



Institute of Actuaries of Australia

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Integrating Financial and Demographic Longevity Risk Models: An Australian Model for Financial Applications

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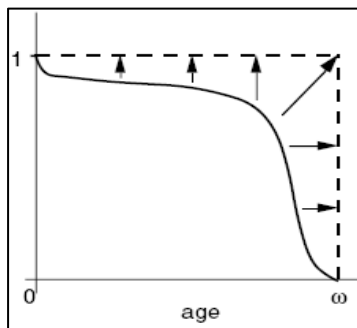
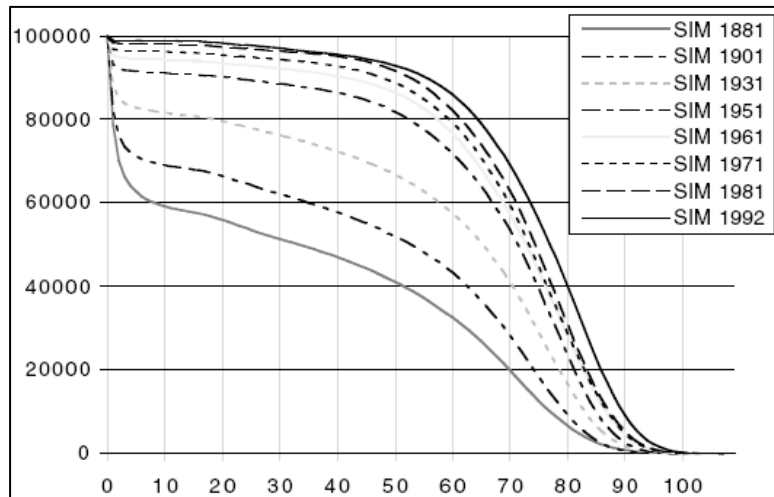
Outline

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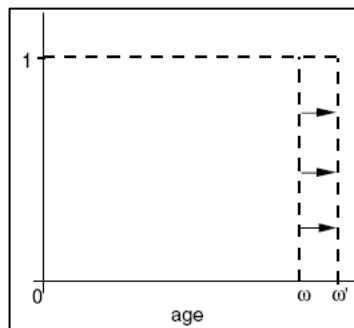


1. Longevity and Mortality Risk

Longevity improvement has seen the survival curve* shift in 2 ways:



Rectangularisation



Expansion

* Survival Functions for Italian Male Populations (1881-1992)
 (Pitacco, 1992)

There is increasing retail exposure to longevity risk..

- Longevity is improving with greater variability
- OECD Male 60-64 Labour Participation:
 - 60-90% (1970s) to 20-50% (today)
- Shift to DC Superannuation
- 3.4m Australians will suffer from insufficient income in retirement**

... and huge potential for investment in life annuities

- Australian Super Industry:
 - \$1,177b assets (Dec 2007)
 - 2/3 DC or Hybrids
- Australian Life Annuities:
 - \$3.9b assets (Dec 2007)

...though currently there are a number of constraints

- Supply/demand constraints (Purcal, 2006)
- Reinsurance:
 - Longevity is “toxic” (Wadsworth, 2005)

** AMP 2007 AMP Superannuation Adequacy Index Report,
 released January 2008



2. Risk Management Strategies

1. Avoidance

- Participating Annuities
- Reverse Mortgages

2. Retention

- **Capital Reserves**
- **Contingent Capital**

3. Transfer

- **Reinsurance**
- **Bulk Purchase Annuities**
- **Securitisation**

4. Hedging

- Natural Hedges
- Survivor Bonds
- Mortality Swaps
- Longevity Options and Futures



3. Longevity Risk Securitisation

Securitisation is a vehicle for risk transfer

- CDOs - late 1980s
- Insurance-Linked Securitization – USD 5.6b issued in 2006*
 - Insurance-Linked Bonds
 - Industry Loss Warranties
 - Sidecars
- Mortality Bond Issues (Vita I-III, Tartan, Osiris, 2003-2007)
- Survivor Bond Issues (BNP Paribas/EIB, 2004)

...with a number of benefits

- Improved capacity for risk transfer as tranching broadens appeal to investors
- Issue can be tailored to manage basis risk vs. moral hazard / info. asymmetry
- Diversification benefits for investors



4. Models for Mortality

a. Lee Carter (1992) Model and Extensions:

$$\ln[m(x, t)] = a_x + b_x k_t + \varepsilon_{x,t} \quad \text{where} \quad \sum_{\text{all } x} b_x = 1 \quad \text{and} \quad \sum_{\text{all } t} k_t = 0$$

- Single time-based index
- Assumes linear trend in k
- Difficult to incorporate risk-neutral pricing

b. Dahl (2004) Model and Extensions:

- Derived from finance theory (see Vasicek, 1977; Cox et al, 1985)

$$d\mu(t, x) = \alpha^\mu(t, x, \mu(t, x))dt + \sigma^\mu(t, x, \mu(t, x))dB_t$$

- Specific form based on Cox et al (1985):

$$d\mu(t, x+t) = (\beta^\mu(t, x) - \gamma^\mu(t, x)\mu(t, x+t))dt + \rho^\mu(t, x)\sqrt{\mu(t, x+t)}dB_t$$

- Readily adapted to risk neutral pricing
- Difficult to calibrate for pricing

c. Forward Rate Models:

- Model the dynamics of the forward mortality surface.
- Based on work by Heath, Jarrow and Morton (1992).

