SM Bonds: A Proposal to Manage, Price and Transfer Longevity Risk

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SM Bonds – a New Product for Managing Longevity Risk

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Abstract

A new type of retirement bond is proposed called an SM bond. SM bonds are long dated government bonds divisible into two parts – a survivorship (S) part and a mortality (M) part.

Each SM bond is associated with a particular age and a particular age SM bonds are only purchasable by (originators) of that age. The SM bond is then splittable into an S and M component. The S part must be retained by the originator, who receives the face value of the bond if he/she is alive at maturity. Originators hedge their own longevity risk by originating a sequence of S bonds at different ages throughout their working life time.

For originators who die prior to the maturity date, the maturity value of the SM bond is assigned to a mortality pool. The holder of the M part of the bond receives the annual bond coupon, and at maturity a pro–rata share of the mortality pool. Hence the holders of M bonds bear the longevity risk for the cohort. Since SM bonds are government–guaranteed, there is minimal default risk for both S and M holders.

M bonds are tradable: holders can sell their M bonds at any time. It is envisaged different age bonds are issued every year for ages say 30 to 64 each with say a 35 year term. The market will be regularly informed about the mortality experience of each age cohort, and the market price of the M bonds will vary over time to reflect that experience.

M market participants will be able to continually adjust positions and arbitrage between bonds of different ages and times to maturity. Arbitrage trading will promote consistent and fair longevity pricing. The market for M bonds will provide a basis for pricing a broader range of longevity products.

Keywords: Longevity risk, retirement income.

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1. Introduction

SM bonds proposed in this paper aim to address key problem areas for individuals planning their retirement; for governments grappling with budgetary problems due to the social security demands of an ageing population; and for the private sector attempting to develop, market and price longevity products. SM bonds will facilitate:

1. The ability of individuals to, in effect, buy government guaranteed retirement income streams at competitive market prices.
2. The ability of governments to directly facilitate and incentivise retirement income planning by individuals whereby incentivised savings can be used to provide for individual retirement income streams that cannot be capitalised before death and cannot be passed to heirs.
3. The ability of governments to absolve themselves of longevity risk.
4. The operation of transparent and deep longevity risk markets.

Individuals planning for retirement face a great deal of uncertainty: uncertainty about their own lifespan, uncertainty about future investment returns on retirement savings, and uncertainty about the future cost of living. Retirees can manage these risks by purchasing indexed lifetime annuities. Economic models, such as those presented in Yaari (1965) and extended by many others (Davidoff et al., 2005), suggest that rational risk averse utility-maximising investors should invest at least part of their savings in lifetime annuities.

Despite this theory, retirees appear reluctant to buy lifetime annuities. This reluctance has been observed in annuity markets in several different countries (Rusconi, 2006; Brown, 2009). The discrepancy between the theoretically optimal behaviour and observed behaviour is commonly described as the "annuity puzzle" (Modigliani, 1988; Benartzi et al., 2011).

Many authors (Lloyd, 2014; Antolin, 2008; Brown, 2009; Rusconi, 2008; Brown et al., 2008) have attempted to explain the annuity puzzle. Possible explanations include:

- Cost. Surveys have shown that many retirees regard lifetime annuities, purchased from life insurance companies, as poor value for money. Lifetime annuities can be expensive (for reasons given in Section 6.2 below).

- Liquidity needs. Lifetime annuities provide a pre-determined sum of money each year, and do not usually allow for ad hoc withdrawals. Retirees may well be concerned about the availability of money to cover unexpected large expenses, e.g. health care costs or the cost of home repairs. Flexibility and control are important for many retirees (Beshears et al., 2014; Stewart, 2007; Echalier et al., 2015)

- Bequest motives. Lifetime annuities usually cease on the death of the annuitant. The retiree may wish to provide bequests to others.

- Health status. Those in poor health (or with shorter life expectancies than the average annuitant) may be especially reluctant to buy lifetime annuities, since the value of benefits received is likely to be lower than the cost of the annuity. Economic models allowing for health uncertainties indicate that this might explain the low level of demand for annuities (Reichling and Smetters, 2013).

- Social security interactions. The retiree may prefer to spend money maintaining a better standard of living in the early years of retirement, and then rely on government provided safety–net benefits to cover income needs at advanced ages. Safety–net systems might "crowd out" annuity products. The potential for "crowding out" will depend on the design of the safety net system, hence demand for annuities may vary across countries (Rusconi, 2008).

- Insolvency risk. The retiree may be concerned about the risk that the annuity provider (a pension fund or insurance company) will become insolvent and hence fail to pay promised benefits. Beshears et al. (2014) found that "worry that the company may not be able to pay me in the future" was a key concern for American survey respondents.

- Financial illiteracy and poor decision–making. People may have trouble understanding annuity products (which are often quite complex) and/or may have a poor understanding of the riska arising from reliance on account–based retirement products (Silcock et al., 2014). There is a great deal of evidence to show that people have difficulty in making rational decisions about retirement savings (Thaler and Sunstein, 2008). For example, the demand for annuities is affected by the "framing" of the alternatives. (Brown et al., 2008; Benartzi et al., 2011; Lloyd, 2014).

A low level of voluntary annuitisation creates problem for government policy–makers. If retirees do not manage their risks effectively, then it is likely that governments will be called upon to deal with the consequences. If retirees outlive their savings, it is likely that governments will need to provide "safety–net" benefits for the elderly, e.g. social security benefits funded from general tax revenue. For countries with ageing populations, this additional cost will increase budgetary pressures and may become increasingly unsustainable. Therefore policy–makers have a strong incentive to improve the market for annuities.

Many policy advisers have argued in favour of government intervention to improve annuities markets (Antolin and Blommestein, 2007; Blake et al., 2014; Sherris and Evans, 2009; Stewart, 2007; Brown and Orszag, 2006; Blake and Burrows, 2001). Blake (1999) proposed two different methods for government involvement. Firstly, the government can sell annuities directly to the public, effectively allowing people to buy state pensions. Secondly, the government can issue longevity bonds (also known as survivor bonds), which would allow life insurance companies to hedge longevity risks more effectively. That is, the government can provide support for private sector provision of annuity products.

Others (Brown and Orszag, 2006) have expressed reservations about government intervention. If a government sells annuities or issues longevity bonds, mortality risk is transferred to the government, i.e. to future generations of taxpayers. In most developed countries, the ageing of the population is already expected to cause government budgetary pressures arising from increased health and social security costs. If governments are also exposed to additional mortality risks as a result of providing longevity bonds, this is likely to exacerbate the problem (Stewart, 2007). Treasury departments are seldom enthusiastic about taking on longevity risks (Rusconi, 2006).

Furthermore, government–provided longevity bonds may increase the risk of inequitable intergenerational risk transfers. Politicians tend to focus on the needs of the current generation of voters, discounting the impact on future generations. This provides an incentive to underprice government–issued annuities (Brown and Orszag, 2006). Again, such behaviour would exacerbate the fiscal problems caused by an ageing population.

SM bonds can be used to address these concerns. The bonds provides some of the above–mentioned advantages of government backed annuities, without imposing longevity risks on future generations of taxpayers.

Many countries presently have "Pillar 2" retirement savings systems with retirement funds set up on a defined contributions basis with each member having an individual retirement savings account. A percentage of the employee's salary is periodically paid into the account. In some countries, such contributions are mandatory; in other countries, contributions are encouraged by a variety of incentives (tax concessions, auto–enrollment, etc). The member's money is invested to earn investment income, which is credited to the account. Management fees are deducted from the account. On death, disablement, or retirement, the member is entitled to receive the balance of the account.

The aim of "Pillar 2" retirement saving systems are typically for members to secure an adequate retirement income while relieving the government of the financial burden of social security costs. In some countries the system permits tax incentivised open–ended saving and wealth creation without any requirement to annuitize. This flexibility may undermine the government's policy objectives.

