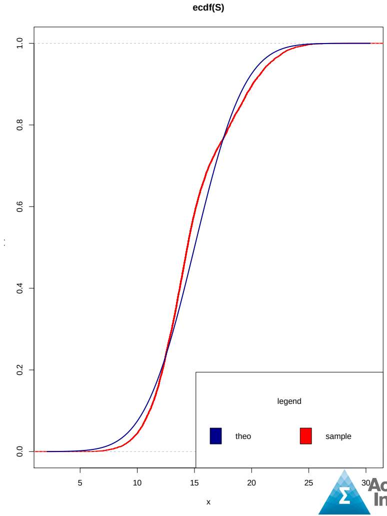
Model the sum with a Normal of mean 15 and variance 12.

What could go wrong?

Histogram of S versus density of a $\mathrm{N}(15, \sqrt{12})$



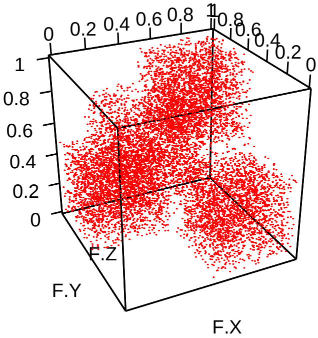
CDF of S versus CDF of a Normal



So what went wrong?

Scatterplot of $F(X), F(Y), F(Z)$:

- All three variables are Normal$(5, 2)$
- Not only pairwise uncorrelated, but actually pairwise independent
- BUT: not mutually independent, AND dependence structure is not "normal" (depends on concordance of X and Y)



$$
\begin{aligned}
X &\sim N(5, 2),\\
Y &\sim N(5, 2),\\
Z &= 5 + |W| \cdot \operatorname{sign}[(X-5)\cdot(Y-5)], \text{ where}\\
W &\sim N(0, 2).
\end{aligned}
$$

Another example

Consider a sample of 20,000 observations of X, Y and Z. These observations yield the following correlation matrix:

	X	Y	Z
X	1.00	-0.00	0.36
Y	-0.00	1.00	0.52
Z	0.36	0.52	1.00

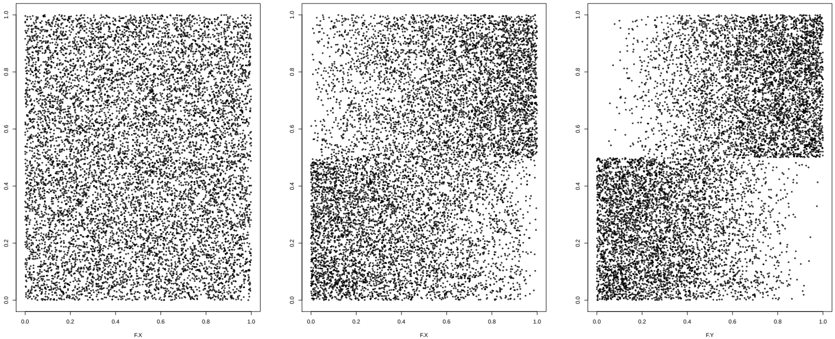
Furthermore, you find that the three random variables look perfectly Normal, with

$$X \sim N(500, 200), \qquad Y \sim N(1000, 300), \qquad Z \sim N(2000, 400)$$

Let us look at their dependence structure

Empirical copulas (scatterplots of respective cdf's of data) of...

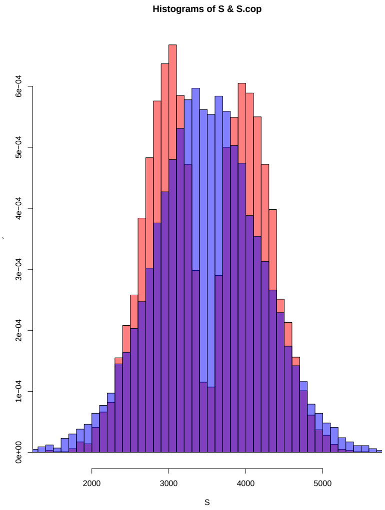
Y vs X Z vs X Z vs Y



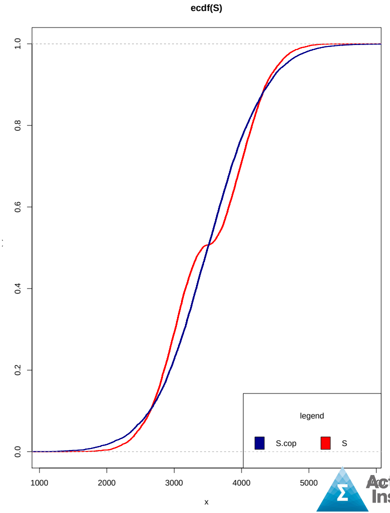
Simulate the sum with a t-copula fitted to the data.