- Less developed in literature than short rate models

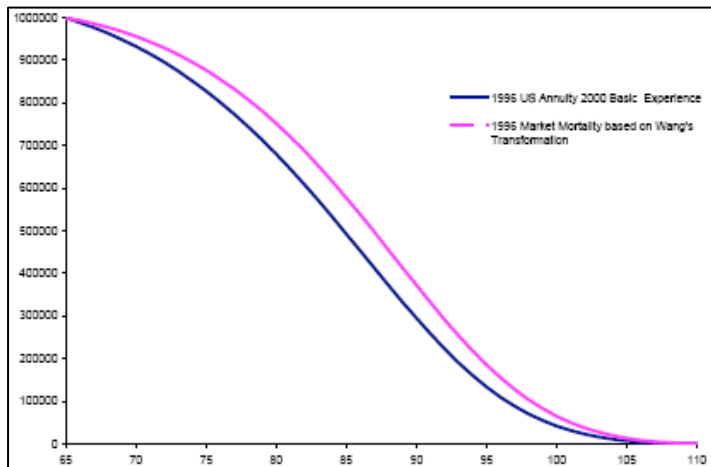


4. Models for Mortality

a. Lee Carter (1992) Model and Extensions:

- Pricing uses the Wang (1996, 2000, 2002) transform → shifts the survival curve using fixed 'price of risk', λ :

$$F^*(t) = \Phi[\Phi^{-1}(F(t)) - \lambda]$$



- Denuit, Devolder and Goderniaux (2007) using stochastic mortality, following Lin and Cox (2005) (deterministic).

Subject to criticism
as it does not
incorporate varying
 λ over age and
time*

* Cairns et al (2006), and Bauer and Russ (2006)



5. The Proposed Model

i) A Multivariate Mortality Process

- For lives at time t , initially aged x , the mortality rate $\mu(x, t)$ is given by:

$$d\mu(x, t) = \left(a(x + t) + b\right)\mu(x, t)dt + \sigma\mu(x, t)dW(x, t) \text{ for all } x.$$

- This falls within the Dahl (2004) family of models.
- To incorporate dependence, we introduce a M.V. random vector $\underline{dW}(t)$, length N :

$$\underline{dW}(t) = \Delta \underline{dZ}(t),$$

with each element

$$dW(x, t) = \sum_{i=1}^N \delta_{x,i} dZ_i(t) \text{ for all } x.$$

- Where $\underline{dZ}(t)$ is a random vector of independent B.M. of length N ; and Δ is a $N \times N$ matrix of constants, such that:

$$\begin{bmatrix} dW(x_1, t) \\ \vdots \\ dW(x_N, t) \end{bmatrix} = \begin{bmatrix} \delta_{11} & \dots & \delta_{1N} \\ \vdots & \ddots & \vdots \\ \delta_{N1} & \dots & \delta_{NN} \end{bmatrix} \begin{bmatrix} dZ_1(t) \\ \vdots \\ dZ_N(t) \end{bmatrix}$$

Note: the dimension of $\underline{dZ}(t)$ can be reduced using PCA.



5. The Proposed Model

i) A Multivariate Mortality Process

- The covariance matrix of $dW(t)$, Σ , has each element:

$$\begin{aligned} \text{Cov}\left(dW(x_n, t), dW(x_m, t)\right) &= \sum_{i=1}^N \delta_{ni} \delta_{im} \text{Var}\left(dZ_i(t)\right) \\ &= \sum_{i=1}^N \delta_{ni} \delta_{im} dt. \end{aligned}$$

such that

$$\Sigma = \left(\Delta \sqrt{dt}\right) \left(\Delta \sqrt{dt}\right)'$$

- This gives the Cholesky decomposition of Σ

ii) Incorporating Age-Dependence

- Using PCA, decompose Σ into its eigenvectors (V), and eigenvalues (diagonal matrix T):

$$\begin{aligned} \Sigma &= VTV' \\ V\sqrt{T} &= \Delta\sqrt{dt} \end{aligned}$$

- Simulations of $dW(t)$ can be generated with the same dependence properties:

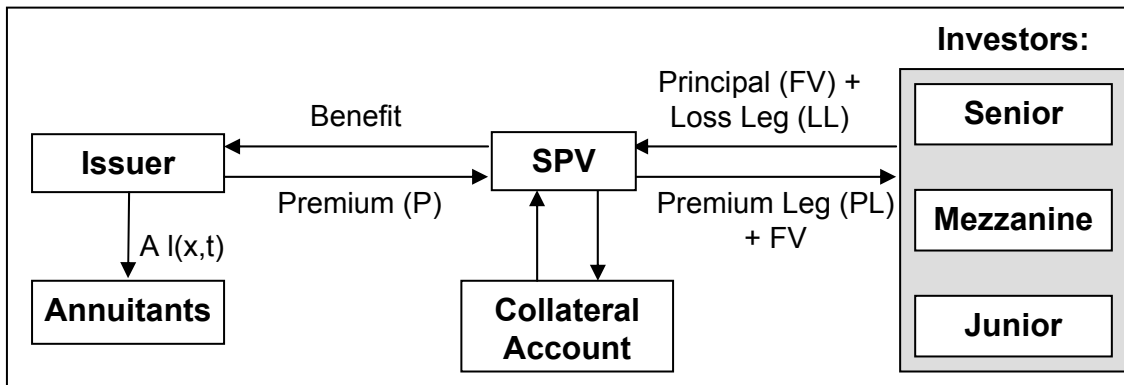
$$d\hat{W}(t) = V\sqrt{T}\underline{\eta}$$