We suggest that the retirement savings accounts should include investment in SM or S bonds. Each year, some minimal amount of every retirement savings account would be allocated to the purchase of SM bonds sufficient to provide an adequate guaranteed retirement income stream.

The further sections of this article are as follows. Section 2 describes the mechanics of SM bonds and the impact on different stakeholders. Section 3 describes the market for M bonds. Section 4 describes the pricing of M bonds and, as a consequence, the price of S bonds. Different patterns of S bond originations and their impact on retirement income streams are discussed in Section 5. Section 6 compares SM bonds to annuity products currently available in the market. Section 7 considers age pooled SM bonds to simplify the operation of the SM bond market. A final section draws conclusions.

2. SM bonds: a public/private collaboration

An SM bond is a long-term fixed-interest government-guaranteed bond with special features. Suppose the term of the government bond on which the SM bond is based is n years, a coupon of c per annum is payable in arrears, and the maturity payment is equal to the face value of 1. Assuming that the government default risk is negligible and r is the continuously compounded risk free rate for a bond with this duration, the present value of the bond payments is

$$ca_{\overline{n}|} + \mathrm{e}^{-rn} \ , \qquad a_{\overline{n}|} \equiv \frac{1 - \mathrm{e}^{-rn}}{\mathrm{e}^r - 1} \ . \qquad (1)$$

So far, the SM bonds looks just like a conventional n year government bond. However SM bonds have added features.

Each calendar year the government issues a series of SM bonds; each issue associated with a particular age x. Age x bonds are only sold to persons aged x last birthday at the date of issue. Hereafter we use the term *originator* to refer to the purchaser of an SM bond.

Each SM bond is divisible into two components: the survivor component (S bond) and the mortality component (M bond). The S bond must be retained by the originator. If the originator survives to the maturity date, then he or she receives the face value of the bond. If the originator is dead at the maturity date, the face value is paid into a "mortality pool," with separate pools for each age x and issue date combination.

M bonds are tradeable and are typically sold by originators immediately upon origination. The market mechanism for trading M bonds is discussed in Section 3. M holders receive coupons of c each year, plus a variable maturity value. The maturity value is a pro–rata share of the mortality pool with the amount in the mortality pool proportional to the number of deaths in the age x cohort. Hence M holders bear all the (pooled) longevity risk for this cohort of originators.

The subsections below consider SM bonds from the perspective of each of the three stakeholders; the government which issues the bonds, the originator who holds the S bond, and the investor who buys the M bond.

2.1. Perspective of SM bond issuer

From the government's perspective, cash flows for SM bond issues are exactly the same as the cash flows for any conventional fixed interest bond. At the date of issue, the government receives the price (1). The government then pays an annual coupon c and at the maturity date it pays the face value of 1 either to the originator (if alive) or into the M pool (if the originator is dead). There is no uncertainty in the amount or timing of the payments: the government bears no longevity or mortality risk.

Since there are restrictions on who can buy SM bonds and the sale of S components, SM bonds might sell at a discount to the price of a conventional government fixed–interest bond with equivalent payments. Factors affecting the

demand for such bonds (and hence the price) are discussed in more detail in Section 3.

The government will incur administrative costs in issuing the bonds and managing payments. We envisage most SM bonds sold by tender on a wholesale basis. Retirement savings funds will buy SM bonds on a bulk basis on behalf of their fund members. The government can also sell SM bonds to individual investors, "on tap." Tap purchases would be based on the wholesale price, subject to a small charge to allow for additional administration costs.

The government will keep a register of S owners and M owners, to facilitate payments of coupons and the face value. The burden associated with this will be comparable to companies keeping a share registry. The government will also need to keep records of deaths of S bond holders. The agency responsible for administering SM bond payments can be integrated with the national register of deaths.

The term n on SM bonds is typically more than 30 years. Proceeds of the sale of SM bonds are thus best invested in projects delivering very long term payoffs such as transport and utility infrastructure. SM bonds are thus a practical financial instrument for the governments to raise funds for projects that may well be too long term for private investors.

2.2. Perspective of SM bond holder

We envisage that SM bonds will be purchased by the employed population throughout their working life, as an integral part of their retirement savings strategy. Each year, some proportion of each person's retirement savings account would be allocated to the purchase of SM bonds. The SM bonds would be purchased and held by retirement savings funds, on behalf of individual members. Coupon payments and maturity payments from the SM bonds would be paid into the member's retirement savings account. After retirement, the member would draw on his account to provide post-retirement income.

The cost of each SM bond is (1). SM bond holders must retain the S components, and may choose to either sell or retain the M components. The investor can make this decision by using the same principles which are applied to any other investment decision for any other type of asset. The decision will depend on the expected returns on the investment and the riskiness of the investment.

Suppose that the originator retains both the S and M components. The return on this investment will depend on the individual's own survival and the mortality experience of the entire cohort. If the originator survives n years then they receive the coupon c for n years plus the face value of 1 as well as a share of the mortality pool. Therefore a survivor will receive a return which is better than the return they would have received from an investment in traditional government bonds; this may be described as a "survivorship premium."

If the SM bond holder dies before the maturity date, then his heirs will continue to receive the coupons payments of c each year, and will also receive a share of the mortality pool at the maturity date. The maturity value may be anywhere between 0 and 1 (depending on the mortality experience of this

cohort). Therefore, for deceased members, the return on their investment will be less than the yield on a traditional government bond.

The maturity values of SM bonds, as a function of the cohort mortality rate, are displayed in the top left panel of Figure 1. This shows that the SM bond results in a transfer of wealth within the cohort of originators: those who die early fund the survivorship benefits for those who survive.
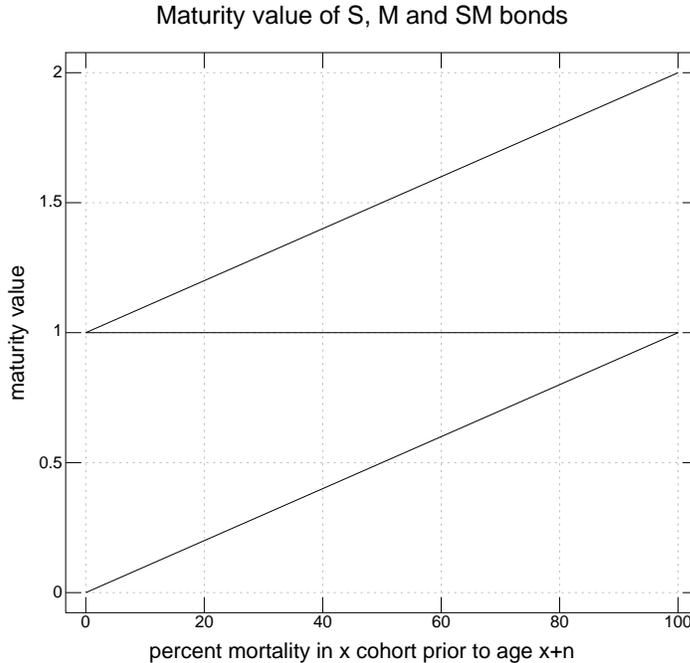
Maturity value of S, M and SM bonds

Figure 1: The top line represents the maturity value for SM bond holders who survive, the middle line represents the maturity value for the holder of a traditional government bond, and the lower line represents the maturity value for the heirs of SM bond holders where the originator dies prior to the maturity date.