What could go wrong?

Histogram of true S (red) and S via t-copula (blue)



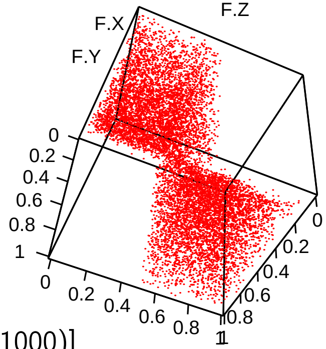
CDF of tre S versus CDF of S via t-copula



So what went wrong?

- All three variables are Normal
- There is dependence, but it not well modelled by correlations
- Z is high (higher than mean) when the sum of $X + Y$ is high (more than its mean), low otherwise

Scatterplot of $F(X), F(Y), F(Z)$:

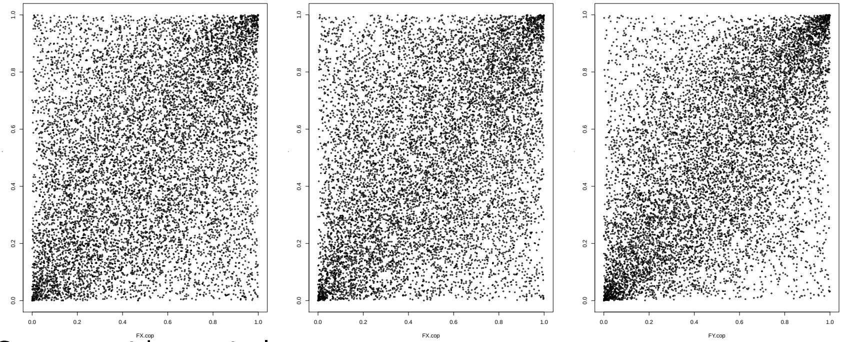


$$
\begin{aligned}
X &\sim N(500, 200), \\
Y &\sim N(1000, 300), \\
Z &= 2000 + |W| \cdot \text{sign}[(X + Y - 500 - 1000)] \\
W &\sim N(0, 400).
\end{aligned}
$$

Simulated (model) dependence structure

Empirical copulas (scatterplots of respective cdfs) of...

Y vs X $\quad$ Z vs X $\quad$ Z vs Y



Compare with emprical ones:

This is not the right shape

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Conclusions

- Correlation is a symptom, not a cause
- Furthermore, correlation sometimes fails to be present when the cause itself is present

Conclusion:

- Look for causes of dependence, and model those first
- Use correlation only as last resort, and only for residual, unexplained 'stuff'
  - and in a way that is consistent with the model that is used
  - and only after having carefully assessed whether correlation is an appropriate implicit dependence structure for this problem

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Implicit modelling

The effect of drivers is approximated via an abstract dependence structure

For example: copulas, correlations being one special case (Gaussian / t-copula)

- often quicker to implement
- often necessary when not all drivers of dependence are known and able to be modelled (the usual case).

Explicit modelling

Drivers are explicitly identified in the model.

For example: common shock models, inflation models, CAT models

- explicit modelling often simplifies projections (for a number of reasons)
- can provide insights about the drivers of dependence which can then be monitored or even controlled.