# of Eigenvectors	% of Observed Variation
1	29.3%
5	69.8%
10	85.1%
15	92.4%
20	96.5%
25	97.1%
30	99.1%
31	99.5%
32	100.0%



6. The Longevity Bond

- The proposed longevity bond has the following structure:



- Both the PL and the LL are based on the percentage cumulative losses incurred on an underlying annuity portfolio:

$$CL(t) = \frac{\sum_{s=1}^t L(s)}{FV}$$

- Where the loss on the portfolio in each period is:

$$L(t) = \left(A \sum_{\text{all } x} l(x, t) - E \left[A \sum_{\text{all } x} l(x, t) \right] \right)^+ \\ \approx \left(A \sum_{\text{all } x} l(x, 0) {}_t p_x - A \sum_{\text{all } x} l(x, 0) {}_t \bar{p}_x \right)^+$$

Differs from existing models as:

- Based on multi-age portfolio
- Allows for variability in ${}_t p_x$
- Provides detailed analysis of longevity bond tranches



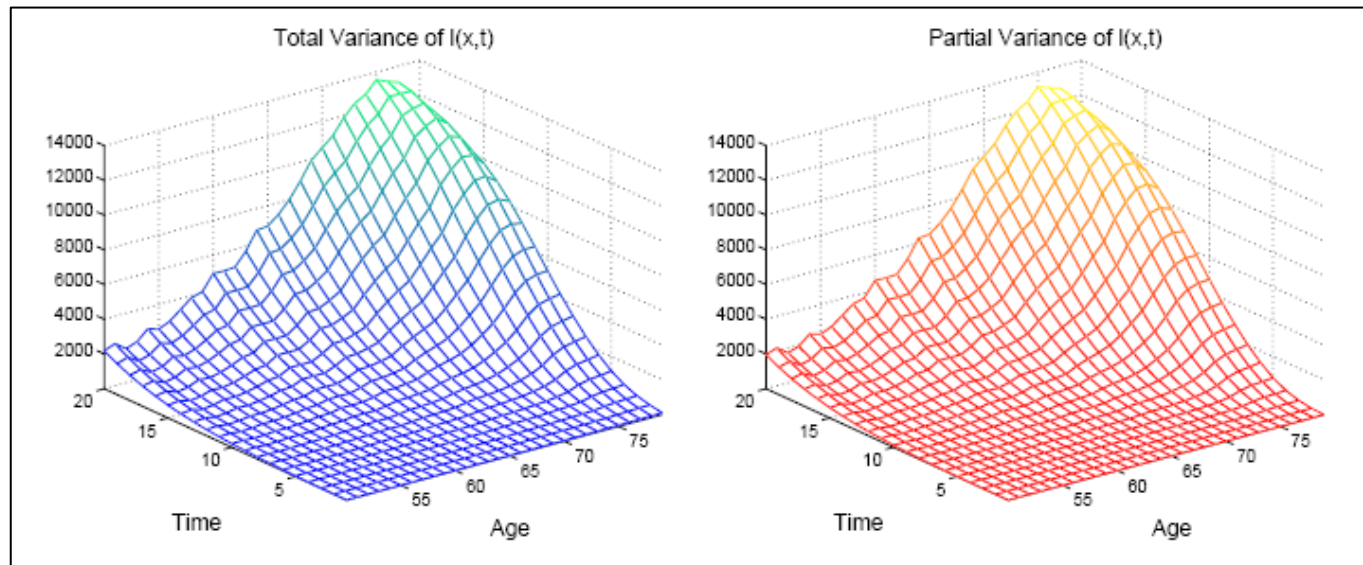
6. The Longevity Bond

- The total variance of the number of lives alive at time t , initially aged x is given by:

$$Var[l(x, t)] = E[Var[l(x, t)|{}_tp_x]] + Var[E[l(x, t)|{}_tp_x]].$$

- The first term gives the binomial variability in the portfolio given a fixed ${}_tp_x$ (the focus of Lin and Cox, 2005).
- The second is the variability due to changes in the mortality rate, which accounts for almost all of the portfolio variance:

Variability in ${}_tp_x$ accounts for almost all the variability in $l(x, t)$.





6. The Longevity Bond - Tranching

- Tranche losses are allocated by the cumulative loss on the portfolio. From this we can find the cumulative tranche loss:

$$CL_j(t) = \begin{cases} 0 & \text{if } L(t) < K_{A,j}; \\ CL(t) - K_{A,j} & \text{if } K_{A,j} \leq L(t) < K_{D,j}; \\ K_{D,j} - K_{A,j} & \text{if } L(t) \geq K_{D,j}, \end{cases}$$