Originators who retain both the S and M components are analogous to annuitants who belong to a tontine annuity scheme: each member's benefits are affected by the mortality experience of the entire cohort. Some SM bond holders will be willing to retain this risk. After all SM bonds allow originators to gamble on their own lifespan. Those who survive receive relatively attractive returns on their investment. Historically, many people have been willing to invest in such products as long as the potential returns are attractive enough. For example, tontine life insurance policies were once very popular in the United States. These policies provided low payments for surrenders and early deaths, while promising that all profits would be reserved to provide additional benefits to those who survived until the end of the term. These policies fell into disrepute when the life insurance companies which sold these products mismanaged

the funds investment and expenses , severely eroding the benefits payable at maturity.

The M component carries the cohort longevity risk: if mortality rates are lower than expected, the returns on the M components will be reduced. Some SM bond originators will be willing to accept this risk and will retain the M components. After all, retirement savers normally invest in a mixture of risky assets, including property, shares, fixed interest, and so on. M bonds can be considered as just another type of risky asset which can be included in the investment portfolio. If there is a low correlation between mortality risks and the financial risks embedded in other risky assets, a holding of M bonds may well improve diversification and hence improve the overall risk-return profile of the investor's portfolio.

On the other hand, some originators will prefer to sell the M components and invest the sale proceeds in other asset classes. This decision will be influenced by the market price of the M components, and in particular will depend on the market valuation of longevity risks.

2.3. Perspective of M bond holder

An M bond holder receives the coupon c each year, plus a share of the mortality pool at maturity. Suppose that each of the originators buys an equal quantity of SM Bonds. If the number of originators aged x is $\ell_{[x]}$ and the number of survivors at maturity is $\ell_{[x]+n}$, then the maturity value for M holders will be proportional to the number of deaths:

$$\frac{\ell_{[x]} - \ell_{[x]+n}}{\ell_{[x]}} = 1 - \frac{\ell_{[x]+t}}{\ell_{[x]}} \times \frac{\ell_{[x]+n}}{\ell_{[x]+t}} \ . \tag{2}$$

If people buy different quantities of SM bonds, it is necessary to modify the calculation. Past experience in the pensions and annuities market suggests that there is likely to be some correlation between mortality risk and the quantity of SM bonds purchased, as a result of selection effects (described in more detail in Section 6.2.4). If such a correlation exists, then maturity value should be calculated using amount–weighted mortality tables. In an amount–weighted mortality table, $\ell_{[x]}$ represents the amount of bonds owned by originators aged x, and $\ell_{[x]+t}$ represents the amount of bonds owned by those who survive until age $x+t$. In other words, the mortality table measures the survival of "dollars" instead of the survival of "persons". Amount–weighted tables are commonly used to price pensions and annuities in the private sector

Throughout this paper, we will assume that $\ell_{[x]+t}$ and associated values are based on suitable amounts–weighted tables.

The number of survivors is unknown when the bond is originated. Therefore investors will bear the cohort mortality risk, and they will demand a reward for doing so, i.e. they will demand an expected rate of return which is higher than the risk free rate r. Let the risk-adjusted rate of return be denoted $r + \delta$. If $\hat{\ell}_{[x]+n}$ is an estimate of the (amount weighted) number of survivors, the price of

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the M bond at origination is:

$$M_{[x]} = ca_{\overline{n}|} + \mathrm{e}^{-(r+\delta)n}\left(1 - \frac{\hat{\ell}_{[x]+n}}{\ell_{[x]}}\right) .\tag{3}$$

The M bond may be sold at any time after origination. The market will be periodically informed about the experience of the mortality pool, so that the value of is known at time t (or shortly thereafter, allowing for some time lag in the notification of deaths). This reduces the uncertainty about the maturity value payable at time n. The value of the M bond at time t is:

$$M_{[x]+t} = ca_{\overline{n-t}|} + \mathrm{e}^{-(r+\delta)(n-t)}\left(1 - \frac{\ell_{[x]+t}}{\ell_{[x]}} \times \frac{\hat{\ell}_{[x]+n}}{\ell_{[x]+t}}\right) ,\tag{4}$$

The value of δ, which represents the market price of longevity risk, may vary over time. The M bond market and pricing are further discussed in Sections 3 and 4.

2.4. Perspective of S bond holders

Selling the M component reduces the up–front cost to originating S bonds. Suppose that an SM bond originator sells the M components at time $t = 0$, retaining the S bond only. The net cost of each S bond to an originator is (1) minus (4) at $t = 0$,

$$S_{[x]} \equiv \mathrm{e}^{-rn}\left\{1 - \mathrm{e}^{-\delta n}\left(1 - \frac{\hat{\ell}_{[x]+n}}{\ell_{[x]}}\right)\right\} .\tag{5}$$

The sequence of costs (5), is independent of the coupon c, declines with age x and depends on the current risk free rate r, the risk premium δ and the projected future proportion of deaths in the x cohort over the next n years.

Suppose an individual buys 35 years S bonds from age 30 through to age 64 at cost (5) (ignoring transaction costs). This provides a government–guaranteed retirement income stream commencing at age 65. The income stream ceases on the death of the originator or at age 99, whichever is earlier. The income steam cannot be capitalised and has no residual value after death. This would provide better security in retirement for the individual and reduce social security safety net costs for the government. Alternative patterns of individual S bond origination are discussed in Section 5.

3. The market for M bonds

SM bond originators may prefer to sell their M component, avoiding exposure to pooled longevity risk while retaining their individual longevity insurance. Selling the M component reduces their cost of longevity insurance.

This raises some questions.

- Who will be interested in buying M bonds?

- How will M bonds be sold?

- Will the market for M bonds be efficient?

- How will M bonds be priced?

Blake et al. (2006) suggests that there are three categories of potential buyers for longevity securities: hedgers, general investors, and speculators. Hedgers are those who will suffer losses if mortality rates are higher than expected, e.g. life insurance companies who might have to pay higher–than–expected death benefits on life insurance policies. Life insurers can partially hedge this mortality risk by buying securities which increase in value when mortality rates are higher than expected. This might be achieved by buying M bonds, since higher mortality rates would result in higher mortality pool payments.

General investors may also be interested in buying M components since these securities provide diversification benefits for their portfolios. As Thomsen and Andersen (2007) points out

> The low correlation between an unexpected increase in life expectancy and the yield on other financial instruments is typically cited as the major reason why longevity bonds would be attractive.

Speculators such as hedge funds may also be willing to trade M bonds, if they believe that they can make a profit by trading: by taking positions based on beliefs about future price movements, or by seeking arbitrage profits in an inefficient market: for example where prices $M_{[x]+t}$ and say $M_{[x]+t+1}$ are inconsistent.

Efforts to develop a private sector market for longevity-based securities have not been successful, for reasons outlined in Blake et al. (2006), and Thomsen and Andersen (2007). Thomsen and Andersen (2007) suggests that at present there is an imbalance between supply and demand: there are many pension funds and annuity providers which would like to hedge their longevity risk, but there are not many hedge–type investors who would benefit from an unexpected rise in life expectancy. In order to develop an efficient market for longevity risk, it is necessary to attract general investors and speculators into the market.

We suggest SM Bonds, and in particular the M components, have characteristics which will be attractive to investors:

- Government guaranteed. Payments of interest and principle are guaranteed by the government. Credit risk is minimised, particularly important for a long–term security. Several researchers have already suggested that government issued longevity securities will be useful for the development of longevity markets (Blake and Burrows, 2001; Blake et al., 2006; Thomsen and Andersen, 2007; Sherris and Evans, 2009).

- Standardisation. At present many longevity securities are traded on the over–the–counter–market: each contract is individually tailored. This impedes the development of a liquid, transparent market. Loeys et al. (2007) has suggested that

  > ... in order for a new market to succeed (1) it must provide effective exposure, or hedging, to a state of the world which is (2) economically important and that (3) cannot be hedged through existing market instruments and (4) it must use a homogeneous and transparent contract to permit exchange between agents.

  SM bonds would be standardised contracts, issued in large volumes especially if origination is tax incentivised.