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Macro modelling

Models directly aggregate quantities corresponding to a certain time frame

Uses aggregate data and random variables

For example: reserving triangles (e.g. quarters or years)

- traditional approach
- can sometimes be inflexible (with exceptions)
- models are very well known and enjoy decades of experience and understanding
- roots development in current practice

Micro modelling

Models payments / claims *dynamic processes*
Uses granular data such as daily individual transactions on all contracts, and stochastic processes

For example: multiple decrement Markov models (in life), see also case studies below for GI applications

- way less developed, still in its infancy
- more flexible
- profits from all the data that is available (also more responsive)
- can be computationally intensive

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Implicit vs Explicit

- As explained earlier, both can (and probably should) be used together
- Explain what you can explicitly, then have an implicit structure for the residuals

Macro vs Micro

- Macro easier, but Micro should always be at least as good (in terms of predictive power/precision)
- The (still open) question is whether micro is *always* worth the effort

Implicit vs Explicit: Claims models

Classical compound Poisson model

$$S_j = \sum_{i=1}^{N_j} X_{ij},$$

has issues:

- you can apply a copula on the aggregate S_j, but that does not differentiate between behaviours of frequency N_j and severity X_{ij}
- the only way to include dependence in frequency N_j is via common shock
- the only way to include dependence in severity X_{ij} is at those common shocks (clearly not adequate for drivers other than "events" such as superimposed inflation)

Micro vs Macro: Advantages...

of stochastic process approach: (over random variable approach)

- individual data can then be used to fit the model, which is particularly useful for dependence modelling (hundreds of thousands of data points vs a few)
- time consistency (adaptable to different timeframes without need of a full recalibration, or even reformulation)
- spatial vs temporal diversification benefits
- in some cases, easier to aggregate (bottom up modelling approach)
- opportunity to better (or at least explicitly) model some realities of the business such as reporting delays, autocorrelation, overdispersion, etc. . .

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Tweedie approach to multivariate loss reserving

Based on Avanzi, Taylor, Vu, and Wong (2016f)

- A macro approach to reserving for dependent lines of business
- Model inputs - standard (aggregated) loss triangles.
- Aim is to produce a multivariate model that provides
  - Sufficient flexibility in the marginal distributions (in particular, beyond lognormal)
  - Transparent introduction of dependence structure

Flexible marginal distributions via the Tweedie family

The Tweedie family of distributions

- Is a major subclass of the EDF
- Has members frequently used for loss reserve modelling: Poisson, gamma, compound Poisson-gamma, etc
- Is a generalisation of the plain vanilla Chain Ladder Poisson model

Multivariate Tweedie model

The multivariate Tweedie distribution for standardised claims

- Introduces cell-wise dependence explicitly through a

  "common shock + idiosyncratic risk"

  structure
- Corresponding decomposition of Mean, Variance, and Covariance
- Availability of cumulants of the sum in closed form

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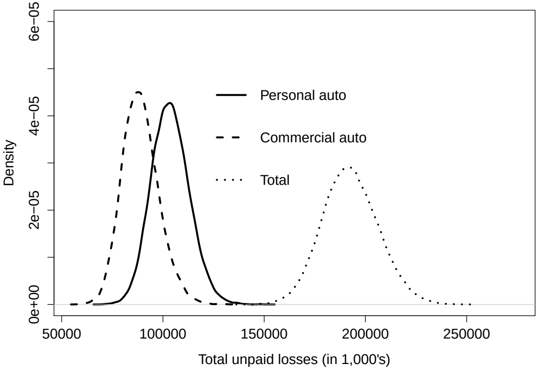
Case study - Pennsylvania National Insurance Group

- Data consists of 2 business lines: personal auto and commercial auto
- Collected from the period 1988 to 1997
- Pearson correlation around 0.39 after accounting for accident and development year effects
- Heatmaps suggest no physical interpretation of any systematic trends beyond correlated noise.

Model fitting and outcome

- Model fitted using MCMC techniques
- Performance of fitting procedure further assessed using simulated data set
- (Fast) simulations used for forecast and quantiles



Key insights

- The multivariate Tweedie framework provides a variety of desirable properties including
  - Flexibility via Tweedie distribution
  - Ease of interpretation of the mean and variance
- The multivariate Tweedie framework provides a general approach to introduce dependence explicitly
- This framework can be extended or modified to capture dependence in other dimensions (e.g. calendar year, accident year, etc.)