where

$$CL(t) = \sum_{j=1}^J CL_j(t).$$

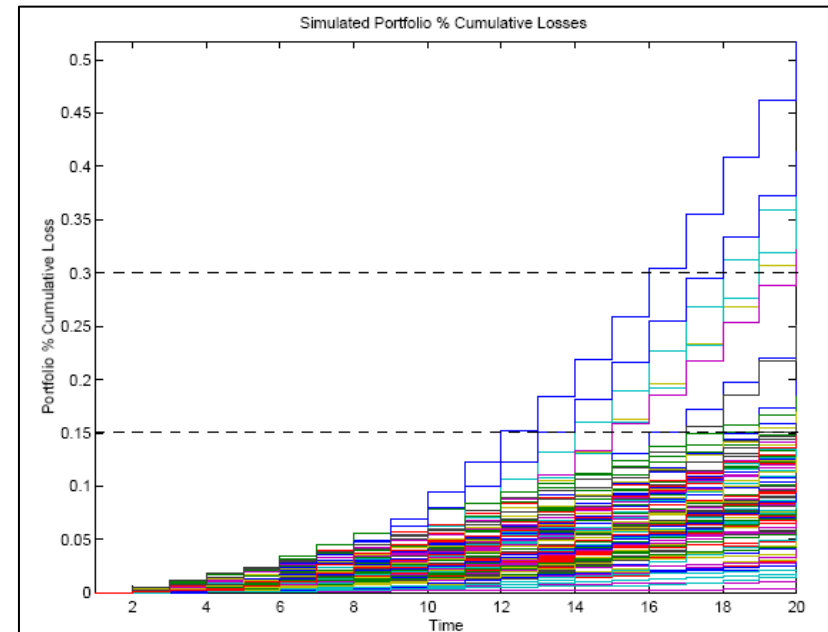
- The tranche loss as a percentage of its prescribed principal is given by:

$$TCL_j(t) = \frac{E[CL_j(t)]}{K_{D,j} - K_{A,j}}.$$

- The assumed tranche thresholds are:

Tranche j	$K_{A,j}$	$K_{D,j}$
1	0%	15%
2	15%	30%
3	30%	100%

Portfolio cumulative loss simulations





7. The Pricing Model

- The premium on tranche j , P_j^* , is set to equate the cashflows on the premium leg (PL_j), and the loss leg (LL_j):

$$PL_j = \sum_{t=1}^T P_j B(0, t-1) [1 - TCL_j(t-1)]$$

$$LL_j = \sum_{t=1}^T B(0, t) [TCL_j(t) - TCL_j(t-1)]$$

such that $PL_j(P_j^*) - LL_j(P_j^*) = 0.$

where:

- $B(0, t)$ is the price of a ZCB.
- $TCL_j(t)$ is the tranche % cum. loss at time t .

- Premiums need to be set under a risk-adjusted \mathbb{Q} mortality measure. Using the Cameron-Martin-Girsanov Theorem:

$$dW^{\mathbb{Q}}(x, t) = \sum_{i=1}^N \delta_{xi} (dZ_i(t) + \lambda_i(t)dt)$$

$$= dW(x, t) + \sum_{i=1}^N \delta_{xi} \lambda_i(t)dt.$$

and for all ages: $d\underline{W}^{\mathbb{Q}}(t) = d\underline{W}(t) + \Delta \underline{\lambda}(t)dt$

where $\Delta \underline{\lambda}(t)$ is a 'risk adjustment' that can differ for each age and time.

and the risk adjusted mortality process is:

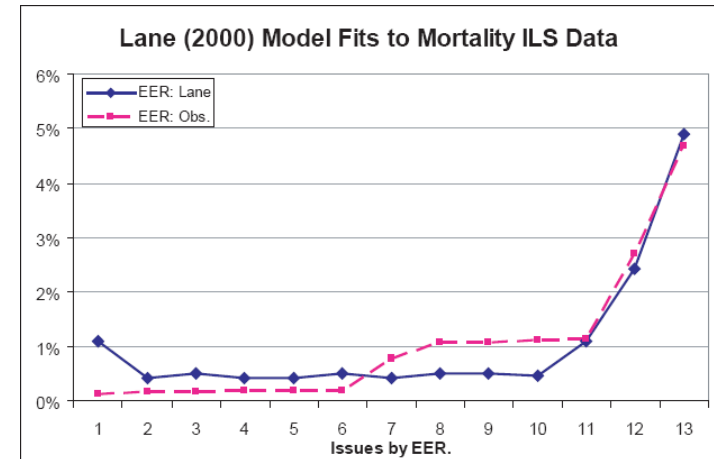
$$d\mu^{\mathbb{Q}}(x, t) = \left(a(x+t) + b + \sum_{i=1}^N \delta_{xi} \lambda_i(t) \right) \mu^{\mathbb{Q}}(x, t)dt + \sigma \mu^{\mathbb{Q}}(x, t) dW(x, t)$$



7. The Pricing Model

- However, the choice of \mathbf{Q} , and thus $\Delta\lambda(t)$ is not unique (like IR derivatives). It thus needs to be calibrated to market prices.
- These are approximated using an empirical model proposed by Lane (2000), fit to the price of 2007 mortality bond issues using non-linear least squares:

$\hat{P}_j^L = EL_j + EER_j$ $EER_j = \gamma(PFL_j)^\alpha (CEL_j)^\beta$	
Parameter	2006-07 Mortality Bonds
γ	0.9980
α	0.8965
β	0.5034
X^2	0.04
χ^2_9 at 99%	2.09



- To facilitate calibration with limited data, simplifying assumptions are made on the risk adjustment:

$$\Delta\lambda(t) = \lambda^* \text{ where } \lambda^* = [\lambda^*, \dots, \lambda^*]'$$

So that for each x and t :

$$d\mu^{\mathbf{Q}}(x, t) = \left(a(x + t) + b - \sigma\lambda^* \right) \mu^{\mathbf{Q}}(x, t) dt + \sigma \mu^{\mathbf{Q}}(x, t) dW(x, t).$$

λ^* is chosen so that:

$$P_j^{\lambda^*} = P_j^L$$

As a result,

$$\lambda_j^* = f(P\hat{F}L_j, C\hat{E}L_j, \gamma, \alpha, \beta)$$



8. Data and Assumptions

Data

- Australian Population Mortality Data, ages 50-99, 1971-2004. Human Mortality Database (www.mortality.org)
- Australian Gov't Treasury Bill and Note rates: maturity 1-12 years. Bloomberg 24/09/2007.
- Market insurance-linked security data: 2007 issues. Drawn from Lane and Beckwith (2007).