- Diversification. The M bonds pool the mortality risk of all the originators who originate SM bonds. We envisage that the M bond mortality pool should include all tax incentivised retirement savers, covering a cross section of employees from a wide range of occupations and industries. Diversification will produce lower volatility than longevity securities based on the experience of a small, select group (such as members of a specific employer sponsored pension fund).

In order for the M bond market to work efficiently, investors must have data about the mortality experience in the M bond mortality pool. Blake et al. (2006) suggests that accurate and up–to–date information is essential for an efficient market:

> For traded securities it is important that potential investors have access to as much relevant information as possible. The less information investors have, or the less forthcoming the issuer appears to be, the more risky the security might seem, and the less likely it is that they will invest. Thus, it is not untypical for offer documents to contain substantial historical data for potential investors to analyse and assess the risks associated with the security as well as its value.

Therefore the SM bond issuer will need to periodically publish accurate up–to–date information about the experience of each M bond mortality pool. This suggests that the SM originator database be linked into the national register of deaths (normally maintained by government agencies). The mortality experience data would feed into pricing models for the M bonds.

In many countries, retirement savings funds have large numbers of members (for example in Australia, several funds have more than one million members). Large funds have already demonstrated the ability to manage wholesale transactions on behalf of their members at reasonable cost e.g. by purchasing group life insurance and/or purchasing lifetime annuities on a bulk basis. Similarly, large pension funds could arrange to originate SM bonds and sell the M components on behalf of their members. Sale proceeds, less transaction costs, would be credited back to the members' accounts.

Smaller pension funds might not be able to manage the process efficiently, so it may be useful to have a centralised tendering agency. The government agency responsible for issuing the SM bonds to originators can also automatically offer the M bonds for sale via tender, (similar to the tendering process used for other government securities). The agency would be acting as an agent for the originators; the net S bond costs would be deducted from the originators' retirement savings accounts. The transaction and administration systems would be similar to the systems which have already been developed for managing exchange-traded government bonds.

At present, a pension fund is likely to have difficulty hedging its longevity risks, because there is a thin market for longevity–based securities, and each fund has idiosyncratic mortality risks. However the standardised nature of the M bonds, based on a large well–diversified mortality pool, should make it easier to do so.

In the early stages of the development of the M bond market the market may be illiquid and underpriced. If so, SM bond originators may well decide retain the M bonds in their own portfolio. As noted previously M bonds are uncorrelated with other asset classes and will provide diversification benefits. If originators retain their M bonds the system becomes a mutual insurance arrangement where each member of the cohort participates in the the longevity risk of other pool members.

Pension funds can also retain M bonds as a separate unitised asset class to hold on behalf of members. This would allow the pension fund members to diversify their risk across a range of ages and across both male and female mortality pools. The pricing of an investment in this portfolio would be based on M bond prices as further discussed in Section 4.

4. M bond pricing

To value M bonds, potential purchasers must estimate the expected number of survivors for each cohort of originators, allowing for both selection effects and mortality improvements. M bond holders bear the risk that the amount–weighted mortality rate will be less than expected (reducing the mortality pool payment). The price of the M bonds will reflect this uncertainty, i.e. it is reasonable to assume that investors will demand a risk premium for bearing mortality risk.

The size of the risk margin is a matter of conjecture. The risk margin will depend on the degree of uncertainty about the risk profile of the entire age x cohort included in the SM mortality pool, and the uncertainty regarding future mortality improvements. Theoretical approaches to the pricing of longevity risk are, at present, still developing. Attempts to model the cost of mortality uncertainty include Bauer et al. (2010), Cairns et al. (2008) Loeys et al. (2007) and Cairns et al. (2006). At present it is difficult to calibrate these models since there is no active and liquid market for mortality risk. Brown and Orszag (2006) attempt to estimate the cost of mortality improvement risk which has been incorporated into existing annuity products.

Cairns et al. (2006) define and discuss the pricing of a zero–coupon longevity bond. This is a bond that pays out an amount equal to the proportion of people aged x who survive to age $x+n$. This payout can be replicated with M bonds by selling one M bond and buy an equivalent term n, coupon c government bond. This position yield a net zero coupon and a net payout at time $t = n$ of

$$1 - {}_nq_{[x]} = {}_np_{[x]} = \frac{\ell_{[x]+n}}{\ell_{[x]}} \ . \tag{6}$$

The pricing methods discussed in Cairns et al. (2006) and references thereto can be used to price M bonds at the time of origination. Denote this price $M_{[x]}$ as in (4) where δ denotes extra discounting on account of risk. The amount of risk discounting will depend on x, n and t.

At time $t > 0$ after origination the uncertain maturity value of the M bond remains (6), rewritten as

$$ {}_tq_{[x]} + (1 - {}_tq_{[x]}){}_{n-t}q_{[x]+t} \ , \tag{7}$$

where ${}_tq_{[x]}$ is the known proportion of deaths in the x cohort within t years. At time t, ${}_{n-t}q_{[x]+t}$ is unknown. Hence the price of an M bond t years after origination is

$$M_{[x]+t} \equiv ca_{\overline{n-t}|} + \mathrm{e}^{-r(n-t)} \left\{ {}_tq_{[x]} + (1 - {}_tq_{[x]}){}_{n-t}\tilde{q}_{[x]+t} \right\} \ , \tag{8}$$

where the tilde denotes the probability after risk discounting. Given the market price $M_{[x]+t}$, the implied estimate of the risk adjusted survival probability is

$$ {}_{n-t}\tilde{p}_{[x]+t} = \frac{1 - \mathrm{e}^{r(n-t)}(M_{[x]+t} - ca_{\overline{n-t}|})}{{}_tp_{[x]}} \ . $$

These estimates can be used to determine one year risk adjusted survival probabilities assuming the probabilities are independent of the cohort. Pricing anomalies between different M bond prices can be identified and arbitraged.

If there is substantial coupon c, the value of M at the time of origination is dominated by value of the coupon stream and interest rate risks will be more important than mortality risks. This is especially true for bonds associated with younger originators, where the expected size of the mortality pool payment is relatively small.

As time progresses, the maturity value payment becomes a relatively higher component of the value of M. But as time goes by, there is also less uncertainty about the number of survivors. For example, for a 35 year bond, the mortality risk will be relatively low after 30 years: the market will be able to make a fairly reliable estimate of the maturity value due to M holders in 5 years time.

5. Origination patterns and retirement income streams

Governments will need to decide on the terms and conditions of SM Bonds: the range of ages x at commencement, terms n, coupons c, and maturity values (fixed or inflation indexed).

Governments are eager to reduce the cost of future social security pension payments. The government can meet this objective by tailoring the S bond tax incentives: each dollar paid to an S bond holder is potentially a dollar saved by the government.

The terms and conditions of SM bond products will depend on expected retirement ages, life expectancies, and social security eligibility rules. Suppose 65 is the normal retirement age and the eligibility age for social security safety–net pension. Further suppose projected mortality rates indicate that, say, 96% of retirees aged 65 die before attaining age 100, i.e. most people spend less than 35 years in retirement. In this case $n = 35$ year SM bonds can be issued every year, for all who are aged between 30 and 64. These S bonds will provide retirement income for the originators when they are between ages 65 and 99. The government may still bear the social security costs for a small proportion of people who survive for a few years beyond age 100. The government has not entirely eliminated its longevity risk, but the safety–net costs would be substantially reduced.

The SM bond system should be flexible, to adapt to changing demographics. As life expectancies increase, the normal retirement age and social security eligibility age will also likely increase. Several countries have already increased social security eligibility ages and/or announced plans to do so in the future. Suppose that the pension eligibility age increases to age 70. The government could issue SM bonds up to age 69. Those who continue working between age 65 and 70 would continue to buy SM bonds. Bonds purchased during the extended working life (at ages 65 to 69) would automatically provide retirement income to cover the extra life expectancy (up to age 104).