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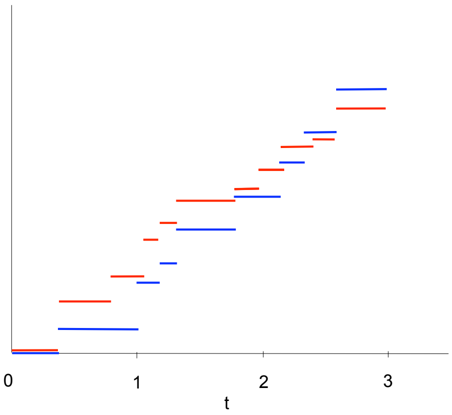
Introduction to Lévy copulas

Based on Avanzi, Cassar, and Wong (2011)

- A micro level approach to dependence modelling between claims processes driven by common events
- Lévy copula approach straddles the advantages of the (parameter intensive) common shock and (non time consistent) distributional copula approaches:
  - Parsimonious
  - Time-consistent
  - Allows for a coherent modelling of dependence in frequency separate to dependence in severity
  - Makes full use of the available data
  - Enables a "bottom-up" approach to model building

Dependent bivariate compound Poisson process



- constituted of unique ($\perp$) and common ($\|$) jumps:

$$\begin{cases} S_1(t) = S_1^{\perp}(t) + S_1^{\|}(t) \\ S_2(t) = S_2^{\perp}(t) + S_2^{\|}(t) \end{cases}$$

- $S_1^{\|}(t)$ and $S_2^{\|}(t)$ with intensity $\lambda^{\|}$
- Joint survival function of common jumps $\overline{F}^{\|}(x_1, x_2)$ (may be dependent)

Sklar's theorem for Lévy copulas (bivariate case)

- For the marginal compound Poisson processes $S_i(t)$ $(i = 1, 2)$, the *tail integral* $U_i(x)$ is given by

$$U_i(x) = \lambda_i \overline{F}_i(x).$$

- The *joint tail integral* measures jumps which occur simultaneously

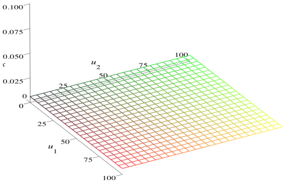
$$U(x_1, x_2) = \lambda^{\|} \overline{F}^{\|}(x_1, x_2).$$

Sklar's Theory for Lévy copulas: A Lévy copula $\mathfrak{C}$ couples the marginal tail integrals and the joint tail integral so that

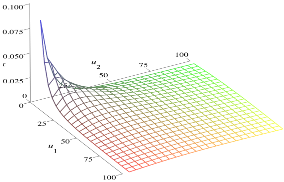
$$U(x_1, x_2) = \mathfrak{C}(U_1(x_1), U_2(x_2))$$

Examples of Lévy copulas

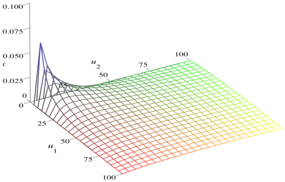
Pure Common Shock



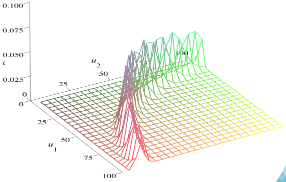
Clayton



Archimedean model I



Archimedean model II



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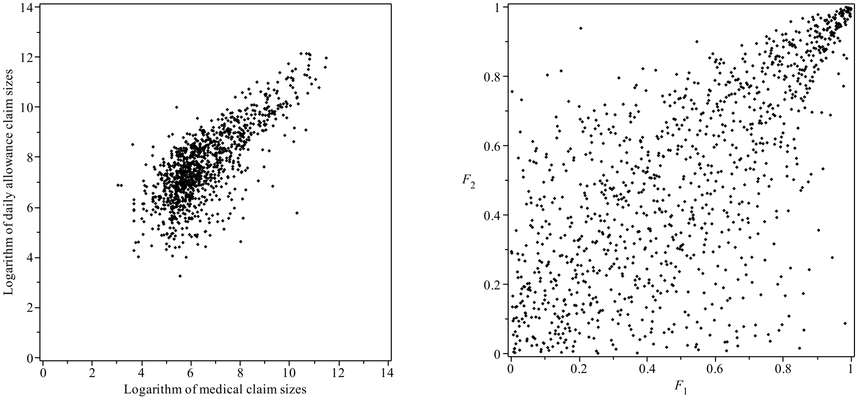
Case study: Swiss Workers Compensation

- We use data provided by SUVA, a Swiss worker's compensation company incorporated under public law.
- The dataset consists of a random sample of 5% of claims from the construction sector for accidents incurred in 1999 (developed as at 2003).
- It features two classes of claims: 2249 medical claims and 1099 daily allowance claims.
- 1089 claims are common to both classes.