Assumptions

Parameter	MLE: Male	MLE: Female
\hat{a}	-9.4398E-04	2.6993E-04
\hat{b}	0.1347	0.0608
$\hat{\sigma}$	0.0906	0.0873

Mortality process parameter estimates.

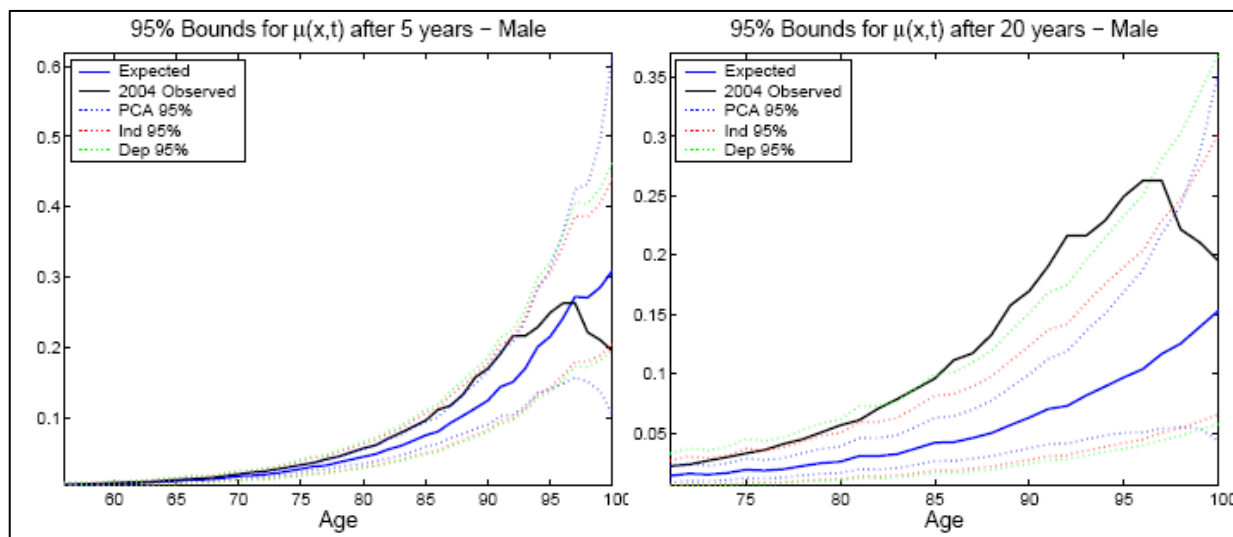
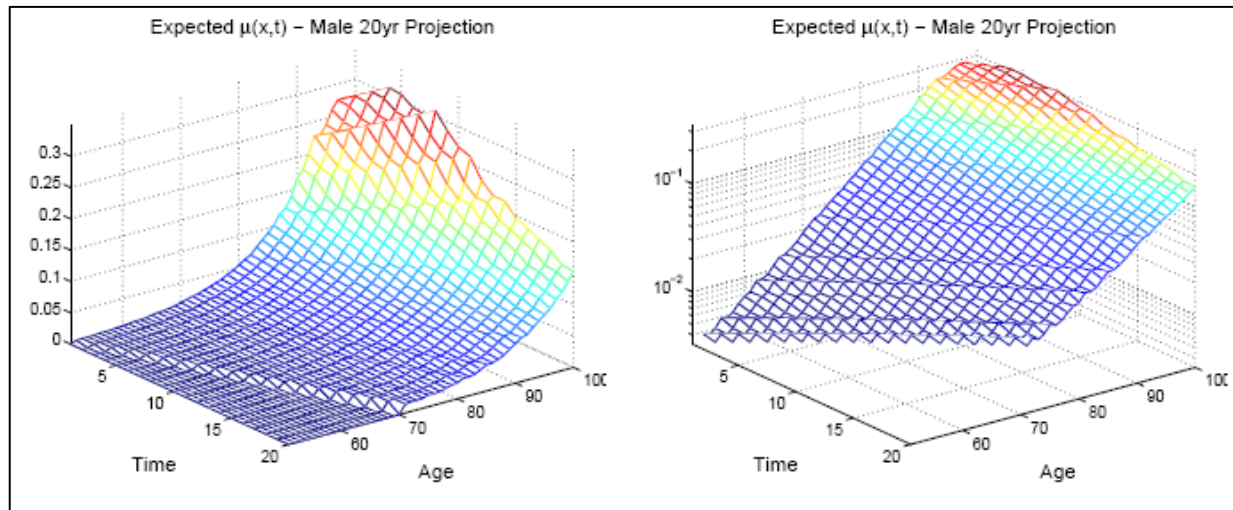
- $dW(t)$ is modeled under 3 assumptions of age dependence:

1. Perfect age independence.
2. Observed age dependence using PCA.
3. Perfect age dependence.

Proposed Longevity Bond Assumptions	
Bond Face Value:	$FV = \$750,000,000$.
Term to Maturity:	$T = 20$ years.
Payment Frequency:	Annually, for both premium and loss payments.
Number of Tranches:	$J = 3$.
Initial Age of Annuitants:	$x = 50, \dots, 79$.
Initial No. of Annuitants:	$n(x, 0) = 60,000$. We assume this is evenly distributed between the 30 ages, with $l(x, 0) = 2,000 \forall x$.
Annuity Payments:	$A = \$50,000$ paid at the end of each year to each living annuitant.