Each originator would originate a certain number of SM Bonds each year, for example via individual or employer sponsored retirement savings accounts. The government might provide a recommended pattern of purchases; and/or retirement savings funds might implement a default pattern of purchases; and/or workers might rely on independent financial advisers to choose a pattern which is more tailored to their own personal circumstances (e.g. allowing for bequest motives which might depend on marital status, number of dependants and so on.) Tax concessions on retirement account contributions and earnings could be conditional on the purchase of a minimal number of SM bonds, sufficient to supply a guaranteed minimal retirement income stream.

The following subsections consider different S bond origination patterns. Suppose n is the term of the SM bond and m is the normal retirement age.

5.1. Originating a fixed number of S bonds at each age

Originating A S bonds at ages $x = m - n$ through to age $x = m - 1$ yields a retirement income stream of A from age m to $m + n - 1$. For example if $m = 65$ and $n = 35$ then there is a constant income stream for ages 65 through to 99.

The cost over time of this income stream is given by (5), displayed in the left panel of Figure 2 for $n = 35$ year S bonds with total face value of $A = 1$ for each age from 30 to 65, based on male mortality derived from Australian

Life Tables 2005–2007, a risk free rate of $r = 1\%$, 2%, 3% and 4%, and a risk adjustment factor $\delta = 0.25\%$. Since n year survival probabilities decrease with age, the cost (5) decreases with age. Higher curves in this panel correspond to smaller r.
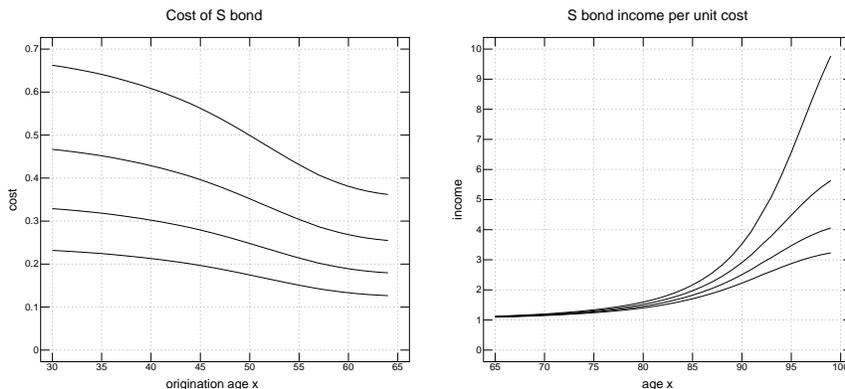

Figure 2: Left panel displays $S_{[x]}$, the cost at age x of 1 unit of retirement income at age $x + 35$ for $r = 1\%$ (highest curve), 2%, 3% and 4%, and $\delta = 0.25\%$, based on males Australian Life Tables 2005-2007 mortality rates and $n = 35$ year bonds sold at par. Right panel displays retirement income streams for given a unit value of origination at each age for the r and mortality assumptions as in the left panel.

This origination pattern only makes sense if there little or no inflation over the n years of each bond. When deciding on the best pattern of S bond purchases, originators must look ahead to estimate their income needs in retirement. This requires some estimates of future cost–of–living increases over a very long time horizon. If inflation is higher than expected, planned retirement incomes may prove inadequate.

In order to overcome this problem, the government can issue index–linked SM bonds. Long term inflation indexed securities have already been issued by some governments including TIPS securities issued in the USA, so this proposal seems feasible. Suppose SM bonds are based on n year government bonds which pay out

$$e^{f_1 + \cdots + f_n} \;,$$

where f_t is the continuously compounded rate of inflation in year t after origination. The coupon can also be adjusted for inflation with the coupon paid after t years being

$$ce^{f_1 + \cdots + f_t} \;.$$

The face value of bonds would also increase in line with inflation so as to deliver a constant value retirement income stream. Under this proposal neither S nor M bond holders carry any inflation risks – it is all carried by the government.

5.2. Originating a fixed value of S bonds at each age

A fixed dollar value of S bonds between ages $x = m - n$ up to age $x = m - 1$ can be originated. Since the cost of each bond decreases with age x, more S bonds are bought as age increases. Ignoring inflation, this origination strategy delivers an increasing retirement income stream. The right panel of Figure 2 illustrates the retirement income stream produced with one dollars worth of S bonds bought at each age for risk free rates $r = 1\%$, 2%, 3% and 4%, and risk premium $\delta = 0.25\%$. Again, higher curves correspond to lower r.

This proposal results in only a slight survivorship premium in the early retirement years since most originators survive. As age increases the survivorship premium increases dramatically. Thus a constant investment in S bonds produces a modest income in the early retirement years, rapidly increasing into old age. Is this desirable? In order to answer this question it would be necessary to assess the costs of living for the old-old. Health–related costs tend to rise with age, and many of those who survive to advanced ages will need expensive long term care. The income generated by the S bonds might help to cover a proportion of these costs, relieving the burden on public health systems and other family members.

An increasing retirement income stream will suit those retirees who have independent funds to draw on in the early years of retirement but are likely to exhaust the same if they live to old age. The increasing income stream is also appropriate for paying increasing insurance costs associated with old age care. Inflation risk can be dealt with as in the final paragraph of Section 5.1.

5.3. S bond purchases as a constant percentage of increasing income

Typically income increase with age even after allowing for inflation. If a fixed fraction of income is allocated to purchasing S bonds then more bonds will be bought at older ages both on account of the increasing income and their decreasing cost.

If an employee invests a constant percentage of salary into buying S bonds each year, what level of retirement income would be produced? Suppose income increases at rate f per annum and that employees apply proportion π of salary to buy S bonds in each year. The face value of the S bonds purchased at age $30 + t$ is then

$$\frac{\pi(1+f)^t R_{[30]}}{S_{[30+t]}} \ , \tag{9}$$

equalling the amount payable to the retiree at age $30+t+n$. Retirement income stream (9) exacerbates the problem of dramatically increased S bond income at the very old ages.

Therefore it might be better to vary the contribution rate, tailoring the pattern of S bond purchases to provide a desirable pattern of retirement income. A desirable pattern of retirement income will reflect the features of the retirement income system in a particular country. For example, it might be dessirable to produce retirement income which increases gradually between age m (the normal retirement age) and age $m + k$ (where k reflects the average life expectancy

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at retirement), and then levels out at a certain percentage of average salary. In the early years of retirement, the retirees can draw on their own accumulated resources. As time passes, their savings are depleted. The income from the S bonds gradually increases and replaces the dwindling investment income provided by other retirement savings.

S bond purchases can be tailored to provide any desirable patten of retirement income. For example, suppose that we want to meet produce retirement income which increases gradually between age 65 and age 85, and then levels out at about 30% of average salary. In the early years of retirement, the retirees can draw on their own accumulated resources. As time passes, their savings are depleted. The income from the S bonds gradually increases and replaces the dwindling investment income provided by other retirement savings.

This pattern of income be achieved by S bond purchases which start at 1% of salary at age 30, increasing by 0.25% each year to reach a maximum contribution rate of 5.75% at say age 49, and then gradually decreasing to about 0.4% of salary by age 64.

This pattern of S bond purchases has much lower initial costs than the level annuity pattern and a much lower proportion of the retirement savings account is invested in S bonds at the earlier work ages. The remainder of the retirement savings account can be invested with an appropriate risk–return profile. This will provide a lump sum at retirement which can be used flexibly – providing both liquidity and bequests for those who die soon after retirement.