Scatterplot of log sizes of 1089 common claims (left) and empirical copula (right):



There is obvious right tail dependence. Back

Model fitting

- The model is fitted using IFM / Likelihood methods.
- Best fit was Gumbel and Gaussian for the logged costs of Medical and Daily allowance
- A1 Lévy copula was judged to be best fitting based on a combination of
  - Common event frequency
  - Common event copula
  - Empirical vs Theoretical Tail Integrals.

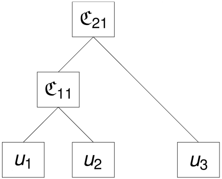
Beyond 2-dimensional Lévy copulas

Based on Avanzi, Tao, Wong, and Yang (2016b)

- Lévy copulas provide a parsimonious approach to modelling dependence between Lévy processes.
- In stochastic processes with at least three components, complex structures and non-exchangeability issues arise.
  - Non-exchangeability occurs when *pair-wise* components exhibit varying structures.
- We use the idea of nesting to provide non-exchangable dependence structures.

Nested Archimedean Lévy copulas

- Nesting essentially means that selected lines are coupled one step at a time



- Alternative nesting options are available when considering 4 or more dimensions
- Implication of construction - components with a higher level of dependence generally need to be combined first

Case Study - Danish fire data

- The Danish fire data contains a trivariate set of fire losses:
  - Building
  - Contents
  - Profit
- Adjustments to data:
  - Removed observations in individual processes less than 1 million.
  - Homogenised the overall Poisson process in the data.
  - Removed two outliers

Danish fire data (continued)

Process	Number of Jumps
Unique to B	472
Unique to C	88
Unique to P	0
Common to B and C but not P	175
Common to B and P but not C	0
Common to C and P but not B	12
Common to B, C and P	56
Total	803

Table: Total number of jumps in each process

Danish fire data (continued)

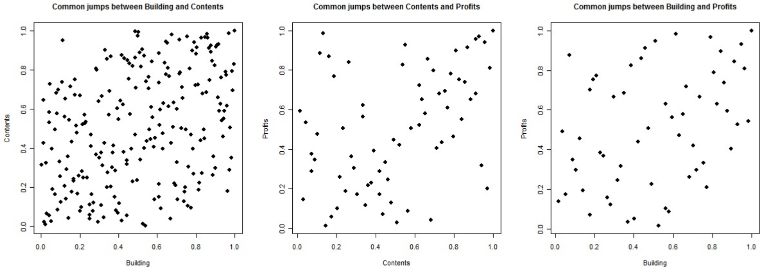


Figure: Empirical severity copula of common jumps between: Building and Contents (left), Contents and Profits (middle), and Building and Profits (right)

Model building

- With a trivariate model, various model choices (and associated parameters) are required. These include
  - Marginal distributions for each line
  - Order of nesting
  - Generator / bivariate Lévy copula to be applied at each level.
  - (with a higher dimension model the type of nesting will also need to be considered)
- We fitted our model using a step-wise, "bottom-up" approach
- Fitting results were good, with a nested A1 Lévy copula being the selected model
- Trade-off of fitting building-contents vs profits

Key insights

- Lévy copulas as a
  - Parsimonious
  - Time-consistent

  approach to modelling dependence between Lévy processes
- Comparison and development of new Lévy copula models
- In three or more dimensions, nesting procedures are available to provide non-exchangable structures
- Fitting / Goodness of Fit procedures available.

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Cox process modelling of insurance claims processes

Based on Avanzi, Wong, and Yang (2016g)

We develop a micro (stochastic process) approach using a Cox process:

- Such processes exhibit:
  - Over-dispersion
  - Serial dependency across time
- Allows for practicalities including reporting delays and changes in exposure
- Extendable to a multivariate setting

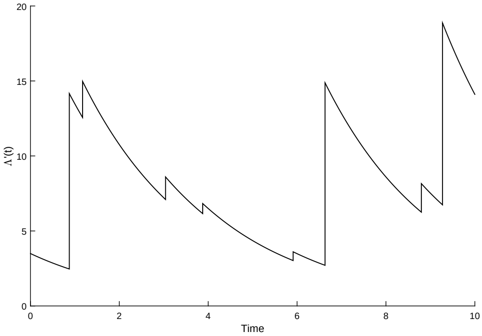
Model development

We design the following model

- The intensity of claim arrival is a stochastic process
  - proportional to the risk exposure
  - subject to external economic and environmental events (that cause jumps of claim frequencies)
- In the following we focus on a shot noise intensity
- Given the stochastic intensity, the arrival of claim follows a Poisson process
- A claim is subject to a reporting delay distribution