9. Results – The Mortality Model



- Mortality expected to continue improving over the next 20 years (except ages 95-100)

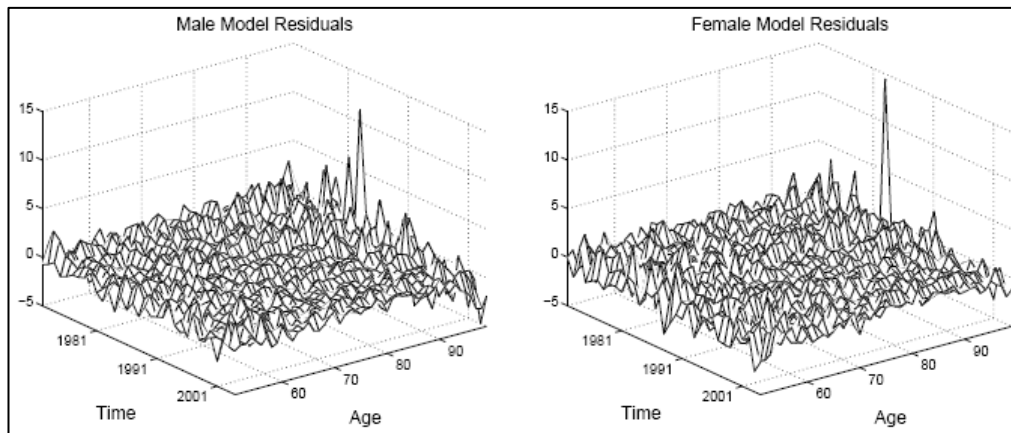
- Passage of cohort through time can be noted

- Volatility highest under perfect dependence, except at the oldest ages



9. Results – The Mortality Model

Analysis of fit shows the model accurately fits observed data



- Fitted residuals are normally distributed, without trend across age or time

	Male	Female
Mean	-3.12E-03	4.84E-08
Standard Error	0.024	0.025
Standard Deviation	0.972	1.000
Minimum	-6.320	-4.396
Maximum	6.277	14.459
Confidence Level(95.0%)	0.047	0.049

- Residuals are distributed with mean 0 and std dev 1

	Male				Female		
	<i>a</i>	<i>b</i>	σ		<i>a</i>	<i>b</i>	σ
<i>a</i>	5.53E-13	4.24E-11	-	<i>a</i>	5.13E-13	3.94E-11	-
<i>b</i>	4.24E-11	3.14E-09	-	<i>b</i>	3.94E-11	2.91E-09	-
σ	5.01E-11	2.84E-07	1.61E-09	σ	-4.48E-11	-2.54E-07	1.50E-09

- Low asymptotic var/covar values suggest high confidence in each parameter estimate