5.4. Tail risk annuities

In many developed countries, the retirement savings system is designed to provide an adequate retirement income for the average worker for the average post–retirement life expectancy. As a result, in the early years of retirement, most people will probably have enough savings to meet their needs and they do not need longevity risk insurance to cover this period. People only become dependent on safety-net benefits at older ages, as they outlive their savings. This is apparent in social security statistics which show that the proportion of people who are dependent on safety–net benefits increases as a function of age.

The SM bond system could be revised to produce a smoother retirement income at older ages, while also covering the longevity risks at the oldest ages. For example, in the final years prior to retirement, say from ages 60 through to 65 , the money allocated for the purchase of S bonds could be split between bonds of different terms. Part of the money could be used to buy S bonds with term n, and part of the money could be used to buy bonds of term $n + 5$. This extends the age range of coverage into the very old ages with the cost falling only on those who survive to the n years prior to retirement.

5.5. Flexibility

If employees originate SM bonds every year, then the S bonds will provide a regular annual income in retirement. But how will this system cope with broken working patterns and other unexpected events?

If an employee is outside the workforce for a relatively short period of time, then it should be possible to continue purchasing SM bonds during the hiatus, by utilising accumulated retirement savings. For example, suppose that the employee has been paying 10% of salary into a retirement savings account between ages 20 and 35. If the employee goes on unpaid maternity leave for a year or two, at age 35, the accumulated amount in the retirement savings account, other than S bonds, should be more than sufficient to fund the purchase of S bonds for a couple of years. The government might even consider subsidising the purchase of SM bonds for people who are out of the workforce due to, for example, family responsibilities.

Longer periods of unemployment, will be more difficult to deal with. Skipping annual S bond purchases, leads to a gaps in the retirement income stream many years later. We suggest that the private sector can step in to fill this need, offering their own variants of S bonds for a range of different ages and terms. Private providers can price and hedge products using the M market.

Such products would be useful for a range of other purposes. For example:

- the private sector could provide "top–up" products. For example, suppose that a employee buys only a small quantity of SM bonds, and later decides that he would like to buy additional bonds. These could be purchased from life insurers who in turn hedge the risk using the M bond market.

- the private sector could provide "extreme tail risk" products. Some people might like to buy a lump sum longevity risk protection to cover the risk of surviving to a very old age (beyond the period covered by the SM bond system). This could also provided by the private sector.

Life insurers will, of course, be able to compete against the government–guaranteed product by offering their own products. For example, the insurers might offer a bond which is very similar to an S bond, but offering higher returns.

Life insurers will be more able and willing to offer such products if there is an active market in longevity risk securities, which would allow them to hedge their risks. As discussed in Section 3, the existence of an active and transparent market in M bonds would promote the development of a broader market for trading longevity risks in the private sector.

6. S bonds versus private sector products

As noted previously, the private sector has been unsuccessful in providing longevity risk protection for retirees: sales of private sector annuities have been relatively low. If there is no change, the government is likely to bear the increasingly–unaffordable costs of longevity risk.

Would the SM bond provide a more successful solution to this problem? SM bonds may provide longevity protection to the public at a lower cost, with lower risk, and with fewer behavioural impediments.

6.1. Default risk

When a retiree buys a lifetime annuity from an insurer, there is the risk that the insurer will become insolvent and may fail to provide the promised payments. Surveys (Beshears et al., 2014) have shown that retirees are aware of this risk and this knowledge affects decisions about the purchase of annuities.

With SM bonds, the government is, in effect, the provider of the retirment income payments. In many countries, the government has a better long–term credit rating than any insurance company. Retirees may be more comfortable with a government–guaranteed annuity than an insurance company's product.

6.2. Costs

There are many studies suggesting commercial annuities are "good value for money." These assertions are usually based on the calculation of Money's Worth Ratios (MWRs). The MWR is the present value of expected benefit payments divided by the price of the annuity. MWRs have been calculated in different countries and over different time frames (Mitchell et al., 1997; James and Vittas, 2000; Doyle et al., 2004; Thorburn et al., 2007; Brown et al., 1999). The majority of these studies show unexpectedly high MWRS – often over 90% – leading some researchers to conclude "annuities seem to be a remarkably good value for consumers in all annuity markets" (Cannon and Tonks, 2008; Rusconi, 2008). Of course, the MWR calculation depends on assumptions about future mortality rates and discount rates, and different assumptions will lead to different results. For example, annuities have high MWRs when the MWRs are calculated using typical annuitant mortality rates, but lower MWRs result when calculated using population mortality rates. So the annuity will be good "value for money" for more healthy and wealthy members of the population, but unattractive to a more typical member of the public.

If annuities are good value for money, then there may be little room for any reduction in the cost of longevity risk insurance. However, there may be room for improvement in four areas which also affect costs: marketing, operations, capital, and adverse selection.

In an international study, James and Song (2001), estimated that the present value of administrative expenses (including marketing and operational costs) over the lifetime of an average annuity was about 7-8% of the premium. After adding in the cost of capital, total costs were estimated to be 10-12% of initial premiums, or 1.2-1.5% of assets per year. Can the SM bond system help to reduce these charges?

6.2.1. Marketing costs

When annuity providers compete on price, then competitive forces are expected to push down prices. However, Rusconi (2008) notes that in some markets there is a wide spread in annuity prices, suggesting insurers are not competing solely on price. The spread in annuity prices varies across different countries (Cannon and Tonks, 2006).

This is consistent with British studies which indicate about half of annuity buyers do not shop around to get the best price and/or or do not obtain any quotes from annuity providers. Although about half the buyers claimed that they did shop around, many admitted that they did not spend much time on this process. Many relied on advice from financial advisers (Wells, 2014).

Historically, life insurers have often competed with each other by controlling the channels of distribution, e.g. by offering the highest sales commission in order to attract the most successful agents; and/or by vertical integration (that is, taking over financial planning companies, which then advise their clients to buy products from the parent company). James and Vittas (2000) asserts that "Price competition in the retail annuity market is limited by the information and other search costs involved, and by the heavy role that selling agents play, leading to a wide dispersion in annuity prices even in the US and other advanced markets."

Naturally, this sort of competition tends to push prices up, rather than down. Life insurers often pay substantial commissions to intermediaries who sell lifetime annuity products; advertising and other marketing activities may also increase costs. James and Song (2001) states that in the United States immediate annuities market, initial commission was about 4% of the annuity premiums; and total distribution costs were 5% to 6% of the premiums. These costs varied across different countries, e.g. generally lower in Canada and higher in Chile.

SM bonds will be sold by a government agency, without commission. The government will still incur marketing and distribution costs: for example customers will need to be educated about the product, especially in the early years. However we expect these costs will be lower than the cost of commission currently paid for traditional annuity products.

We envisage that most SM bonds would be purchased on a wholesale basis (via the employee's retirement savings fund). Wholesale distribution costs should be lower than retail distribution costs – just as group life insurance is cheaper than individual life insurance policies.

6.2.2. Operational costs

Commercial annuity prices are also loaded to cover the insurer's operational costs. For example, Reichling and Smetters (2013) state that in the United States, yearly management fees for a typical annuity range from 0.80% to 2.0% of underlying assets, not including any initial commission charges.

Under the SM bond scheme, the government will also incur operational costs. These costs will include (i) costs related to issuing the SM bonds by tender or tap; (ii) maintaining records of the owners of the bonds, and making coupon payments to M bond–holders; (iii) managing the mortality pool, which includes keeping accurate records of the deaths of originators; (iv) making the correct maturity payments; and (v) periodically providing death information to the market to facilitate the accurate pricing of M bonds.

Operational costs for SM bonds are likely to be lower than the costs for traditional annuity product, for two reasons. Firstly, the scheme benefits from

economies of scale. Sherris and Evans (2009) point out that there are significant economies of scale in the provision of annuities. A large centralised scheme should have lower operating costs than multiple smaller private sector schemes.