Shot noise intensity

- A shot noise intensity process is non-negative and exhibits mean reversion:



Alternative - Regime switching intensity

Based on Avanzi, Taylor, Wong, and Xian (2016c)

- Intensity could be driven by a Markov chain consisting of two types of components:
  - λ_i, the claim intensity in regime i
  - q_{ij}, the transition rate from regime i to j
- The number of regimes can be chosen using various statistical techniques, or they can be left up to the user
- Computationally efficient

Back to Shot Noise Cox: Model calibration

- Development of likelihood-based estimation (via an EM approach); see also Avanzi, Liu, and Wong (2016a)
- Allowing for the discrete nature of real data
- Joint estimation of both the reporting delay and claim arrival process.

Case study - AUSI building and contents

We apply our model to a real dataset that

- correponds to the Building and Contents business of a major Australian general insurer
- includes observations from 01/July/2013 to 04/January/2015

Furthermore, we

- randomly subset the data
- allowed for exposure (measured by the number of policyholders)
- removed catastrophe claims
- aggregated the data into weekly intervals (resulting in a 104 by 104 counts triangle).

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Model forecast

- Algorithm filters the (unobserved) intensity process
- Forecast based on fitted model below (aggregated for illustration)
- Associated quantiles also available.

	reporting quarter						
accident quarter	1	2	3	4	5	≥ 6	IBNR
2013-Q3	2039	70	37	12	5	29.86	29.86
2013-Q4	3884	153	50	24	13.85	57.83	71.68
2013-Q1	5931	147	47	32.03	20.46	85.41	137.91
2013-Q2	5196	121	51.15	28.25	18.05	75.34	172.79
2013-Q3	4808	118.95	47.21	26.08	16.66	69.54	278.43

Key insights

- Development of a Cox model for the arrival process of claims
  - over-dispersion
  - serial dependency
  - risk exposures and reporting delays
- Much more realistic micro level model
- Estimation and prediction:
  - good performance of estimation and prediction
  - simultaneous calibration both the arrival and reporting models leads to better results
  - filtering leads to interesting insights

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Dependence modelling using Cox processes

Based on Avanzi, Taylor, Wong, and Yang (2016d)

- The Cox approach readily lends itself to modelling dependency bewteen multiple lines (on top of common events)
- There are two sources of apparent dependency
  - systematic effect
  - stochastic noise
- Research questions
  - how to allow for the systematic effect?
  - how to create dependent stochastic noise?

Dependent intensities

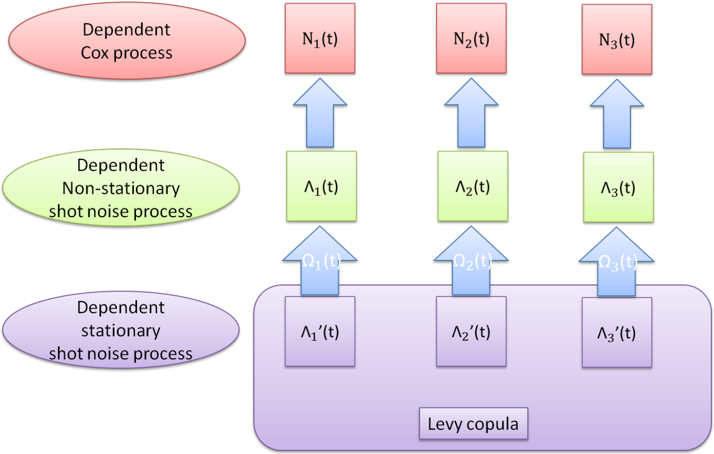
The idea of a common shock in intensities in different Lines of Business

- some shots arrival simultaneously on more than one LoB's
- such a common shot triggers dependent sizes of jumps
- dependence structure impacts the likelihood of claiming, not the claiming itself

We adopt a bottom-up approach in model construction

- each LoB is modelled separately with a shot noise Cox process
- we use a Lévy copula model to create the dependence