X^2 Male	X^2 Female	χ^2_{388} at 99%
71.08	23.16	326.15

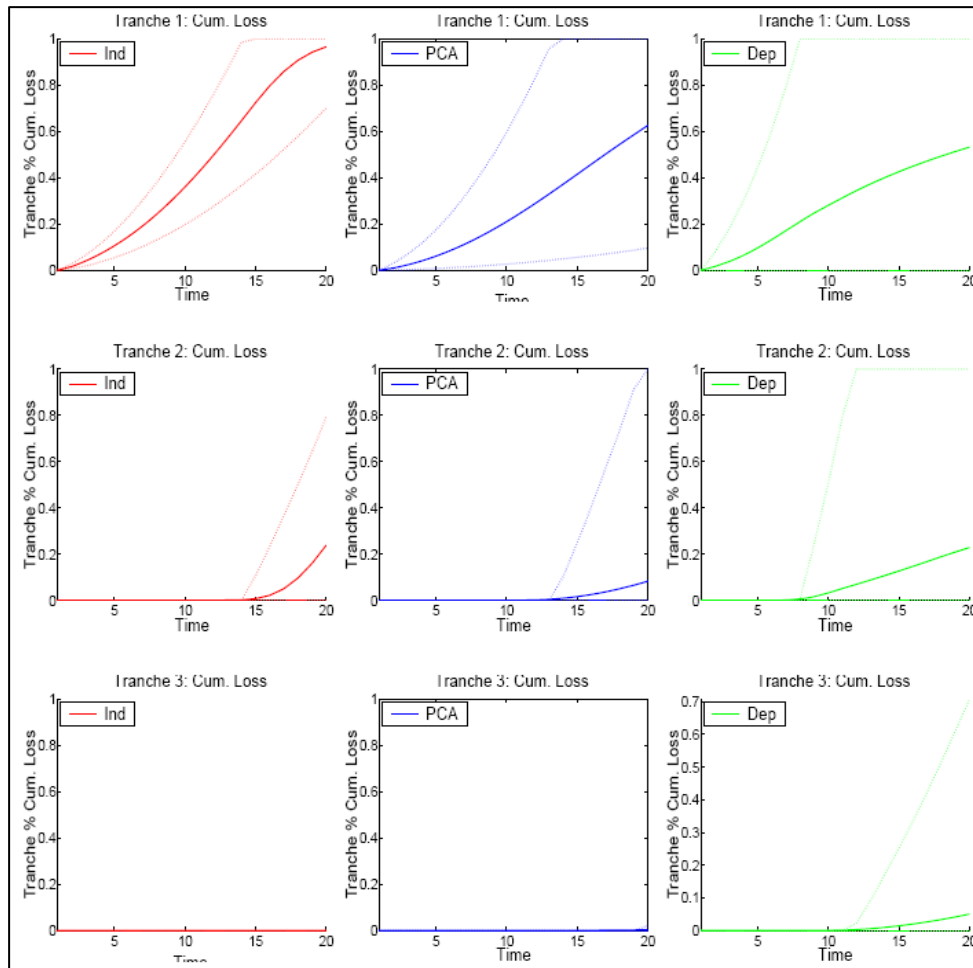
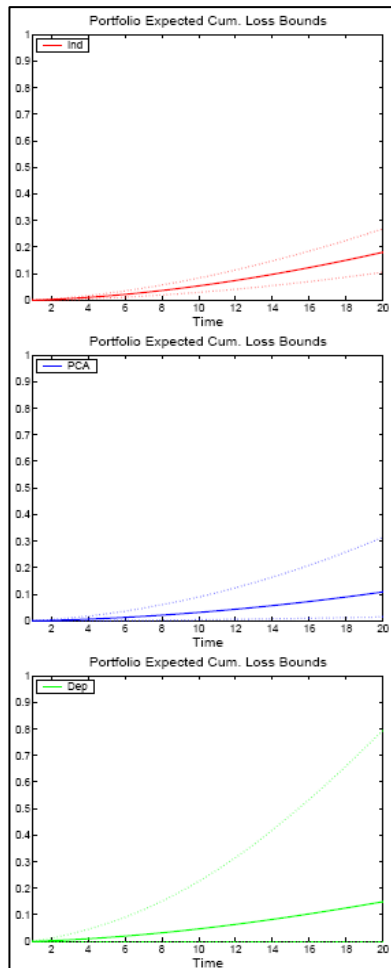
- Pearson's chi-square shows that the model fits the observed data very well



9. Results – The Longevity Bond

Portfolio expected cum. loss and 95% bounds

Tranche expected cum. loss and 95% bounds under 3 age-dependence assumptions.



- Variability of portfolio loss increases with age dependence

- Expected loss higher under dep., due to option-like payoff

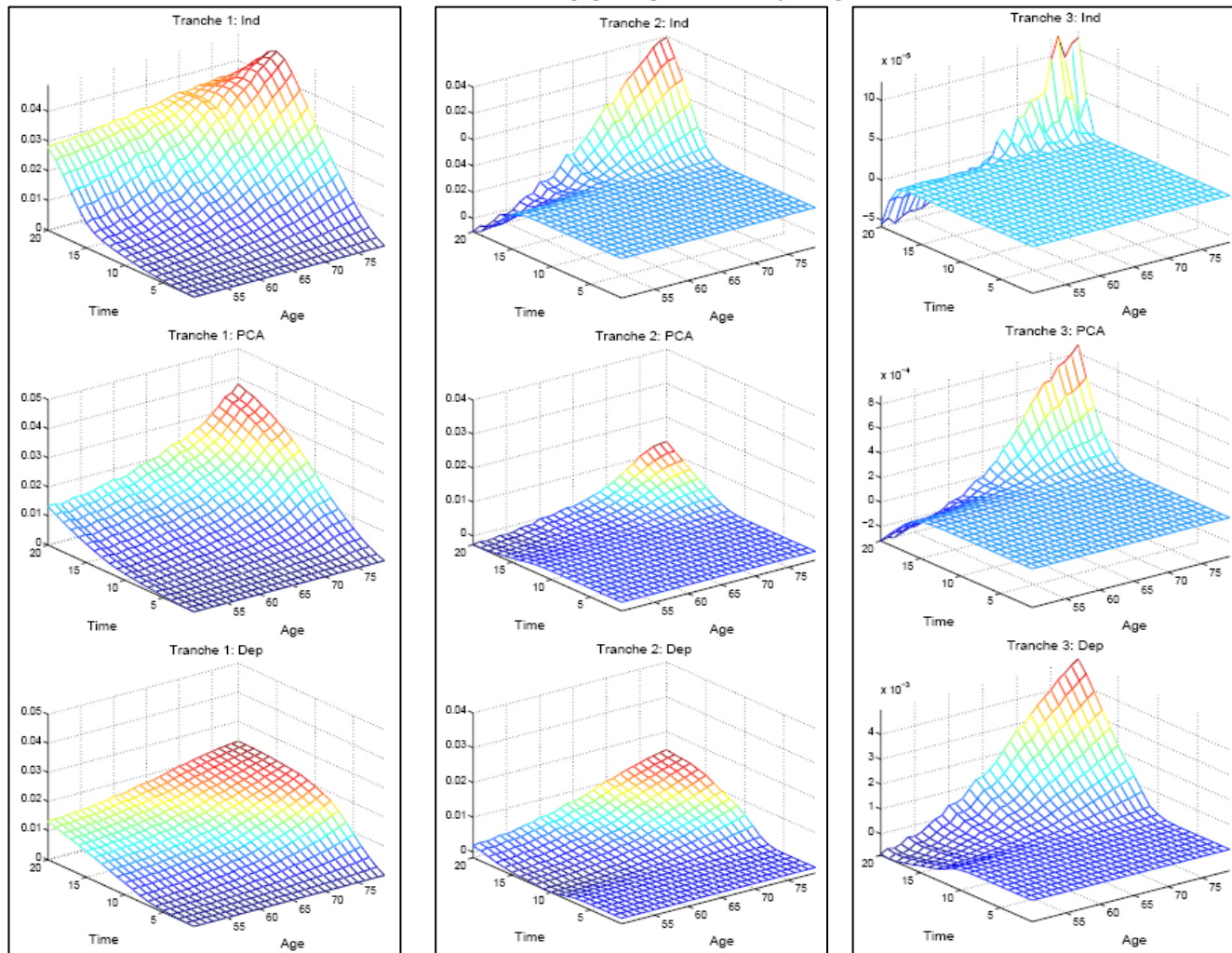
- Tranches losses are over/under-estimated due to dependence