Secondly, the government already has some of this infrastructure in place. Treasury Departments routinely issue fixed interest securities, maintain records of investors, and make payments of coupon and interest. The government might contract out the operations of the scheme to the private sector, if this is more cost-effective. For example, in Australia, Treasury Bonds are normally sold on a wholesale basis, but individual investors can buy and sell Exchange-Traded Treasury Bonds via the Australian Stock Exchange. Exchange–Traded Treasury Bonds can be purchased in small quantities. The Stock Exchange records all transfers of ownership.

Over the longer term, technological innovations are likely to improve the efficiency of trading, driving down operational costs. Nasdaq is already using blockchain technology on its private share-trading market, claiming that this will "streamline financial record keeping while making it cheaper and more accurate."

The SM Bond system will impose some operational costs on the administrators of retirement savings funds. These funds will be responsible for tendering for the SM Bonds on behalf of their members, selling M bonds at the behest of the fund members, collecting coupon and maturity payments, and allocating these payments to the fund members' accounts. It is difficult to quantify these costs, which will probably be passed on to the fund members via fees. This may be a problem for smaller funds. In order to improve efficiency and reduce these costs, the sale of M bonds at origination could be conducted by the same government agency which issues SM bonds, with the net proceeds automatically remitted to the designated member's retirement savings account.[1]

6.2.3. Capital costs

For commercial annuity products, the annuity providers (life insurers) are required to hold capital to cover investment and longevity risks. The cost of this capital must be passed on to customers through higher annuity prices. The cost of capital represents a reward to shareholders in return for their acceptance of longevity risk.

Under the SM bonds system, there is no direct capital charge. However, the SM bonds do not provide a "free lunch". Those who bear the longevity risk - the M bond holders - will demand financial reward for doing so. Longevity risks will be incorporated into the sale price of the M bonds. If the purchasers of the M bonds demand a large reward for bearing these risks, then this will be reflected in higher net costs for S bonds.

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[1]This is certainly a practical possibility. In Australia, many low-income fund members are eligible for supplementary contributions from the Australian government. Every year the Australian Taxation Office calculates the correct amount and pays it directly into each individual member's retirement savings account.

However, the longevity risk for M bonds is likely to be lower than for traditional annuity products, for two reasons. Firstly, if there is widespread investment in SM bonds, there will be diversification benefits. The membership of the centralised M bond mortality pool should be larger and more diversified than any individual pension fund or life insurance portfolio. This would reduce random fluctuations and enable greater reliability of forecasts.

Secondly, the existence of standardised tradable government–guaranteed M bonds would promote the development of a more efficient market for longevity risk transfer. Such markets would allow longevity risks to be spread and dynamically adjusted across a wider group of risk–takers (such as individuals and reinsurers), who may hold these longevity bonds as part of a diversified portfolio of uncorrelated or negatively correlated risks. Such diversification allows (at least theoretically) for a reduction in annuity pricing. Sherris and Evans (2009) explains:

> Whilst sharing of risks across the economy does not reduce the risk, what it can do is reduce the capital required, and hence the return required, if members of the economy can be found where some of these risks would be negatively correlated with their existing risks. Diversification of the risks inherent in annuities should therefore reduce costs, and should be encouraged, subject to consideration of the credit risk then introduced.

6.2.4. Adverse selection costs

Commercial annuities are expensive because those who buy these annuities are healthier than the average population. When MWRs are calculated using annuitant mortality tables, they show that annuities are good value for the people who are typical purchasers, i.e. those who are in good health. When MWRs are calculated using population mortality tables, they are much lower: the reduction is between 3% and 12%, according to a range of studies (Cannon and Tonks, 2006).

Mortality pools may be affected by both adverse selection and passive selection. Adverse selection occurs because people who are in poor health are less likely to purchase annuities. Traditional annuities are particularly vulnerable to adverse selection because people usually make purchasing decisions when they retire. As people age, the variation in health status increases; by retirement age, that a significant proportion of the population may be aware of their own health problems and hence less likely to buy an annuity.

The SM bond system will be less vulnerable to adverse selection because people buy the bonds progressively during their working lifetime, e.g. over age 30 to 64. Since most people are in good health at the younger ages, the risk of adverse selection is reduced. If adverse selection is reduced, leading to higher average mortality rates in the risk pool, then the price of longevity protection will fall.

Passive selection also affects the mortality experience of a risk pool. Passive selection occurs if wealthy people are more likely to buy annuities, and to

spend larger sums in buying annuities. Since wealth is positively correlated with longevity, the dollar-weighted life expectancy of annuitants increases (Finkelstein and Poterba, 2002). This makes commercial annuities more expensive and less attractive to the average retiree.

The same problem might also affect SM bonds. The extent of passive selection will depend on the rules for participation in the scheme: passive selection is reduced by encouraging and tax incentivising wide–spread investment in SM bonds and possibly limiting the amount of tax incentivised bonds which can be purchased.

6.3. Annuity timing risks

When a retiree purchases a traditional lifetime annuity at the date of retirement, the purchase price will depend on the interest rates prevailing at that time. If interest rates are low, then the annuity prices will be unattractively high and annuity sales correspondingly low. For example, Cannon and Tonks (2004) found that much of the variation in UK annuity prices over the period 1957 to 2002 was due to changes in government bond rates.

With the SM system, longevity insurance purchases via S bonds are automatically spread over many years – this helps to diversify the risk of interest rate fluctuations at the time of purchase.

6.4. Incremental purchases

Behavioural economists have suggested that some people find it daunting to make once–and–for–always decisions regarding large sums of money. Hence they may be reluctant to spend their life savings on the purchase of a lifetime annuity at retirement.

The proposed SM Bond system allows savers to make smaller incremental purchases, building up their investment over time. This alleviates one of the psychological barriers to the purchase of longevity protection.

6.5. Reframing purchasing decisions

Studies have shown that "framing" affects pension purchase decisions. Under the SM bond system, people will be encouraged to regard the SM bond as a type of investment providing very attractive government–guaranteed returns to survivors. This may be psychologically more attractive than the traditional system, where people see annuities as an expensive product purchased from a life insurance company with insurance companies profiting from deaths.

SM bonds allow originators to gamble on their own lifespan. Those who survive can receive relatively attractive returns on their investment. Historically, many people have been willing to invest in such products – as long as the potential returns are attractive enough. For example, tontine life insurance policies were once very popular in the United States. These policies provided low payments for surrenders and early deaths, while promising that all profits would be set aside in a special fund to provide additional benefits to those who survived until the end of the term. These policies fell into disrepute when

the life insurance companies which sold these products mismanaged the funds' investment and expenses, severely eroding the benefits payable at maturity.

6.6. Nudges and Tweaks

SM bonds have many advantages compared to traditional annuity products. Ideally, these advantages will attract more people to voluntarily buy such bonds. However, the demand for SM bonds will still be adversely affected by some of the same problems which affect the sale of annuities: bequest motives, crowding out by the social security system, and financial illiteracy.

The government has a financial incentive to promote the purchase of SM bonds, especially by low and middle income earners, since this will reduce future safety–net costs. Therefore the government might well decide to promote the purchase of SM bonds by using "nudges." Thaler and Sunstein (2008) provide many examples where small "nudges" can help to improve decision–making.

7. Age pooled SM bonds

SM bonds are associated with each age and calendar year. If the age range is 30 through to 64 then 35 new bonds are issued in each calendar year. If the term of the bonds is 35 years, then there will be $35^2 = 1225$ bonds in existence at any point of time. Separate bonds for male and females doubles the number. With separate mortality pools for each group, there will be 2450 separate mortality pools. While this is a large number, it is comparable to the number of companies traded on a typical large scale stock exchange. Nevertheless it may be useful to simplify the SM bond market.