A multivariate Cox model



Key insights

- Development of a dependency model for the claim arrival processes of multiple LoB's
- Allowing for both systematic factors and dependent stochastic noise
- Empirical correlation can be very misleading with the presence of systematic drivers

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Summary

Our team is interested in:

- better representing the fine, complex structures driving the dependencies relevant to ERM of insurers
- the stochastic process approach, but only if the extra effort is warranted
- extending methods currently used in practice

In this presentation we

- discussed the relevance of correlation for dependence modelling
- discussed approaches for the modelling of dependence
- illustrated those with some of our research and real data

Conclusions: Beyond correlation

At our last Insights session, we showed that most correlation could be ‘modelled out’ of the AUSI dataset. Such an approach does not solve the problem entirely, because:

- correlation is a symptom, not a cause
- correlation can work as a proxy for unexplained dependency drivers, but
- it can also fail: not all dependence structures are well represented by correlation
- preference for explicit modelling of the “fine, complex dependence structures”

Conclusions: Recent developments

Recent literature:

- Macro literature still focuses on implicit dependence structures
- Micro literature still trying to get the univariate case right
- Multivariate micro models very scarce

Recent developments and case studies:

- We developed an explicit macro reserving extension of chain ladder (with Tweedie)
- We developed implicit and explicit methodologies for combining micro level models, especially in reserving (with the Cox process)

What next?

Major open questions:

- Modelling of severity for micro models
- Modelling of major drivers, such as superimposed inflation
- Micro vs Macro question

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Macro models with implicit dependence structures I

Non-parametric reserving models with correlations

- Multivariate chain ladder: introduced by Schmidt (2006); Merz and Wüthrich (2008) derived the estimator of prediction error of outstanding claims in this model. See also Zhang (2010)
- Multivariate additive loss reserving model, introduced by Hess et al. (2006); see Merz and Wüthrich (2009b) for the prediction error
- Merz and Wüthrich (2009a) combine the two above

Parametric reserving models with copulas:

- Shi and Frees (2011); Shi (2014) develop regression model with cell-wise and calendar year dependence. Uses mainly Gaussian copula (first introduced by De Jong, 2012).

Macro models with implicit dependence structures II

- Abdallah et al. (2015) recently extended the above model by adding an (explicit) random calendar year effect to the mean structure, and by considering hierarchical Archimedean copulas
- Zhang and Dukic (2013) develop a flexible Bayesian copula framework for cell-wise dependence between lines.
- multivariate lognormal on incremental claims (Shi et al., 2012) or log-link ratios Merz et al. (2013).

Parametric reserving models not with copulas:

- Taylor and McGuire (2007): synchronous bootstrap with GLMs

Macro models with explicit dependence structures

Common shock models

- Abdallah et al. (2016) use bivariate Sarmanov distributions on top of a standard regression model
- Avanzi, Taylor, Vu, and Wong (2016f): use Tweedie marginals with common shocks to model cell-wise dependence

Micro models with implicit dependence structures

Lévy copulas:

- with exchangeable structures: Avanzi, Cassar, and Wong (2011), Esmaeili and Klüppelberg (2011), Esmaeili and Klüppelberg (2013)
- with non-exchangeable structures: Grothe and Hofert (2015), Avanzi, Tao, Wong, and Yang (2016b)

Univariate Micro models for reserving

Marked Poisson (continuous time):

- Jin (2013), Antonio and Plat (2014), Ekberg (2015), Van Oirbeek et al. (2015)
- link to cluster theory: Matsui (2015) and references therein

Marked Poisson (discrete time):

- Pigeon et al. (2013, 2014), de Souza and Veiga (2014), Alm (2015)
- with GLM: Tao (2014)
- with a focus on claim counts: Charpentier and Pigeon (2016)

Non-parametric:

- Rosenlund (2012), Godecharle and Antonio (2015)

Marked Cox:

- Shot noise: Avanzi, Wong, and Yang (2016g)
- Markov modulated intensity: Avanzi, Taylor, Wong, and Xian (2016c)
- Marked Cox with discrete time Markov chain: Badescu et al. (2015, 2016)

On micro vs macro models:

- Jin and Frees (2013), Huang et al. (2015b,a, 2016), Avanzi, Taylor, Wong, and Yang (2016e)

Micro models with explicit dependence structures

Using Shot noise Cox processes:

- Avanzi, Taylor, Wong, and Yang (2016d)

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