- Dependence has a strong impact on the size of tranche expected losses



9. Results – The Longevity Bond

Tranche cumulative losses, disaggregated by age.

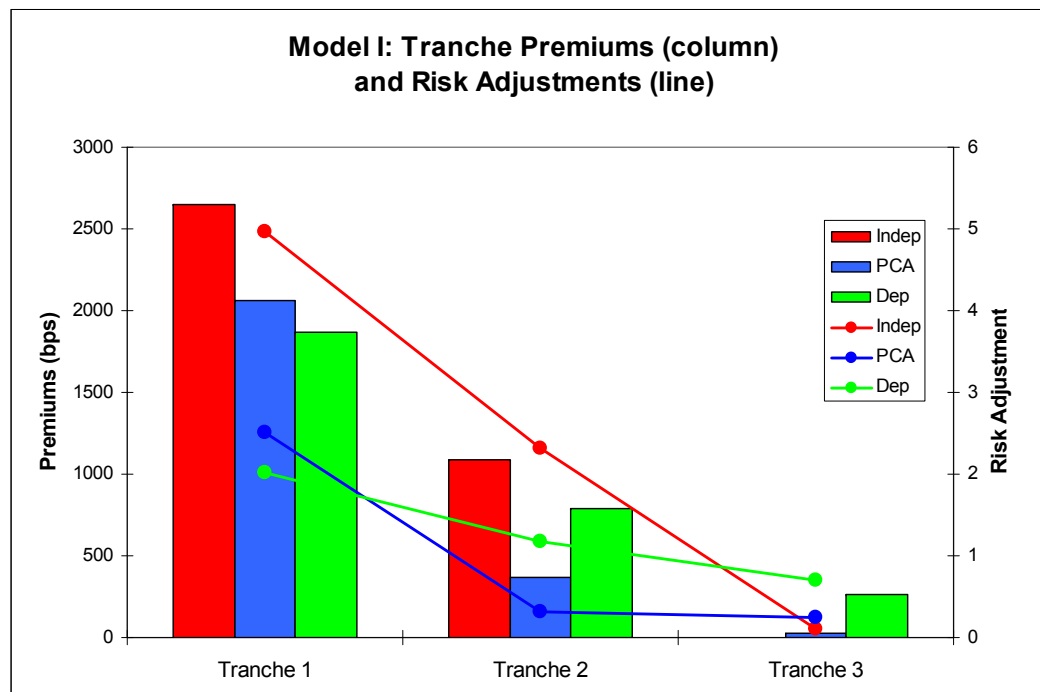


- Tranche losses not equally incurred across all ages

- Lower losses in young cohorts offset high losses in old cohorts



9. Results – The Pricing Model



- Calibrated tranche premiums and associated 'prices of risk' λ are consistent with risk averse investors

- λ sensitivities* show the model is very sensitive to the choice of data and the fit of the Lane (2000) model

λ sensitivities:

(observed dependence)

$$\lambda_j^* = f(P\hat{F}L_j, C\hat{E}L_j, \gamma, \alpha, \beta)$$

	Premium	λ_j^*	Sensitivities				
			$P\hat{F}L_j$	$C\hat{E}L_j$	γ	α	β
Tranche 1	2058	2.52	-	1.39	2.04	-	-3.52
Tranche 2	371	0.31	1.24	0.76	1.21	-1.76	-2.33
Tranche 3	31	0.25	0.29	0.17	0.31	-0.95	-0.79

*In the absence of a closed form



9. Results – The Pricing Model

Implications of Results

- Mortality can effectively be modelled as a dynamic, multi-age process.
- Tranched longevity bonds provide an effective vehicle for managing longevity risk.
- Dynamic mortality models are well suited to pricing longevity-linked securities.

Further Research

- Calibration of the risk-adjusted mortality process.
- Application of the proposed mortality model to a broader range of ages
- Alternative definitions for portfolio loss, eg. change in future annuity obligations (Sherris and Wills, 2007).



10. Conclusion

The Mortality Model

- Fit Dahl (2004) framework successfully to changes in mortality by age and time simultaneously
- Verified age-dependence as crucial
- Facilitated modelling of mortality-linked securities on multi-age portfolios

The Longevity Bond

- Investigated longevity-linked security on multiple ages
- Performed detailed analysis of the impact of tranching, under a range of age dependence assumptions

The Pricing Model

- Calibrated price of risk was consistent with risk averse investor with non-linear risk/return tradeoff
- 'Price of risk' able to vary by age and time, to incorporate range of investor sentiments



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Questions and Comments