The number of mortality pools may be reduced by pooling SM bonds across age or sex or both for each calendar year. The discussion below focusses on age pooling: extension to pooling by sex is obvious.

With age pooling, there is one SM bond for each calendar year, purchasable by any one from say age 30 through to 64. As before, the bond is an ordinary long term government bond splittable into two parts: S and M and where the S part must be retained by the original purchases (originator). The S part pays the originator the face value in n years if the originator is then alive. The M part is sold into the market at the time of origination with the sale proceeds going to originators. At maturity M holders receive the face value of the bond multiplied by the fraction of deaths amongst all originators, irrespective of age.

The price of an age pooled M bond is

$$M = ca_{\overline{n}|} + \mathrm{e}^{-(r+\delta)n}\hat{\mathcal{E}}(_nq_{[x]}) \ , \tag{10}$$

where $\mathcal{E}$ denotes averaging with respect to the age distribution $\ell_{[x]}$ and $\hat{\mathcal{E}}(_nq_{[x]})$ is the market estimate of age pooled mortality. The higher the average age of the originators, the higher the price M. Potential investors will need to know the age composition of the cohort at each point of time so as to make an assessment as to expected future payoff. In order to avoid adverse selection, all originators would be required to sell their M components immediately after origination.

After selling the age–pooled M bonds the sale proceeds would need to be allocated amongst the originators, on an equitable basis. We put forward two alternative methods of equitable allocation.

7.1. M bond proceeds distributed on basis of estimated future mortality

Originators can be given an amount for their M bond on the basis of estimated relative mortality,

$$ ca_{\overline{n}|} + \frac{{}_n\hat{q}_{[x]}}{\mathcal{E}({}_n\hat{q}_{[x]})}(M - ca_{\overline{n}|}) \ , \tag{11} $$

where the ${}_n\hat{q}_{[x]}$ are estimates, at the time of origination, made by a Government Actuary or some other official and independent source. As a result, the net cost of an S bond to an originator aged x is

$$ \mathrm{e}^{-rn}\left\{ 1 - \mathrm{e}^{-\delta n}{}_n\hat{q}_{[x]}\frac{\hat{\mathcal{E}}({}_nq_{[x]})}{\mathcal{E}({}_n\hat{q}_{[x]})} \right\} \ , $$

where the numerator and denominator in the fraction are the market and official estimate of exposure weighted mortality, respectively. If these estimates match, the net S bond cost for the age–pooled system matches (5).

The weighted average of official estimates $\mathcal{E}({}_n\hat{q}_{[x]})$ might not match the market–based estimate $\hat{\mathcal{E}}({}_nq_{[x]})$. There is a risk that the official mortality estimates might be adjusted to meet government objectives, such as redistribution of wealth between age groups. Although some redistribution might be considered to be socially desirable, this also creates the risk of distortions in the demand for SM bonds. If for a particular age x the estimate ${}_n\hat{q}_{[x]}$ is judged to be too low, those in this age category would receive less than their fair share of the M bond proceeds. This would discourage purchase of SM bonds in this age group.

The total amount payable to the originators will match the total proceeds from the sale of the M bonds, producing a zero net cost to the government (other than administration costs). The government will administer the mortality pool, but will not bear any longevity risk; these risks are borne by age–pooled M bond holders.

7.2. M bond proceeds distributed equally with survivor clawback

Alternatively, we could adopt a different method for the equitable treatment of originators in an age–pooled system. Instead of age x originators receiving (11) suppose each originator receives the common pool price M. Equitable treatment is then achieved by adjusting the S maturity value payable to each survivor.[2]

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[2]We are grateful to a referee for suggesting age based S bond payouts

If an age x originator buys an SM bond for (1), and sells the M component for (11), then the initial net cost of an S bond is the same for all ages

$$\mathrm{e}^{-rn}\{1 - \mathrm{e}^{-\delta n}\hat{\mathcal{E}}(_nq_{[x]})\} \; .$$

Under an equitable treatment of originators, this outlay should have provided age x originators with a total of

$$\frac{\mathrm{e}^{-rn}\{1 - \mathrm{e}^{-\delta n}\hat{\mathcal{E}}(_nq_{[x]})\}}{\mathrm{e}^{-rn}(1 - {_nq_{[x]}})} = 1 + \frac{{_nq_{[x]}} - \mathrm{e}^{-\delta n}\hat{\mathcal{E}}(_nq_{[x]})}{1 - {_nq_{[x]}}} \; , \tag{12}$$

S bonds, where $_nq_{[x]}$ is the cohort x mortality. Amount (12) is the equitable payment made to the $\ell_{[x]+n}$ survivors given the initial equal investment. From (12) the survivors receive more than 1 if their historical mortality is more than the risk discounted market estimate of average mortality. Age dependent payout thus produces an income stream that increases with age, reminiscent of the constant investment option discussed in Section 5.2.

In each calendar year, the total S bond payment equals the total available for S bond holders if

$$\mathcal{E}(_nq_{[x]}) - \mathrm{e}^{-\delta n}\hat{\mathcal{E}}(_nq_{[x]}) = 0 \; , \tag{13}$$

requiring likely minor pro–rata adjustment. Hence there is minor risk transfer of risk between the different age cohorts but no additional risk for either M holders or the government.

Under this approach, the S bond owners face some uncertainty in the size of their maturity payments, since the payment depends on the mortality experience of their own age–group, relative to other age groups. The left hand side of (13) is the risk premium retained by originators for retaining age specific longevity risk.

7.3. M bond reimbursement rules

The above reimbursement rules and S bond payouts do not affect the pooled M bond payoff and hence price M. The rules do not transfer any longevity risk either to the government or the market. The market carries the overall mortality risk and is only concerned with the age composition $\ell_{[x]}$, announced at the start of M trading. In other words, the management and pricing of longevity risk, and the financial markets' contribution thereto, is focussed on aggregate mortality as opposed to relative age specific mortality. Financial markets are likely to be more effective in pricing overall longevity rather than age specific mortality.

8. Conclusion

The SM Bond is a new product for individuals, markets and governments to manage longevity risk. New products are needed, because currently available private sector products are not providing satisfactory outcomes. For reasons outlined in Section 1, sales of traditional annuity products are disappointingly

low. As a result, many of those who survive to advanced ages become increasingly reliant on safety–net benefits. As populations age, this will impose an unaffordable burden on future generations of taxpayers.

In response to this market failure, the government might intervene to provide a public sector solution: for example, the government might offer to sell an entitlement to a government guaranteed lifetime pension. However experience shows that such social security systems are susceptible to underfunding, as a result of both political pressures and increasing longevity. Once again, future generations of taxpayer would be expected to cover the deficit.

At present, neither the private sector nor the public sector offers a satisfactory solution to the problem of longevity risk. We suggest that a public–private partnership – the SM bond – would have marked advantages.

The SM Bond system enables an individual to incrementally buy a government–guaranteed retirement income stream, without imposing longevity risks onto the government. Originators hedge their own individual longevity risk via ownership of the S component. Originators share in the financial effects of their own cohort's mortality risk via investment in the M component. Originators may retain their M component, or sell them.

In contrast to the longevity securities which are currently available in the market, M bonds will be government–guaranteed standardised contracts. These characteristics of the M bonds will make them more attractive to hedgers, speculators, and other investors. Longevity risks will be priced by market forces. The creation of a more liquid and transparent market for longevity risk will facilitate and expedite the development of a wider range of private sector longevity products, complementary to S bonds.

SM bonds can be integrated into the existing retirement savings system, as one component of each individual's retirement savings portfolio. Originators will choose the pattern of SM bond investments suitable for their own individual circumstances, allowing for wealth, risk preferences, liquidity needs, and bequest motives. Governments can mandate or incentivise specified SM bond purchases in order to meet social policy objectives, such as minimisation of social security safety–net costs for the elderly.

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